Supplementary Materials for "Unconventional Nonreciprocal Voltage Transition from Edge Channels in Ag_2Te Nanobelts"*

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Content:

I. Magnetoresistance and Hall resistance

Figure 2(b) introduces the nonreciprocal voltage of eight channels in Ag_2Te nanobelt #2. The corresponding resistance as a function of the magnetic field is shown in Fig. S1. The two channels in the left panel with diagonal connections show strong asymmetry because of the mixture of magnetoresistance and Hall resistance, which can be linked with the asymmetry of the nonreciprocal voltage in these two channels. The three channels in the middle panel are all connected along a longitudinal direction. Symmetric magnetoresistances can result in symmetric nonreciprocal voltage in Fig. 2(b). The three channels in the right panel show the Hall resistance, while linear response under an external magnetic field has zero nonreciprocal voltage.

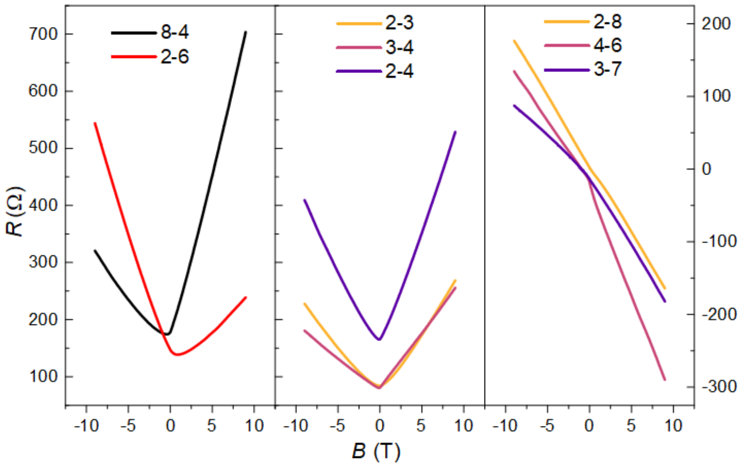

Fig. S1. Magnetoresistance and Hall resistance in Ag_2Te nanobelt #2.

II. The principle of linear superposition

The voltage and resistance in any channels measured in traditional Hall bar devices should all satisfy the principle of linear superposition[1], in other words, the sum of the two adjacent channel signals should be equal to the signal of the entire channel measured alone. Indeed, the resistance in Ag_2Te nanobelts #2 follows as mentioned, the corresponding curves overlap each other. However, the nonreciprocal voltage violates the principle of linear superposition. For the edge channels in Ag_2Te nanobelts can give rise to a giant but infinite nonreciprocal voltage when the voltage grows exceeding the breakdown voltage between two electrodes[2]. As a consequence, these unidirectional nonreciprocal voltage violates the principle of linear superposition.

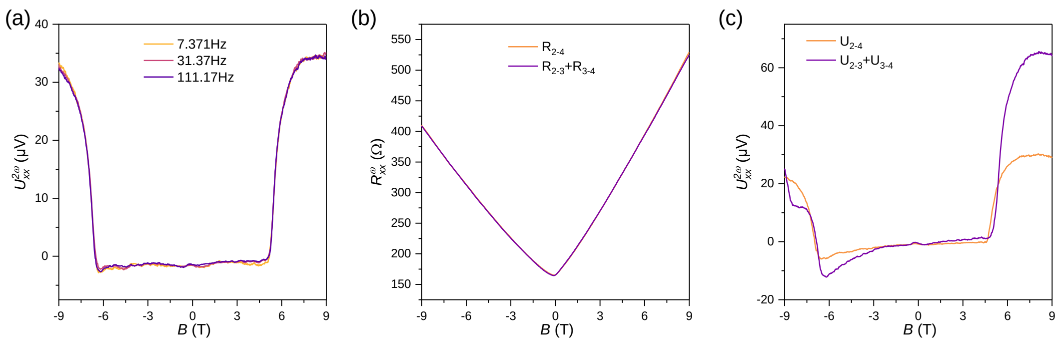

Fig. S2. The nonlinearity of the nonreciprocal voltage. (a). The symmetric nonreciprocal voltage under current I_{ac} with three different frequencies. (b) The resistance of the channel R_{2-4} and the sum of the channel R_{2-3} and R_{3-4} overlap each other, satisfying the principle of the linear superposition. (c) The nonreciprocal voltage of the channel U_{2-4} and the sum of the channel U_{2-3} and U_{3-4} misalign each other, indicating a violation of the principle of linear superposition.

III. The nonreciprocal voltage under various current I_{dc}

With $I_{ac} = 1$ μA, the current is too small that a magnetic field up to 9T cannot induce the nonreciprocal voltage transition, even though the various I_{dc} from 1 μA to 20 μA generate large thermoelectrical voltages. With $I_{ac} =$ 4 μA and $I_{ac} = 5$ μA, the nonreciprocal voltage transition can be induced by the large magnetic field, the I_{dc} is applied from -28 μA to 20 μA and from -20 μA to 10 μA, respectively. All current I_{dc} cannot change the transition position, but affect the final nonreciprocal voltage because of the Joule heating.

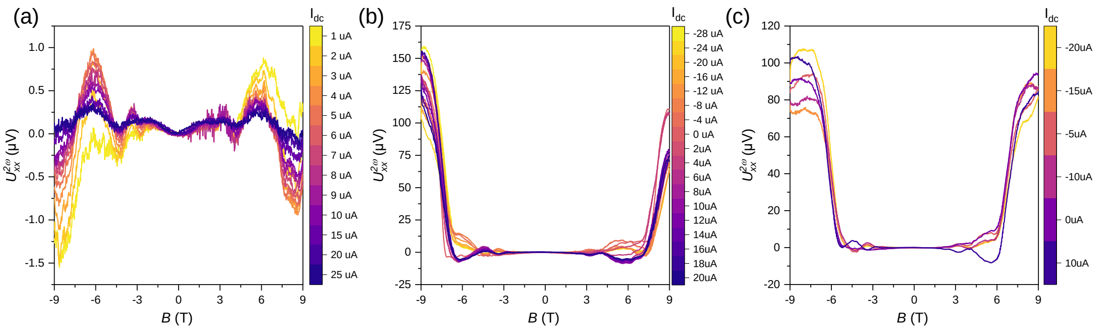

Fig. S3. The nonreciprocal voltage under various current I_{dc} when the current $I_{ac} = 1$ μA (a), $I_{ac} = 4$ μA (b), $I_{ac} =$ 5 μA (c).

IV. The $\mathrm{d}V/\mathrm{d}I$ measurement in symmetric channels

In a channel with symmetric nonreciprocal voltage transition, as shown in Fig. S4(a), both the positive and the negative magnetic field can induce the transition, and the corresponding $\Delta\mathrm{d}V/\mathrm{d}I$ curves under various magnetic fields show antisymmetry behavior with the current I_{dc}, as displayed in Fig. S4(b). Under positive magnetic fields, the $\Delta\mathrm{d}V/\mathrm{d}I$ decreases rapidly to a minimum and grows linearly with the increased current I_{dc}, as shown in purple curves in Fig. S4(b). While under negative magnetic fields, the $\Delta\mathrm{d}V/\mathrm{d}I$ grows linearly with the current I_{dc} gradually to maximum at small magnetic fields and then decrease linearly with the current I_{dc}. The behavior of $\Delta\mathrm{d}V/\mathrm{d}I$ with external magnetic fields fits well with the magnetic field dependent nonreciprocal voltage in Fig. S4(a).

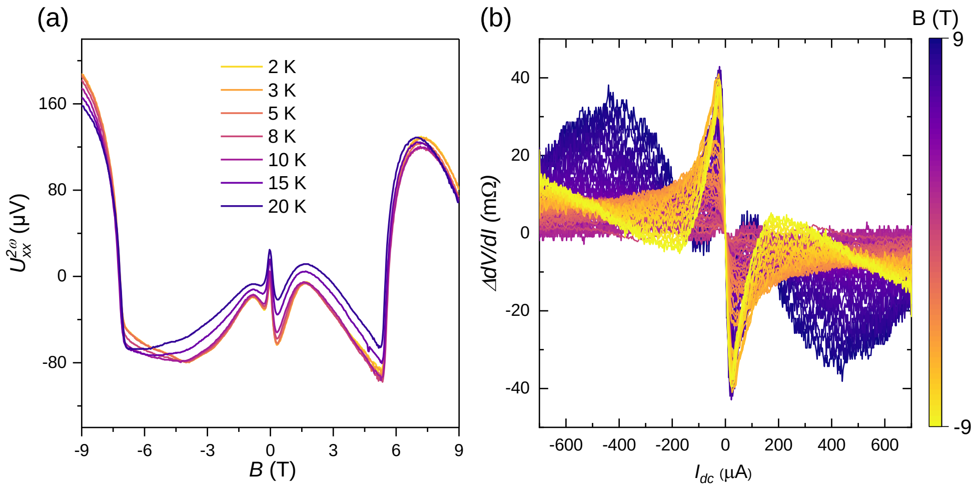

Fig. S4. The $\mathrm{d}V/\mathrm{d}I$ measurement in symmetry channel. (a) The nonreciprocal voltage conceives a giant response under negative and positive magnetic fields. (b) The $\Delta\mathrm{d}V/\mathrm{d}I$ as a function of the applied current shows large signals under negative and positive magnetic fields.

V. Angle-dependent nonreciprocal voltage

The nonreciprocal voltage $U_{xx}^{2\omega}$ current as a function of the magnetic field is shown in Fig. S5(a), the corresponding critical magnetic field is captured as points in Fig. S5(b), the function of $1/\cos(\theta)$ fits well with the data points as a function of the magnetic field, pointing to the two-dimension properties. When the current grows from the $I_{ac} = 20$ μA in Fig. S5(a) to $I_{ac} = 28$ μA in Fig. S5(b), the transition is destroyed by the large current. However, the persistent transition under various titled magnetic fields can still show two-dimension properties.

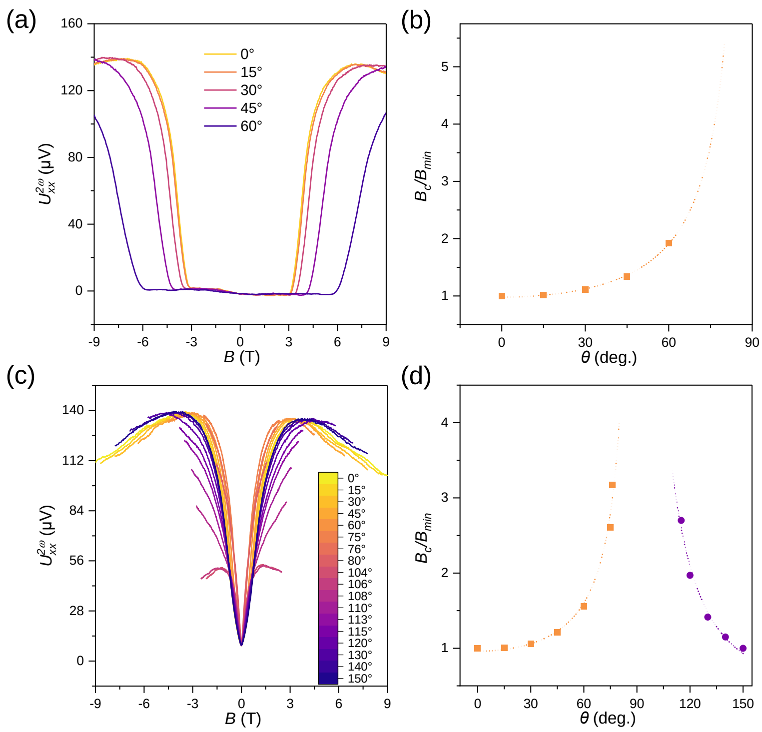

Fig. S5. Angle-dependent nonreciprocal voltage. (a) The angle-dependent nonreciprocal responses under a tilted magnetic field when the current $I_{ac} = 20$ μA. (b) The fitting of the critical magnetic field that happens the transition with the function of the $1/\cos(\theta)$. (c) The angle-dependent nonreciprocal responses under a tilted magnetic field when the current $I_{ac} = 28$ μA. (d) The fitting of the critical magnetic field that happens the transition with the function of the $1/\cos(\theta)$.

VI. The nonreciprocal voltage under high magnetic fields

The high magnetic field is not necessary to induce the nonreciprocal voltage transition as long as the current is large enough from the analysis in main text, while the large current I_{ac} involves thermoelectrical effect, it would be better that the nonreciprocal voltage transition can be induced under a high magnetic field with a small current I_{ac}. In Ag_2Te nanobelts with two second harmonic measurement channels, we perform the high magnetic field measurement, the data are shown in Fig S5. In the symmetric channel, as shown in Fig. S5(a), the quantum oscillations are still visible. As mentioned in the main text, the critical magnetic fields keep decreasing when the applied current changes from 1.5 μA to 25 μA. When the current is the largest, a very small magnetic field can induce the transition, and the nonreciprocal voltage reduces to zero gradually with the increased magnetic field. In asymmetric channels, a moderate current $I_{ac} = 5$ μA can push the system into the transition process, and the larger current can destroy the transition state when the magnetic field is very small.

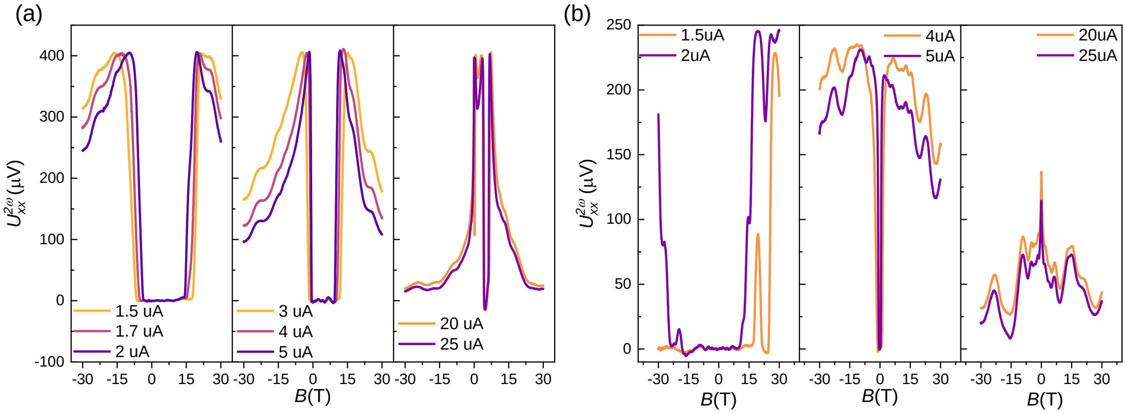

Fig. S5. The nonreciprocal voltage under high magnetic field. (a) The magnetic field-dependent nonreciprocal voltage when the current varies from 1.5 μA to 25 μA in the symmetric channel. (b) The magnetic field-dependent nonreciprocal voltage when the current varies from 1.5 μA to 25 μA in the asymmetric channel.

VII. References

[1] Tokura Y and Nagaosa N 2018 Nonreciprocal responses from non-centrosymmetric quantum materials *Nat Commun* **9** 3740

[2] Yasuda K, Morimoto T, Yoshimi R, Mogi M, Tsukazaki A, Kawamura M, Takahashi K S, Kawasaki M, Nagaosa N and Tokura Y 2020 Large non-reciprocal charge transport mediated by quantum anomalous Hall edge states *Nat. Nanotechnol.*