## **Supplementary Material for "Photonic generation of reconfigurable**

### chirped microwave waveforms using temporal cavity solitons with

# agile repetition-rate"

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#### Part I. The measured spectrum of resulted dual OFCs

In the experiment, two optical frequency combs (OFCs) with slightly different repetition rate were obtained by temperature tuning the wavelength interval of the external dual-pump laser. Obviously, the repetition rate difference  $\delta$  of the dual OFCs was also continuously adjustable in a wide range. A programmable optical filter (Finisar 4000A) was used to obtain a tunable channelized bandwidth. Figure S1 shows the optical spectrum of resulted dual OFCs. The repetition rate of OFC1 and OFC2 were set to 100 GHz and 112 GHz, respectively. The DFB-FL1was used as the frequency reference of the dual-comb system, and is also the 0th order comb. Assume the reference frequency line at the center is zero order, then the m<sup>th</sup> comb line at either right or left side will generate the same RF frequency of f<sub>R</sub> = m $\delta$  when dual comb beats. In order to facilitate the demonstrate, only three pairs of comb teeth on the left side of the frequency comb were filtered simultaneously by programmable optical filter in this paper. The frequency of the beat signal of each order is 12 GHz, 24 GHz and 36 GHz respectively. In this case, the central frequency of the beat signal is fixed and there is no chirp. Unfortunately, in the subsequent experiment, the repetition rate difference  $\delta$  of the dual- comb is set to 1 GHz due to the limited measurement range of electrical spectrum analyzer (measurement range: 0~3 GHz).



Fig. S1. Dual OFCs with 100 GHz, 112 GHz mode spacing respectively.

#### Part II. The generation of multi-band reconfigurable microwave waveforms (MRMWs)

In the proposed system, fast frequency sweeping of the extra-cavity pump laser can also flexibly manipulate the repetition-rate difference of the dual-comb. Then, performance of the multi-band triangular

microwave waveforms generation by established dual-chirped fiber comb is investigated. A 1-kHz control signal S(t) which had a triangular profile and an amplitude of ~10V (positive bias of 10 V) was applied to the PZT. After optical filtering, by beating the paired comb lines of the same order at a high-speed photodetector (PD), the bandwidth, central frequency and repetition rate of the chirped microwave waveforms were obtained. Fig.S2 (a) shows the electrical spectrum of the generated 1th and 2th band microwave waveforms, in which the bandwidth remains 0.48 GHz and 0.952 GHz respectively, and ranges from 0.85 GHz to 1.33 GHz and 1.65 GHz to 2.602 GHz. It is noteworthy that the central frequency and bandwidth of the 2th band transmission signal become twice as large as the 1th band frequency modulation signal. As can be seen from Fig.S2 (a), the out of band signal-to-noise ratio of the 1th band is significantly higher than that of the 2th band. This is mainly because flatness of the OFC determines the power uniformity of the output microwave signals in each channel. The 3th frequency component whose central frequency and bandwidth are predicted as 3.27GHz and 1.92 GHz respectively was unable to be measured due to the limited measurement range of electrical spectrum analyzer.

Fig.S2 (b) shows the measured waveforms of the generated MRMWs in two periods. A total of three bands of microwave waveforms were captured by the oscilloscope. Their instantaneous frequency was calculated by performing short-time Fourier transformation. The recovered instantaneous frequency of three bands triangular microwave waveforms is shown jointly in Fig. S2(c), where the chirp rates of the 1th, 2th and 3th bands are 1 GHz/ms, 2 GHz/ms and 3 GHz/ms respectively. Under the same control signal, the chirp rate of the m<sup>th</sup> band is always m times that of the 1th band. As shown in Fig. S2(c), the frequency of generated chirped waveforms rapidly increases with sweeping time. Due to the PZT module's narrower response bandwidth, there is local hysteresis at the rising edge of the triangle wave.



Fig. S2. The measured multi-band triangular microwave waveform (a) the electrical spectrum of three bands, (b) waveform, (c) the recovered instantaneous frequencies corresponding to the (b).

Finally, the reconfigurability of multi-band microwave waveforms was also demonstrated by adjusting the pattern of control signals. The sinusoidal signal and sawtooth signal with a repetition rate of 1kHz were applied to PZT for frequency sweep respectively. The waveform of the electrical control signal is shown in Fig.S3. Fig.S4 (a) and (c) show the measured nonlinearly (sinusoidal waveform of driving-signal) and linearly (sawtooth waveform of driving-signal) multi-band chirped waveforms, which were recorded on the real-time scope in the receiver-end. According to our measurement results, Fig.S4 (b) and (d) show the recovered instantaneous frequencies of the MRMWs with sinusoidal modulation and sawtooth modulation, respectively, in which the frequency of generated higher-order band rapidly increases with sweeping time. However, the instantaneous frequency of multi-band sawtooth microwave waveform increases nonlinearly, which was mainly due to the long response time of PZT module and the inability to realize fast voltage agility.

In conclusion, we have demonstrated the photonic generation of multi-band reconfigurable microwave

waveforms based on dual-chirped TCSs. This all-optical method is expected to achieve more wavebands and wider bandwidth of reconfigurable chirped microwave signals. In the future, the flatness of the OFC needs to be improved, which can enhance the power uniformity of the output microwave signals in each channel.



Fig. S3. The measured control signal S(t). (a) sinusoidal signal with a repetition rate of 1kHz, (b) sawtooth signal with a repetition rate of 1kHz.



Fig. S4. The multi-band chirped microwave waveform. (a) and (c): the measured waveform of MRMWs with sinusoidal frequency modulation and sawtooth modulation, respectively. (b) and (d): the recovered instantaneous frequencies corresponding to the (a) and (c), respectively.