Supplementary Material: Nonlinear Hall Effect in Antiferromagnetic Half-Heusler Materials

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(Dated: April 26, 2021)

TABLE S1. Parameters for Fig. 2(a), where a is a real parameter with the unit of length.

$\frac{E_v+\xi_0}{\xi_0}$	γ_1	γ_2	γ_3	$\frac{C}{\beta_c/a}$
0	0	0.5	0.1	0.2

TABLE S2. Parameters for Fig. 2(b) and Fig. 3(a). Here, $\xi_1 = 0$, $\xi_2 = 0$, $\xi_3 = \xi_4 = \xi_5 = \xi$. The energy unit ϵ_0 is defined as $\epsilon_0 \equiv \beta_c/a^2$.

$\frac{E_v+\xi_0}{\xi_0}$	γ_1	γ_2	γ_3	$\frac{C}{\beta_c/a}$	$\frac{\xi}{\epsilon_0}$
0	0	0.5	0.1	0.2	−0.025

TABLE S3. Parameters for Fig. 2(c3) and Fig. 3(b). Here, $\xi_2 = 0$, $\xi_4 = \frac{\sqrt{\sqrt{3}\gamma_3\xi_1\xi_3+\gamma_2\xi_3^2}}{\sqrt{\gamma_2}}$, $\xi_5 = \xi_4$.

$\frac{E_v+\xi_0}{\xi_0}$	γ_1	γ_2	γ_3	$\frac{C}{\beta_c/a}$	$\frac{\xi_1}{\epsilon_0}$	$\frac{\xi_3}{\epsilon_0}$
0	0	0.5	0.1	0.2	−0.16	−0.01

TABLE S4. Parameters for Fig. 2(c4), $\xi_4 = \xi_3$, $\xi_1 = \gamma_2\frac{\xi_5^2-\xi_3^2}{2\sqrt{3}\gamma_3\xi_5}$, $\xi_2 = -\sqrt{3}\xi_1$.

$\frac{E_v+\xi_0}{\xi_0}$	γ_1	γ_2	γ_3	$\frac{C}{\beta_c/a}$	$\frac{\xi_3}{\epsilon_0}$	$\frac{\xi_5}{\epsilon_0}$
0	0	0.5	0.1	0.02	0.001	−0.002

I. PARAMETERS IN THE $k \cdot p$ MODEL

This section is a summary of parameters listed in Table S1, S2, S3, and S4 for the models used in the main text.

II. EXPLICIT FORM OF SPIN-3/2 MATRICES AND ξ_i PARAMETERS IN THE HAMILTONIAN

The matrix form of the spin-3/2 matrices is given by

$$J_x = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}, \tag{1a}$$

$$J_y = \frac{\mathrm{I}}{2}\begin{pmatrix} 0 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & -2 & 0 \\ 0 & 2 & 0 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}, \tag{1b}$$

$$J_z = \frac{1}{2}\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}. \tag{1c}$$

The expressions for ξ_i ($i = 0, 1, 2, 3, 4, 5$) are polynomials of magnetic moment $\mathbf{M}$, listed as follows: [1]

$$\begin{aligned}
\xi_0 =&\alpha_1\left(M_x^2+M_y^2+M_z^2\right)+\beta_1\left(M_yM_z+M_zM_x+M_xM_y\right),\\
\xi_1 =&\alpha_3\left(2M_z^2-M_x^2-M_y^2\right)+\beta_3\left(2M_xM_y-M_yM_z-M_zM_x\right),\\
\xi_2 =&\alpha_3\sqrt{3}\left(M_x^2-M_y^2\right)+\beta_3\sqrt{3}\left(M_yM_z-M_xM_z\right),\\
\xi_3 =&\alpha_2\left(M_x^2+M_y^2+M_z^2\right)+\beta_2\left(M_yM_z+M_zM_x+M_xM_y\right)+2[\alpha_4\left(2M_z^2-M_x^2-M_y^2\right)+\beta_4\left(2M_xM_y-M_yM_z-M_zM_x\right)],\\
\xi_4 =&\alpha_2\left(M_x^2+M_y^2+M_z^2\right)+\beta_2\left(M_yM_z+M_zM_x+M_xM_y\right)-\left[\alpha_4\left(2M_z^2-M_x^2-M_y^2\right)+\beta_4\left(2M_xM_y-M_yM_z-M_zM_x\right)\right]\\
&-\sqrt{3}\left[\alpha_4\sqrt{3}\left(M_x^2-M_y^2\right)+\beta_4\sqrt{3}\left(M_yM_z-M_xM_z\right)\right],\\
\xi_5 =&\alpha_2\left(M_x^2+M_y^2+M_z^2\right)+\beta_2\left(M_yM_z+M_zM_x+M_xM_y\right)-\left[\alpha_4\left(2M_z^2-M_x^2-M_y^2\right)+\beta_4\left(2M_xM_y-M_yM_z-M_zM_x\right)\right]\\
&+\sqrt{3}\left[\alpha_4\sqrt{3}\left(M_x^2-M_y^2\right)+\beta_4\sqrt{3}\left(M_yM_z-M_xM_z\right)\right].
\end{aligned} \tag{2}$$

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[1] J. Yu, B. Yan, and C.-X. Liu, Phys. Rev. B **95**, 235158 (2017).