Supplemental Materials for "Inverse design of phononic crystal with desired transmission via a gradient-descent approach"

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THE TRANSMISSION IN TERMS OF NORMAL MODES

From the motion equation $M\ddot{X}+KX=0$, we know that the amplitude X of the normal mode satisfies $\omega^2 MX=KX$. Let $U=M^{\frac{1}{2}}X$, then we can get $\omega^2 U=M^{-\frac{1}{2}}KM^{-\frac{1}{2}}U$ which satisfies

$$F\Omega^2F^{\top}=M^{-\frac{1}{2}}KM^{-\frac{1}{2}}, \tag{S1}$$

where eigenmatrix $F_{ji}=e^i_j$ and eigenfrequencies $\Omega_{ii}=\omega_i$.

Define $R=\Omega^2-\omega^2 I$ and $Z'=Z+\Sigma(\omega)=K-\omega^2 M$ where $Z'^{*}_{1N}=Z^{*}_{1N}$ and I is identity matrix, then we obtain $Z'=M^{\frac{1}{2}}FRF^{\top}M^{\frac{1}{2}}$. Thus $Z^{*}_{1N}=\mathrm{Det}(MR)(M^{-\frac{1}{2}}FR^{-1}F^{\top}M^{-\frac{1}{2}})_{1N}$. Then one can find

$$Z^{*}_{1N}=\frac{\mathrm{Det}(MR)}{\sqrt{\gamma_{\mathrm{L}}\gamma_{\mathrm{R}}}}S_1=0, \tag{S2}$$

where S_1 is given by

$$S_1=\sum_{i=1}^{N}\frac{E^i_1E^i_N}{\omega_i^2-\omega^2}. \tag{S3}$$

Next, we simplify $\mathrm{Det}(Z)$. Due to the property of determinant, one has $\mathrm{Det}(Z)=\mathrm{Det}(MF^{\top}M^{-\frac{1}{2}}ZM^{-\frac{1}{2}}F)$ and

$$F^{\top}M^{-\frac{1}{2}}ZM^{-\frac{1}{2}}F=R+A+B, \tag{S4}$$

where $A=-i\frac{\gamma_{\mathrm{L}}}{m_1}F^{\top}_{(1,:)}F_{(1,:)}$, $B=-i\frac{\gamma_{\mathrm{R}}}{m_N}F^{\top}_{(N,:)}F_{(N,:)}$. The $F_{(i,:)}$ means taking the ith row of matrix F as a vector.

Since R is a diagonal matrix and the rank of matrices A and B is 1, one can find

$$\mathrm{Det}(R+A+B)=\mathrm{Det}(R)\left[1+\sum_{i=1}^{N}\frac{A_{ii}+B_{ii}}{R_{ii}}+\sum_{1=i<j}^{N}\frac{\mathrm{Det}\begin{pmatrix}A_{ii} & B_{ij}\\ A_{ji} & B_{jj}\end{pmatrix}+\begin{pmatrix}B_{ii} & A_{ij}\\ B_{ji} & A_{jj}\end{pmatrix}}{R_{ii}R_{jj}}\right]. \tag{S5}$$

It results in

$$\mathrm{Det}(Z)=\mathrm{Det}(MR)(1-i\omega S_3-\omega^2 S_2), \tag{S6}$$

where S_2 and S_3 is given by

$$\begin{aligned}S_2&=\sum_{1=i<j}^{N}\frac{(E^i_1E^j_N-E^j_1E^i_N)^2}{(\omega_i^2-\omega^2)(\omega_j^2-\omega^2)},\\ S_3&=\sum_{i=1}^{N}\frac{(E^i_1)^2+(E^i_N)^2}{\omega_i^2-\omega^2}.\end{aligned} \tag{S7}$$

So far we have obtained Z^{*}_{1N} and $\mathrm{Det}(Z)$. Finally, in terms of $G_{1N}=Z^{*}_{1N}/\mathrm{Det}(Z)$ and $\mathcal{T}(\omega)=4\gamma_{\mathrm{L}}\gamma_{\mathrm{R}}\omega^2\left|G_{1N}\right|^2$, one can get the transmission with respect to the normal mode as given by

$$\mathcal{T}(\omega)=\frac{4\omega^2S_1^2}{(1-\omega^2S_2)^2+\omega^2S_3^2}. \tag{S8}$$

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