Supplemental Materials

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SOLUTION OF THE SLAVE-SPIN THEORY WITHIN WEISS MEAN-FIELD APPROXIMATION

In this part, we show the saddle-point solution of the slave spin theory within a Weiss mean-field decomposition in Eq. (7) of the main text obtained by diagonalization.

First, we find the dMIMT taking place at a finite h_c with

$$h_c \simeq U/2 \times \sqrt{U_c(U_c^{-1} - U^{-1})}. \tag{S1}$$

Note that in the single-orbital Hubbard model, one can always set $h = -\mu$, where μ is the electron chemical potential. This makes the ratio $a = h/U$ proportional to doping δ, as shown in Figs. Fig. (S1a) and Fig. (S1b). Actually,

$$a = a_c(1 + b\,\delta). \tag{S2}$$

where the factor b decreases as U increases, and the critical value $a_c = \sqrt{U_c^{-1} - U^{-1}}/2$.

The ratio a is shown for both its bare values in Fig. (S1c) and its changes from the critical point a/a_c in Fig. (S1d). The change in a as a percentage of a_c is about 4 times of δ, hence leads to the increase of J_0 as δ increases. We consider that it is an artifact of mean field theory since h accounts also for effects due to the hopping terms.

The dMIMT is of Brinkman-Rice type, as we find that $Z \propto \delta$ which is shown in Fig. (S2a). To support the survival of the Nagaoka-ferromagnetic interaction in the $U \to \infty$ limit, we plot Z/δ as a function of U^{-1}. As shown in Fig. (S2b), Z/δ converges to 1 in the limit of $U \to \infty$, which indicates that the FM exchange coupling in Eq.(18) of the main text keeps finite in this limit.

PERTURBATIVE SCHWINGER'S EQUATION-OF-MOTION APPROACH FOR QUANTUM SPINS

A full description of the perturbative Schwinger's equation-of-motion approach for spin-1/2 quantum spins is given in Ref. [32] independently. Here we briefly present the approach and the solutions for the slave spins.

The Schwinger's equation-of-motion theory converts the operator Heisenberg-equations-of-motion (HEoM) into equations of motion for the Green's functions. For the quantum spins that obey the $SU(2)$ Lie-algebra, we introduce both a bosonic and a fermionic Green's functions as the follows:

$$\begin{aligned} iG_\eta^{OO'}[i,f] = \langle\langle \hat{O}_i[t_i]\hat{O}'_f[t_f]\rangle\rangle_\eta =& \langle \mathcal{T}_\pm \left[\hat{O}_i[t_i]\hat{O}'_f[t_f]\right]\rangle - C_\eta\langle\hat{O}_i\rangle\langle\hat{O}'_f\rangle \\ =& \langle \theta(t_i - t_f)\hat{O}_i[t_i]\hat{O}'_f[t_f] + \eta\theta(t_f - t_i)\hat{O}'_f[t_f]\hat{O}_i[t_i]\rangle - C_\eta\langle\hat{O}_i\rangle\langle\hat{O}'_f\rangle, \end{aligned} \tag{S3}$$

where $\eta = B,\ F$ as subscripts while $\eta = \pm$ correspondingly in the equations and $C_{B(F)} = 2(0)$. Whereas $G_{B(F)}^{OO'}[i,f]$ are considered to constitute a complete set, we consider both here since sometimes it is more convenient to use one not the other for computing certain quantities of interests. Details of such consideration is available in Ref. [30] of the main text.

Atomic limit solution

In the atomic limit, since the slave spin Hamiltonian

$$H_{S,at} = \frac{U}{2}\sum_i(\sum_s S_{is}^z)^2 + h\sum_{is} S_{is}^z \tag{S4}$$

is purely Ising-type, we only need to consider

$$G_{\eta,S,ss'}^{\alpha\bar{\alpha}'}[i,f] = \langle\mathcal{T}[S_{is}^\alpha(t_i)S_{fs'}^{\bar{\alpha}'}(t_f)]\rangle - 2\langle S_{is}^\alpha\rangle\langle S_{fs'}^{\bar{\alpha}'}\rangle, \tag{S5}$$

with $\alpha = +$ or $-$.

First, we obtain the HEoM

$$-i\partial_t S_{is}^\alpha = [H_{int}^S, S_{is}^\alpha] = \alpha U S_{i\bar{s}}^z S_{is}^\alpha + \alpha h S_{is}^\alpha. \tag{S6}$$

Correspondingly, the SEoM is

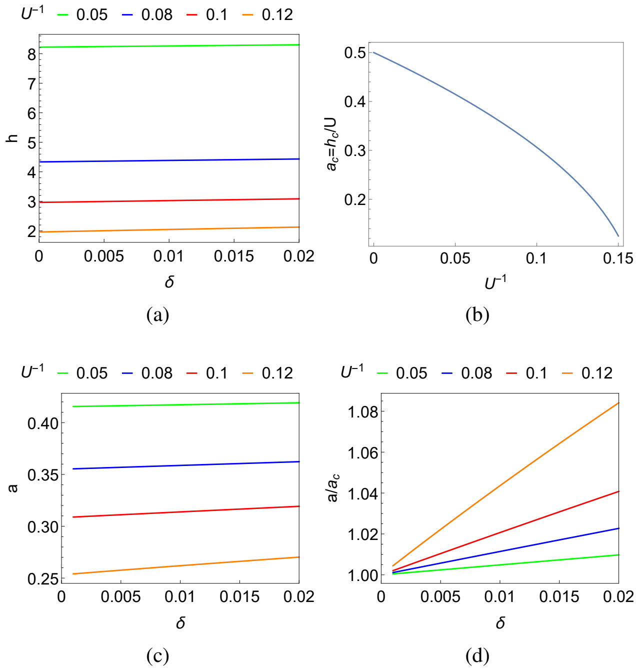

FIG. S1. (S1a) bare values of h shown at different U's as functions of δ; (S1b) $a_c = h_c/U$ shown as a function of U^{-1} over the full range; (S1c) bare values of $a = h/U$; (S1d) the relative change of a from a_c

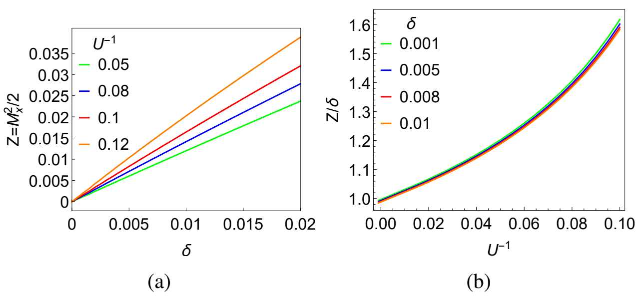

FIG. S2. (S2a) $Z = M_x^2/2$ shown as functions of δ at different U's; (S2b) Z/δ plotted as a function of U^{-1} down to zero which converges to 1.

$$-i\partial_{t_i} G^{\alpha\bar{\alpha}'}_{B,S,\sigma\sigma'}[i,f] = \alpha\left(2\langle S^z_\sigma\rangle\delta_{\alpha\alpha'}\delta_{\sigma\sigma'}\delta[i,f] - hG^{\alpha\bar{\alpha}'}_{B,S,\sigma\sigma'}[i,f] + J\,\Gamma^{z\alpha;\bar{\alpha}'}_{B,S,\bar{\sigma}\sigma;\sigma'}[i,f]\right), \tag{S7}$$

$$-i\partial_{t_i} G^{\alpha\bar{\alpha}'}_{F,S,\sigma\sigma'}[i,f] = \delta_{\alpha\alpha'}\delta_{\sigma\sigma'}\delta[i,f] + \alpha\left(-hG^{\alpha\bar{\alpha}'}_{F,S,\sigma\sigma'}[i,f] + J\,\Gamma^{z\alpha;\bar{\alpha}'}_{F,S,\bar{\sigma}\sigma;\sigma'}[i,f]\right), \tag{S8}$$

where $\Gamma^{\alpha\alpha';\alpha''}_{B(F)ss';s''}[i,f]$ denotes the vertex functions defined as

$$i\Gamma^{\alpha\alpha';\alpha''}_{B(F),S,ss';s''}[i,f] = \langle\langle S^{\alpha}_{s}[t_i]S^{\alpha'}_{s'}[t_i]S^{\alpha''}_{\boldsymbol{s''}}[t_f]\rangle\rangle_{B(F)}. \tag{S9}$$

In the Ising limit, the vertex function can be simplified as

$$i\Gamma^{z\alpha;\alpha'}_{B(F),S,ss';s''}[i,f] = \langle S^z_{is}\rangle G^{\alpha\alpha'}_{B(F),S,s's''}[i,f]. \tag{S10}$$

To simplify the notation, we shall drop the slave spin index s so that $G^{\alpha\bar{\alpha}'}_{S}$ indicates a 2×2 matrix. Here σ_i denotes the Pauli matrices (σ_0 being the identity matrix). Denoting

$\langle S_a^z + S_b^z \rangle = M$, $\langle S_a^z - S_b^z \rangle = m$, which are good quantum numbers, we obtain

$$G_{B,S}^{\alpha\bar{\alpha}'}[\omega] = \frac{\alpha\delta_{\alpha\alpha'}(M\sigma_0 + m\sigma_z)}{\omega - \alpha(h + U(M\sigma_0 - m\sigma_z)/2)}, \quad \text{(S11)}$$

$$G_{F,S}^{\alpha\bar{\alpha}'}[\omega] = \frac{\delta_{\alpha\alpha'}\sigma_0}{\omega - \alpha(+h + U(M\sigma_0 - m\sigma_z)/2)}. \quad \text{(S12)}$$

Expressions for arbitrary states

For an arbitrary state $|\psi\rangle$, we can always expand it in terms of eigenstates of H. In this Ising limit, it is easy to prove that no crossing propagators for $\langle\langle S^+[t_i]S^-[t_f]\rangle\rangle$ and $\langle\langle S^-[t_i]S^+[t_f]\rangle\rangle$. Therefore, for any other states $|\psi(M,\ m,\ \Delta m)\rangle$ as given below,

$$|\psi_{M,m,\Delta m}\rangle = a|\uparrow\downarrow\rangle + b|\downarrow\uparrow\rangle + c|\uparrow\uparrow\rangle + d|\downarrow\downarrow\rangle, \quad \text{(S13)}$$

where

$$M = c^2 - d^2, \quad m = a^2 - b^2,$$
$$\Delta m = \sqrt{\langle \hat{m}^2 \rangle} = \sqrt{a^2 + b^2}. \quad \text{(S14)}$$

We find that the arbitrary $G_{B(F),para}^{\alpha\alpha'}$ can be constructed as

$$\begin{aligned} &G_{B(F),S,para}^{\alpha\alpha'} = \\ &a_1^2 G_{B(F),S,M=1,m=0} + a_2^2 G_{B(F),S,M=-1,m=0} \\ &+ a_3^2 G_{B(F),S,M=0,m=1} + a_4^2 G_{B(F),S,M=0,m=-1}, \end{aligned} \quad \text{(S15)}$$

where $para = (M, m, \Delta m)$ is the complete parameter set that describes the underlying state. With Eq. (S15), we can plug the $G_{B(F),S}$ back into the SEoM to obtain solutions for the vertex functions.

To prepare for the perturbation calculation of transverse field, we write down the explicit expressions for arbitrary states with physical parametrization (use $(M,\ m,\ \Delta m)$ instead of a_is).

First, the solution for real a_is is not unique. For later purpose, here we pick a solution that gives us a positive and uniform $\langle S^x \rangle$:

$$a_1 = \sqrt{\frac{1 + M - \Delta m^2}{2}}, \quad a_2 = \sqrt{\frac{1 - M - \Delta m^2}{2}}, \quad \text{(S16)}$$

$$a_3 = \sqrt{\frac{m + \Delta m^2}{2}}, \quad a_4 = \sqrt{\frac{\Delta m^2 - m}{2}}, \quad \text{(S17)}$$

which gives

$$\begin{aligned} &\langle S_a^x \rangle = \langle S_b^x \rangle = 1/2(\sqrt{\Delta m^2 + m}\sqrt{1 - \Delta m^2 - M} \\ &+ \sqrt{\Delta m^2 - m}\sqrt{1 - \Delta m^2 + M}). \end{aligned} \quad \text{(S18)}$$

Now we can write Eq. (S15) as

$$\begin{aligned} G_{B(F)}^{\alpha\bar{\alpha}'} = \frac{1}{2}\Big(&(1 - \Delta m^2)(G_{B(F),S,(1,0)}^{\alpha\bar{\alpha}'} + G_{B(F),S,(-1,0)}^{\alpha\bar{\alpha}'}) \\ &+ M(G_{B(F),S,(1,0)}^{\alpha\bar{\alpha}'} - G_{B(F),S,(-1,0)}^{\alpha\bar{\alpha}'}) \\ &+ m(G_{B(F),S,(0,1)}^{\alpha\bar{\alpha}'} - G_{B(F),S,(0,-1)}^{\alpha\bar{\alpha}'}) \\ &+ \Delta m^2(G_{B(F),S,(0,1)}^{\alpha\bar{\alpha}'} + G_{B(F),S,(0,-1)}^{\alpha\bar{\alpha}'})\Big), \end{aligned} \quad \text{(S19)}$$

where (M, m) denotes the base states parameters.

The $G_{B(F)}^{\alpha\alpha'}$ for arbitrary state with parameters $(M,\ m,\ \Delta m)$ read

$$G_{B,S,0}^{\alpha\bar{\alpha}}[\omega] = \frac{\alpha((1 - \Delta m^2)\sigma_0 + m\sigma_z)(\omega + \alpha h) + (M + \alpha\Delta m^2)J\sigma_0/2}{(\omega + \alpha h)^2 - U^2/4}, \quad \text{(S20)}$$

$$G_{F,S,0}^{\alpha\bar{\alpha}}[\omega] = \frac{(\omega + \alpha h)\sigma_0 + \alpha(M\sigma_0 - m\sigma_z)J/2}{(\omega + \alpha h)^2 - U^2/4}. \quad \text{(S21)}$$

Weiss mean-field approximation to the hopping term of the slave spin Hamiltonian

In this part, we solve $H_{S,eff}$ for finite doping at the mean-field level. The mean-field approximation is to decouple $H_{S,hopping}$ as

$$\begin{aligned} &H_{S,hopping} \to H_{S,hMF} \\ &= -Q_f \sum_{is} \big((\sum_{\langle ij \rangle_{s'}} \langle S_{js'}^- \rangle) S_{is}^+ + h.c. \big), \end{aligned} \quad \text{(S22)}$$

Since the emergence of $\langle S_{is}^{\pm} \rangle \neq 0$ is from spontaneous-symmetry-breaking, we can choose the direction of the magnetization at our convenience: $\langle S_{is}^+ \rangle = \langle S_{is}^- \rangle = \langle S_{is}^x \rangle = M_x$. On a 2D square lattice with only nearest neighbor (nn) hopping, $H_{S,MF}$ becomes

$$\begin{aligned} &H_{S,MF} = H_{S,at} + H_{S,hMF} \\ &= \sum_i \Big(U S_{ia}^z S_{ib}^z + h(S_{ia}^z + S_{ib}^z) - h_x(S_{ia}^x + S_{ib}^x) \Big), \end{aligned} \quad \text{(S23)}$$

where $h_x = DM_xQ_f$, $D = 4$ is the number of nn bonds for a two dimensional square lattice. This is a local Hamiltonian and can be solved by exact diagonalization. Here we adopt an alternative analytic calculation using a perturbation theory in terms of the Green's functions.

The corresponding HEoM reads

$$-i\partial_t S_s^\alpha = \alpha(US_{\bar{s}}^z S_s^\alpha + hS_s^\alpha + h_x S_s^z), \tag{S24}$$

and hence the SEoM becomes

$$-i\partial_{t_i} G^{\alpha\bar{\alpha}'}_{B,S,ss'}[i,f] = \alpha\left(2\langle S_s^z\rangle\delta_{\alpha\alpha'}\delta_{ss'}\delta[i,f] - hG^{\alpha\bar{\alpha}'}_{B,S,ss'}[i,f] + J\,\Gamma^{z\alpha;\bar{\alpha}'}_{B,S,\bar{s}s;s'}[i,f] + h_x G^{z\bar{\alpha}'}_{B,S,\bar{s}s'}[i,f]\right), \tag{S25}$$

$$-i\partial_{t_i} G^{\alpha\bar{\alpha}'}_{F,S,ss'}[i,f] = \delta_{\alpha\alpha'}\delta_{ss'}\delta[i,f] + \alpha\left(-hG^{\alpha\bar{\alpha}'}_{F,S,ss'}[i,f] + J\,\Gamma^{z\alpha;\bar{\alpha}'}_{F,S,\bar{s}s;s'}[i,f] + h_x G^{z\bar{\alpha}'}_{F,S,\bar{s}s'}[i,f]\right). \tag{S26}$$

According to the result of diagonalization, we know that the ground state is a singlet/triplet with the onset of an infinitesimal transverse field.

The effects of the perturbation term are twofold: i) modifying the ground state wavefunction(s); ii) altering the evolution of the states (altering the SEoM). So we first consider the change in wavefunction, which gives G_0 with renormalized parameters. Then we consider the revised SEoM hence the further correction to G's and Γ's.

In the presence of a transverse field $h_x\hat{x}$, a magnetization $M_{x,s}$ along the field direction is induced. Now with the new correlators $G^{z\bar{\alpha}'}_{B(F)}$ entering the SEoM, we need to consider their HEoM and SEoM as well. The HEoM of S_s^z reads

$$-i\partial_t S_s^z = ih_x S_s^y = \frac{h_x}{2}(S_s^+ - S_s^-), \tag{S27}$$

which leads to the following SEoM in frequency space

$$\omega G^{z\bar{\alpha}'}_B[\omega] = \frac{1}{2}(\alpha' I_x + h_x \sum_\alpha \alpha G^{\alpha\bar{\alpha}'}_B[\omega]), \tag{S28}$$

$$\omega G^{z\bar{\alpha}'}_F[\omega] = \frac{h_x}{2}\sum_\alpha \alpha G^{\alpha\bar{\alpha}'}_F[\omega], \tag{S29}$$

where $I_x = M_x\sigma_0$ since $\langle[S_s^z, S_{s'}^{\bar{\alpha}'}]\rangle = \alpha'\delta_{ss'}\langle S_s^{\bar{\alpha}'}\rangle = \alpha'\delta_{ss'}\langle S_s^x\rangle$ and the second equal sign is because we apply a uniform field along $\hat{x}$.

Note that

$$\begin{aligned}\langle S_s^x\rangle &= -i\langle[S_s^z, S_s^y]\rangle = \frac{1}{2i}(G_F^{z+}[i,i] - G_F^{z-}[i,i]) \\ &= -\int\frac{d\omega}{2\pi}\frac{h_x}{2i\omega}(G^{++}_{F,S,ss}[\omega] + G^{--}_{F,S,ss}[\omega] \\ &\quad - G^{-+}_{F,S,ss}[\omega] - G^{+-}_{F,S,ss}[\omega]).\end{aligned} \tag{S30}$$

To the lowest order, the latter two terms can be computed as

$$\langle S_s^x\rangle = \int\frac{d\omega}{2\pi}\frac{h_x}{2i\omega}(G^{-+}_{F,S,0,ss}[\omega] + G^{+-}_{F,S,0,ss}[\omega]), \tag{S31}$$

where $G^{-+}_{F,S,0,ss}[\omega]$ is the Green's function without transverse field in Eq. (S21).

The transverse magnetization, i.e. the quasiparticle weight, the magnetization, i.e. the hole density, and Δm^2 correction, to the lowest order in h_x, are found to be

$$\langle S_a^x\rangle \simeq h_x\left[\frac{U\sigma_0 - 2hM\sigma_0 + 2hm\sigma_z}{U^2 - 4h^2}\right]_{aa}, \tag{S32}$$

$$M = 2\langle S_a^z\rangle \simeq \frac{-4hh_x^2}{U(U^2/4 - h^2)}, \tag{S33}$$

$$\delta\Delta m^2 \simeq \frac{-h_x^2}{U^2/4 - h^2}. \tag{S34}$$

which leads to $Z = M_x^2/2 \propto \delta$ by solving the self-consistency equation $h_x = DM_xQ_f$. All are consistent with the numerical calculations.

In our SEoM theory, the dynamical spin Green's functions of $H_{S,MF}$ can be written as

$$G^{\alpha\bar{\alpha}'}_{B(F)}[\omega] \simeq G^{\alpha\bar{\alpha}'}_{B(F),S,0}[\omega] + G^{\alpha\bar{\alpha}'}_{B(F),S,1}[\omega], \tag{S35}$$

where $G^{\alpha\bar{\alpha}'}_{B(F),S,1}[\omega]$ is the lowest order correction of $G^{\alpha\bar{\alpha}'}_{B(F)}[\omega]$ (other than the change in the wavefunction under the evolution of H_0).

$$G^{\alpha\bar{\alpha'}}_{B,S,1,ss'}[\omega]=\frac{\alpha}{\omega+\alpha h}\Big(\frac{\alpha'(U\langle S^x_s\rangle+h_x)I_x}{2}+\frac{Uh_x}{2\omega}\langle S^x_s\rangle\sum_{\alpha''}\alpha''G^{\alpha''\bar{\alpha'}}_{B,S,0,\bar{s}s'}[\omega]+\frac{h_x^2}{2\omega}\sum_{\alpha''}\alpha''G^{\alpha''\bar{\alpha'}}_{B,S,0,ss'}[\omega]\Big), \tag{S36}$$

$$G^{\alpha\bar{\alpha'}}_{F,S,1,ss'}[\omega]=\frac{\alpha}{\omega+\alpha h}\Big(\frac{Uh_x}{2\omega}M_x\sum_{\alpha''}\alpha''G^{\alpha''\bar{\alpha'}}_{F,S,0,\bar{s}s'}[\omega]+\frac{h_x^2}{2\omega}\sum_{\alpha''}\alpha''G^{\alpha''\bar{\alpha'}}_{F,S,0,ss'}[\omega]\Big). \tag{S37}$$

SPECTRAL FUNCTIONS IN k-SPACE

The rigorous spinon spectral function $\rho_f[\omega',\boldsymbol{k}] = \sum_{\boldsymbol{k}}\delta(\mu^*+M_x^2\epsilon(\boldsymbol{k})-\omega')$ does have $k-$dependence where $\epsilon(\boldsymbol{k})$ is the bare dispersion given by the hopping t_{ij}. Therefore, so does $\rho_d^{MF}[\omega,\boldsymbol{k}]$. But since $\rho^{-+}_{B,Sss'}[\omega]$ is still strictly local, $\rho_d^{MF}[\omega,\boldsymbol{k}]$ just splits into bands determined by the poles of $\rho^{-+}_{B,Sss'}[\omega]$ according to $\rho_d[\omega,\boldsymbol{k}] = \int d\omega'(\sum_{ss'}\delta(\omega+\omega'-\omega_i)W_i)\rho_f[\omega',\boldsymbol{k}] = \int d\omega'(\sum_{ss'}\delta(\omega+\omega'-\omega_{ss',i})W_{ss',i})\delta(M_x^2\epsilon(\boldsymbol{k})-\omega') = \sum_{ss'}\delta(\omega+(\mu^*+M_x^2\epsilon(\boldsymbol{k}))-\omega_{ss',i})W_{ss',i}$. Here $\omega_{ss',i}$ are the poles, $W_{ss',i}$ are their corresponding spectral weights and $\mu^*=-h-\mu$ is the Lagrangian multiplier for fixing the spinon density. In our perturbative regime, both M_x^2 and μ^* are significantly smaller than the spacing between poles. Hence, these bands will not mix and each carries a uniform weight $W_{ss',i}$.