Supplemental Material: Search for ultralight dark matter with a frequency adjustable diamagnetic levitated sensor

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I. OPTICAL CALCULATION

The light emitted from the incident fiber is assumed to be Gaussian, taking the light propagation direction as the z-axis, the incident Gaussian light intensity distribution at waist can be written as [1]:

$$I_1(r) = I_0 \exp\left(-\frac{2r^2}{\omega_{01}^2}\right)$$

And the waist radius of incident Gaussian beam is ω_{01}, which satisfies relation:

$$\omega_{01} = \sqrt{\frac{a_0^2\lambda^2}{\lambda^2 + \pi^2 a_0^2 tan^2\alpha}}$$

where a_0 is the radius of fiber core, and sinα = N.A, N.A. is the numerical aperture of the fiber. In there $a_0 = 5\mu$m and N.A.=0.13 for single-mode fiber. The incident optical power is:

$$P_{\text{in}} = \int_0^\infty I_1(r)2\pi r dr = \frac{\pi}{2}\omega_{01}^2 I_0$$

The response of the light to the micro-sphere is calculated using the standard optical ABCD ray matrix[2]. Under the par-axial approximation, the transmission matrix $\mathbf{T}$ is:

$$\mathbf{T} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

which has the equation:

$$\begin{pmatrix} r_f \\ \theta_f \end{pmatrix} = \mathbf{T} \begin{pmatrix} r_i \\ \theta_i \end{pmatrix}$$

In calculating the transmission matrix $\mathbf{T}$, we neglected the reflection of light at the interface and the absorption in the micro-sphere. Here A, B, C, D are

$$A = \frac{2}{n} - 1, B = \frac{2R}{n}, C = \frac{1-n}{n}\frac{2}{n}, D = \frac{2}{n} - 1, \beta_0 = \frac{\lambda}{\pi\omega_{01}^2}$$

with the parameters $\lambda = 1550nm$, n=1.45, the we get the d_2 and ω_{02} satisfy

$$d_2 = \frac{AC/\beta_0^2 + ACd_1^2 + ADd_1 + BCd_1 + BD}{C^2/\beta_0^2 + C^2d_1^2 + 2CDd_1 + D^2}$$

$$\omega_{02} = \omega_{01}\sqrt{(A + Cd_2)^2 + \beta_0^2(Ad_1 + B + Cd_1d_2 + Dd_2)^2}$$

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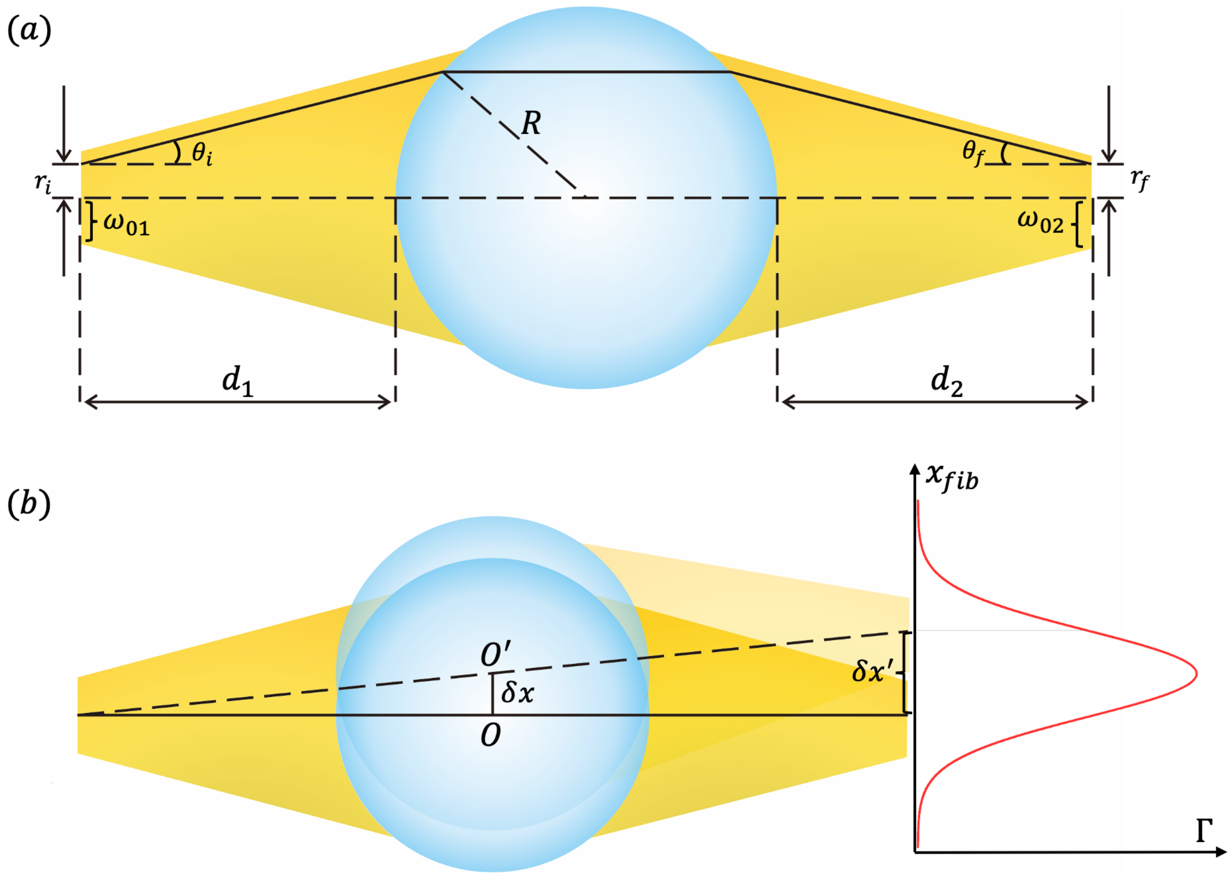

FIG. 1: (a) Optical ray of the laser. θ_i (θ_f) and r_i (r_f) are used to characterize the optical ray coming from incident fiber and reaching the detection fiber, d_1 (d_2) are the distance between the incident fiber (detection fiber) and the optical axis, R is the radius of the micro-sphere. (b) Dependence of the light field distribution with the microsphere position. The position of the image on the incident fiber core $\delta x'$ in x axis depends on the position of the micro-sphere position δx. The transmission coefficient Γ changes with δx.

d_2 and ω_{02} are functions of d_1, choose a suitable d_1 so that $\omega_{02} \approx a_0$. The coupling efficiency Γ, of the laser beam and the single-mode optical fiber can be written as:

$$\Gamma = \Gamma_0 \exp\left(-\Gamma_0 \cdot \frac{x_{\text{fib}}^2}{2} \left(\frac{1}{\omega_{02}^2} + \frac{1}{a_0^2}\right)\right), \Gamma_0 = \frac{4\omega_{02}^2 a_0^2}{(\omega_{02}^2 + a_0^2)^2}$$

x_{fib} indicate the fiber shift from the x direction, when $x_{\text{fib}} = 0$, $\Gamma = \Gamma_{max} = \Gamma_0$. In the experiment, fix x_{fib} at the place where $\partial\Gamma/\partial x_{\text{fib}}$ is the largest. As $x_{\text{fib}} = 2.51\mu m$ and $\Gamma(x_{\text{fib}}) = 0.604$ in Fig.1(b).

δx is the displacement of the micro-sphere vertically to the optical axis (similar result for y direction), while $\delta x'$ is the projection on the incident fiber surface. Under par-axial approximation, $\delta x = \zeta \cdot \delta x'$ for small displacement δx of the micro-sphere, with the displacement magnification factor:

$$\zeta = \frac{d_1 + d_2 + 2R}{d_1 + R}, \varsigma = \frac{\partial \Gamma}{\partial x} = \frac{\partial \Gamma}{\partial x'} \cdot \frac{\partial x'}{\partial x} = \zeta \cdot \frac{\partial \Gamma}{\partial x'}$$

II. MEASUREMENT NOISE

The relationship between the average power P and the photon number N is:

$$N_{\text{in}} = \frac{P_{\text{in}} T_{\text{mea}}}{\hbar\omega_{\text{op}}}, N_{\text{dec}} = \frac{P_{\text{dec}} T_{\text{mea}}}{\hbar\omega_{\text{op}}}$$

where $\omega_{\rm op}$ is the light frequency. The photons satisfy the Poisson distribution and the corresponding photon number fluctuation is $\delta N_{\rm in} = \sqrt{N_{\rm in}}$ and $\delta N_{\rm dec} = \sqrt{N_{\rm dec}}$. Such fluctuation brings a imprecise detection noise of displacement $\delta x_{\rm imp}$:

$$\begin{aligned}\delta x_{\rm imp} &= \frac{\partial x}{\partial \Gamma}\sqrt{\left(\frac{\partial \Gamma}{\partial N_{\rm in}}\delta N_{\rm in}\right)^2 + \left(\frac{\partial \Gamma}{\partial N_{\rm dec}}\delta N_{\rm dec}\right)^2} \\ &= \frac{1}{\varsigma}\sqrt{\frac{\Gamma+\Gamma^2}{N_{\rm in}}}\end{aligned}$$

Thus the power density of displacement noise is:

$$S_{\rm xx}^{\rm imp} = \frac{1}{\varsigma^2}\frac{(\Gamma+\Gamma^2)\hbar\omega_{\rm op}}{P_{\rm in}}$$

On the other hand, the photon passes through the micro-sphere which changes the direction and therefore generated a back-action force $\delta f_{\rm ba}$ with the strength also proportional to the fluctuation of the incident photon $\delta N_{\rm in}$. The back-action force $\delta f_{\rm ba}$ can be written as:

$$\delta f_{\rm ba} = \sqrt{N_{\rm in}}\hbar\Delta k/T_{\rm mea}$$

where Δk is the change of the wave vector.

Here we suppose that the direction of light wave vector is along the direction of the Gaussian light wavefront, and the probability of photon appearing is proportional to the intensity of Gaussian light. Δk is the average change of light wave vector pass through the micro-sphere. It is calculated by $\sqrt{(\Delta k_{\rm in})^2 + (\Delta k_{\rm out})^2}$, where $\Delta k_{\rm in}$ is the average light wave vector go to the micro-sphere, $\Delta k_{\rm out}$ is the average light wave vector go out of the micro-sphere. We obtain

$$\begin{aligned}(\Delta k)^2 =& k^2\beta \\ =& k^2\int_0^\infty \frac{k^2r^3}{k^2r^2 + ((1-\frac{z_r^2}{z_l^2})\frac{kR^2}{2\rho(z_l)} + \frac{z_r}{z} - k\rho(z_l))^2}. \\ & \frac{1}{\omega_1^2(z_l)}\exp\left(-\frac{2r^2}{\omega_1^2(z_l)}\right)dr\end{aligned}$$

where $k = \omega_{\rm op}/c$, $z_l = d_1 + R - \sqrt{R^2 - r^2}$, $\omega_1(z_l) = \omega_{01}\sqrt{1 + (z_l/z_r)^2}$, $z_r = 2\pi\omega_{01}^2/\lambda$ and $\rho(z_l) = z_r(z_l/z_r + z_r/z_l)$.

The power density of back-action noise is thus:

$$S_{\rm ff}^{\rm ba} = \frac{P_{\rm in}\hbar\omega_{\rm op}\beta}{c^2}$$

and the product of imprecision noise and back-action noise is:

$$S_{\rm xx}^{\rm imp}\cdot S_{\rm ff}^{\rm ba} = \frac{1}{\varsigma^2}(\Gamma+\Gamma^2)(\omega_{\rm op}/c)^2\beta^2\hbar^2$$

The quantum efficiency of the measurement is defined as:

$$\eta = \frac{\varsigma}{4(\Gamma+\Gamma^2)\beta k^2}$$

where $\eta = 1$ corresponding standard quantum limit (SQL). The total measurement noise is

$$S_{\rm aa}^{\rm mea}(\omega) = \frac{S_{\rm xx}^{\rm imp}}{|\chi_{\rm m}(\omega,\omega_0)|^2} + \frac{S_{\rm ff}^{\rm ba}}{m^2}$$

$S_{\rm aa}^{\rm mea}$ is minimized by tuning the incident laser power $P_{\rm in}$ under the product constraint of the imprecision noise and backaction noise. The optimized power is:

$$P_{\rm opt}(\omega,\omega_0) = \sqrt{\frac{\Gamma+\Gamma^2}{\beta}}\frac{mc}{\varsigma|\chi_{\rm m}(\omega,\omega_0)|}$$

with the minimised total acceleration measurement noise as:

$$S_{\mathrm{aa,min}}^{\mathrm{mea}} = \frac{2\hbar\omega_{\mathrm{op}}}{m\varsigma c|\chi_{\mathrm{m}}(\omega,\omega_0)|}\sqrt{\beta(\Gamma+\Gamma^2)}$$

And in order to simplify the experiment process, we choose $P_{\mathrm{in}} = P_{\mathrm{opt}}(\omega_0,\omega_0)$, with the optimized acceleration measurement noise at this time:

$$S_{\mathrm{aa,opt}}^{\mathrm{mea}} = \frac{\hbar\omega_{\mathrm{op}}\sqrt{\beta(\Gamma+\Gamma^2)}}{m\varsigma c\gamma\omega_0}\cdot\left(\frac{1}{|\chi_{\mathrm{m}}(\omega,\omega_0)|^2}+\gamma^2\omega_0^2\right)$$

[1] O. Svelto, “Properties of laser beams,” in Principles of Lasers (Springer New York, NY, 2010) p.153.
[2] H. Kogelnik and T. Li, Appl. Opt. 5, 1550 (1966).