

Supplementary Materials for “Chiral anomaly enhanced Casimir interaction between Weyl semimetals”

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Here we show the detailed derivation of the reflection matrices, Eqs. (7) and (8) in the main text, by using the Berreman matrix method. The components of the electric field \mathbf{E} and the magnetic field \mathbf{H} parallel to the interface, denoted as $\psi(\mathbf{k}_\parallel, \omega, z) = (E_x, E_y, H_x, H_y)^\top$, have the following solutions in the vacuum,

$$\psi(\mathbf{k}_\parallel, \omega, z) = \mathbb{W} e^{\mathbb{K}z} \begin{pmatrix} V_\downarrow \\ V_\uparrow \end{pmatrix}, \quad (\text{S-1})$$

where

$$\mathbb{W} = \begin{pmatrix} k_3 \mathbb{I} & k_3 \mathbb{I} \\ \mathbb{Q} & -\mathbb{Q} \end{pmatrix}, \quad (\text{S-2})$$

is consists of the four eigenvectors of the Berreman matrix and $\mathbb{Q} = i \frac{\omega}{c} \begin{pmatrix} -\frac{c^2 k_x k_y}{\omega^2} & -1 + \frac{c^2 k_x^2}{\omega^2} \\ 1 - \frac{c^2 k_y^2}{\omega^2} & \frac{c^2 k_x k_y}{\omega^2} \end{pmatrix}$, $\mathbb{K} = \text{diag}(k_3, k_3, -k_3, -k_3)$ is the matrix consists of the four eigenvalues, $k_3 = \sqrt{k_x^2 + k_y^2 - \omega^2/c^2}$. \mathbb{I} is a 2×2 identity matrix. $(V_\downarrow, V_\uparrow)^\top$ is a column vector, the subscripts \downarrow and \uparrow refer to the propagation directions of EM waves, downward and upward, respectively.

In WSMs, the solution of Maxwell's equations take the same form of Eq. (S1), however, the eigenvector matrix \mathbb{W} and eigenvalue matrix \mathbb{K} take different forms for different situations. Here we consider two different cases, (1) the separation of Weyl nodes in the Brillouin zone, $2be^{(j)}$, is perpendicular to the interface, i.e., $\mathbf{e}^{(1)} = \mathbf{e}^{(2)} = \mathbf{e}_z$; (2) the separation of the two Weyl nodes is parallel to the interface, i.e., $\mathbf{e}^{(j)} = \mathbf{e}_x \cos \theta_j + \mathbf{e}_y \sin \theta_j$ ($j = 1, 2$). For the first case, the Berreman matrix is,

$$\mathbb{B} = i \begin{pmatrix} 0 & 0 & \frac{ck_x k_y}{\varepsilon \omega} & \frac{\omega}{c} - \frac{ck_x^2}{\varepsilon \omega} \\ 0 & 0 & -\frac{\omega}{c} + \frac{ck_y^2}{\varepsilon \omega} & -\frac{ck_x k_y}{\varepsilon \omega} \\ i\eta - \frac{ck_x k_y}{\omega} & -\frac{\varepsilon \omega}{c} + \frac{ck_x^2}{\omega} & 0 & 0 \\ \frac{\varepsilon \omega}{c} - \frac{ck_y^2}{\omega} & i\eta + \frac{ck_x k_y}{\omega} & 0 & 0 \end{pmatrix}. \quad (\text{S-3})$$

The eigen equation, $\mathbb{B}\mathbb{W} = \mathbb{W}\mathbb{K}$, has the following solutions,

$$\mathbb{W} = (W_1, W_2, W_3, W_4), \quad (\text{S-4})$$

$$W_1 = \begin{pmatrix} (-\epsilon \frac{\omega^2}{c^2} + k_x^2)p_3 \\ (k_x k_y - \sqrt{\varepsilon} \frac{\omega}{c} p_3)p_3 \\ -i\sqrt{\varepsilon}(-\varepsilon \frac{\omega^2}{c^2} + k_x^2)\lambda_1 \\ -i(\sqrt{\varepsilon} k_x k_y - \varepsilon \frac{\omega}{c} p_3)\lambda_1 \end{pmatrix}, \quad W_2 = \begin{pmatrix} (-\epsilon \frac{\omega^2}{c^2} + k_x^2)p_3 \\ (k_x k_y + \sqrt{\varepsilon} \frac{\omega}{c} p_3)p_3 \\ i\sqrt{\varepsilon}(-\varepsilon \frac{\omega^2}{c^2} + k_x^2)\lambda_2 \\ i(\sqrt{\varepsilon} k_x k_y + \varepsilon \frac{\omega}{c} p_3)\lambda_2 \end{pmatrix}, \quad (\text{S-5})$$

$$W_3 = \begin{pmatrix} (-\epsilon \frac{\omega^2}{c^2} + k_x^2)p_3 \\ (k_x k_y + \sqrt{\varepsilon} \frac{\omega}{c} p_3)p_3 \\ i\sqrt{\varepsilon}(-\varepsilon \frac{\omega^2}{c^2} + k_x^2)\lambda_3 \\ i(\sqrt{\varepsilon} k_x k_y + \varepsilon \frac{\omega}{c} p_3)\lambda_3 \end{pmatrix}, \quad W_4 = \begin{pmatrix} (-\epsilon \frac{\omega^2}{c^2} + k_x^2)p_3 \\ (k_x k_y - \sqrt{\varepsilon} \frac{\omega}{c} p_3)p_3 \\ -i\sqrt{\varepsilon}(-\varepsilon \frac{\omega^2}{c^2} + k_x^2)\lambda_4 \\ -i(\sqrt{\varepsilon} k_x k_y - \varepsilon \frac{\omega}{c} p_3)\lambda_4 \end{pmatrix}, \quad (\text{S-6})$$

where the corresponding eigenvalues $\lambda_1 = -\lambda_4 = \sqrt{p_3[p_3 - i\eta/\sqrt{\varepsilon}]}$, $\lambda_2 = -\lambda_3 = \sqrt{p_3[p_3 + i\eta/\sqrt{\varepsilon}]}$, $p_3 = \sqrt{\mathbf{k}_\parallel^2 - \varepsilon \omega^2/c^2}$, $\varepsilon = \varepsilon(\omega)$ is the dielectric function of WSM. The eigenvalue matrix,

$$\mathbb{K} = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4). \quad (\text{S-7})$$

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For the second case, the Berreman matrix is,

$$\mathbb{B}^{(j)} = \begin{pmatrix} \eta \frac{ck_x}{\varepsilon\omega} \sin \theta_j & -\eta \frac{ck_x}{\varepsilon\omega} \sin \theta_j & i \frac{ck_x k_y}{\varepsilon\omega} & i \frac{\omega}{c} - i \frac{ck_x^2}{\varepsilon\omega} \\ \eta \frac{ck_y}{\varepsilon\omega} \sin \theta_j & -\eta \frac{ck_y}{\varepsilon\omega} \sin \theta_j & -i \frac{\omega}{c} + i \frac{ck_y^2}{\varepsilon\omega} & -i \frac{ck_x k_y}{\varepsilon\omega} \\ -i \frac{ck_x k_y}{\omega} - i \frac{\eta^2 c}{\varepsilon\omega} \sin \theta_j \cos \theta_j & -i \frac{\varepsilon\omega}{c} + i \frac{ck_x}{\omega} + i \frac{\eta^2 c}{\varepsilon\omega} \cos^2 \theta_j & \eta \frac{ck_y}{\varepsilon\omega} \cos \theta_j & -\eta \frac{ck_x}{\varepsilon\omega} \cos \theta_j \\ i \frac{\varepsilon\omega}{c} - i \frac{ck_x^2}{\omega} - i \frac{\eta^2 c}{\varepsilon\omega} \sin^2 \theta_j & i \frac{ck_x k_y}{\omega} + i \frac{\eta^2 c}{\varepsilon\omega} \sin \theta_j \cos \theta_j & \eta \frac{ck_y}{\varepsilon\omega} \sin \theta_j & -\eta \frac{ck_x}{\varepsilon\omega} \sin \theta_j \end{pmatrix}. \quad (\text{S-8})$$

The eigen equation, $\mathbb{B}^{(j)} \mathbb{W}^{(j)} = \mathbb{W}^{(j)} \mathbb{K}^{(j)}$, has the following solutions,

$$\mathbb{W}^{(j)} = (W_1^{(j)}, W_2^{(j)}, W_3^{(j)}, W_4^{(j)}), \quad (\text{S-9})$$

$$W_1^{(j)} = \begin{pmatrix} i[\eta(\eta + \delta^{(j)}) + 2\varepsilon k_y^2] \frac{\omega}{c} \sin \theta_j + i[\lambda_1^{(j)}(\eta + \delta^{(j)}) + 2\varepsilon \frac{\omega}{c} k_y \cos \theta_j] k_x \\ -i[\eta(\eta + \delta^{(j)}) + 2\varepsilon k_x^2] \frac{\omega}{c} \cos \theta_j + i[\lambda_1^{(j)}(\eta + \delta^{(j)}) - 2\varepsilon \frac{\omega}{c} k_x \sin \theta_j] k_y \\ [\eta(\eta + \delta^{(j)}) + 2\varepsilon k_x^2] \lambda_1^{(j)} \cos \theta_j + \varepsilon[\frac{\omega}{c}(\eta + \delta^{(j)}) + 2\lambda_1^{(j)} k_x \sin \theta_j] k_y \\ [\eta(\eta + \delta^{(j)}) + 2\varepsilon k_y^2] \lambda_1^{(j)} \sin \theta_j - \varepsilon[\frac{\omega}{c}(\eta + \delta^{(j)}) - 2\lambda_1^{(j)} k_y \cos \theta_j] k_x \end{pmatrix}, \quad (\text{S-10})$$

$$W_2^{(j)} = \begin{pmatrix} i[\eta(\eta - \delta^{(j)}) + 2\varepsilon k_y^2] \frac{\omega}{c} \sin \theta_j + i[\lambda_2^{(j)}(\eta - \delta^{(j)}) + 2\varepsilon \frac{\omega}{c} k_y \cos \theta_j] k_x \\ -i[\eta(\eta - \delta^{(j)}) + 2\varepsilon k_x^2] \frac{\omega}{c} \cos \theta_j + i[\lambda_2^{(j)}(\eta - \delta^{(j)}) - 2\varepsilon \frac{\omega}{c} k_x \sin \theta_j] k_y \\ [\eta(\eta - \delta^{(j)}) + 2\varepsilon k_x^2] \lambda_2^{(j)} \cos \theta_j + \varepsilon[\frac{\omega}{c}(\eta - \delta^{(j)}) + 2\lambda_2^{(j)} k_x \sin \theta_j] k_y \\ [\eta(\eta - \delta^{(j)}) + 2\varepsilon k_y^2] \lambda_2^{(j)} \sin \theta_j - \varepsilon[\frac{\omega}{c}(\eta - \delta^{(j)}) - 2\lambda_2^{(j)} k_y \cos \theta_j] k_x \end{pmatrix}, \quad (\text{S-11})$$

$$W_3^{(j)} = \begin{pmatrix} i[\eta(\eta - \delta^{(j)}) + 2\varepsilon k_y^2] \frac{\omega}{c} \sin \theta_j + i[\lambda_3^{(j)}(\eta - \delta^{(j)}) + 2\varepsilon \frac{\omega}{c} k_y \cos \theta_j] k_x \\ -i[\eta(\eta - \delta^{(j)}) + 2\varepsilon k_x^2] \frac{\omega}{c} \cos \theta_j + i[\lambda_3^{(j)}(\eta - \delta^{(j)}) - 2\varepsilon \frac{\omega}{c} k_x \sin \theta_j] k_y \\ [\eta(\eta - \delta^{(j)}) + 2\varepsilon k_x^2] \lambda_3^{(j)} \cos \theta_j + \varepsilon[\frac{\omega}{c}(\eta - \delta^{(j)}) + 2\lambda_3^{(j)} k_x \sin \theta_j] k_y \\ [\eta(\eta - \delta^{(j)}) + 2\varepsilon k_y^2] \lambda_3^{(j)} \sin \theta_j - \varepsilon[\frac{\omega}{c}(\eta - \delta^{(j)}) - 2\lambda_3^{(j)} k_y \cos \theta_j] k_x \end{pmatrix}, \quad (\text{S-12})$$

$$W_4^{(j)} = \begin{pmatrix} i[\eta(\eta + \delta^{(j)}) + 2\varepsilon k_y^2] \frac{\omega}{c} \sin \theta_j + i[\lambda_4^{(j)}(\eta + \delta^{(j)}) + 2\varepsilon \frac{\omega}{c} k_y \cos \theta_j] k_x \\ -i[\eta(\eta + \delta^{(j)}) + 2\varepsilon k_x^2] \frac{\omega}{c} \cos \theta_j + i[\lambda_4^{(j)}(\eta + \delta^{(j)}) - 2\varepsilon \frac{\omega}{c} k_x \sin \theta_j] k_y \\ [\eta(\eta + \delta^{(j)}) + 2\varepsilon k_x^2] \lambda_4^{(j)} \cos \theta_j + \varepsilon[\frac{\omega}{c}(\eta + \delta^{(j)}) + 2\lambda_4^{(j)} k_x \sin \theta_j] k_y \\ [\eta(\eta + \delta^{(j)}) + 2\varepsilon k_y^2] \lambda_4^{(j)} \sin \theta_j - \varepsilon[\frac{\omega}{c}(\eta + \delta^{(j)}) - 2\lambda_4^{(j)} k_y \cos \theta_j] k_x \end{pmatrix}. \quad (\text{S-13})$$

The corresponding eigenvalues are, $\lambda_1^{(j)} = -\lambda_4^{(j)} = \sqrt{p_3^2 + \frac{\eta}{2\varepsilon}(\eta + \delta^{(j)})}$, $\lambda_2^{(j)} = -\lambda_3^{(j)} = \sqrt{p_3^2 + \frac{\eta}{2\varepsilon}(\eta - \delta^{(j)})}$, $\delta^{(j)} = \sqrt{\eta^2 + 4\varepsilon(k_x \cos \theta_j + k_y \sin \theta_j)^2}$. The eigenvalue matrix for the j -th WSM slab is given by,

$$\mathbb{K}^{(j)} = \text{diag}(\lambda_1^{(j)}, \lambda_2^{(j)}, \lambda_3^{(j)}, \lambda_4^{(j)}). \quad (\text{S-14})$$

For finite thickness WSMs, there are two boundaries, the wave function ψ is continuous on the boundaries. For the first WSM, these boundary conditions are,

$$\mathbb{W} \begin{pmatrix} 0 \\ E_{\uparrow}^{[\text{inj}]} \end{pmatrix} + \mathbb{W} \begin{pmatrix} E_{\downarrow}^{[\text{ref}]} \\ 0 \end{pmatrix} = \mathbb{W}^{(1)} \begin{pmatrix} E_{\downarrow}^{[\text{WSM}]} \\ E_{\uparrow}^{[\text{WSM}]} \end{pmatrix}, \quad (\text{S-15})$$

$$\mathbb{W}^{(1)} e^{\mathbb{K}^{(1)} d} \begin{pmatrix} E_{\downarrow}^{[\text{WSM}]} \\ E_{\uparrow}^{[\text{WSM}]} \end{pmatrix} = \mathbb{W} \begin{pmatrix} 0 \\ E_{\uparrow}^{[\text{tran}]} \end{pmatrix} \quad (\text{S-16})$$

where $E_{\uparrow}^{[\text{inj}]}$ and $E_{\downarrow}^{[\text{ref}]}$ denote the injection and the reflection EM wave from the vacuum to the first WSM, $(E_{\downarrow}^{[\text{WSM}]}, E_{\uparrow}^{[\text{WSM}]})^T$ is the EM wave in the first WSM, $E_{\uparrow}^{[\text{tran}]}$ is the transmission EM wave. The reflection matrix is defined as,

$$E_{\downarrow}^{[\text{ref}]} = \mathbb{R}_{\text{Cart}}^{(1)} E_{\uparrow}^{[\text{inj}]} . \quad (\text{S-17})$$

Solving Eqs. (S15) and (S16), we get Eq. (7) in the main text. Utilizing the same method, we can get Eq. (8).