SUPPLEMENTAL MATERIAL FOR "SYMMETRY-PROTECTED SCATTERING IN NON-HERMITIAN LINEAR SYSTEMS"

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A: Proof of the constraints on the scattering coefficients for discrete symmetries

Symmetric scattering coefficients.—The transmission and reflection coefficients for the input-output in the leads m and n are

$$t_L = \frac{\Delta_{nm}^{-1} J^{-1}(e^{ik} - e^{-ik})}{(J^{-1} + \Delta_{mm}^{-1} e^{ik})(J^{-1} + \Delta_{nn}^{-1} e^{ik}) - \Delta_{mn}^{-1}\Delta_{nm}^{-1} e^{2ik}}, \tag{1}$$

$$r_L = \frac{\Delta_{mn}^{-1}\Delta_{nm}^{-1} - (J^{-1}e^{ik} + \Delta_{mm}^{-1})(J^{-1}e^{-ik} + \Delta_{nn}^{-1})}{(J^{-1} + \Delta_{mm}^{-1} e^{ik})(J^{-1} + \Delta_{nn}^{-1} e^{ik}) - \Delta_{mn}^{-1}\Delta_{nm}^{-1} e^{2ik}}. \tag{2}$$

$$t_R = \frac{\Delta_{mn}^{-1} J^{-1}(e^{ik} - e^{-ik})}{(J^{-1} + \Delta_{mm}^{-1} e^{ik})(J^{-1} + \Delta_{nn}^{-1} e^{ik}) - \Delta_{mn}^{-1}\Delta_{nm}^{-1} e^{2ik}}, \tag{3}$$

$$r_R = \frac{\Delta_{mn}^{-1}\Delta_{nm}^{-1} - (J^{-1}e^{ik} + \Delta_{nn}^{-1})(J^{-1}e^{-ik} + \Delta_{mm}^{-1})}{(J^{-1} + \Delta_{mm}^{-1} e^{ik})(J^{-1} + \Delta_{nn}^{-1} e^{ik}) - \Delta_{mn}^{-1}\Delta_{nm}^{-1} e^{2ik}}, \tag{4}$$

where $\Delta = H_c + (\omega_0 - \omega)\,\mathbf{1} = H_c - (2J\cos k)\,\mathbf{1}$, Δ^{-1} indicates the inverse of Δ, and Δ_{mn}^{-1} indicates the element on the m-th row and the n-th column of Δ^{-1}.

It is obvious that

$$t_L = t_R \text{ for } \Delta_{nm}^{-1} = \Delta_{mn}^{-1}, \tag{5}$$

$$r_L = r_R \text{ for } \Delta_{mm}^{-1} = \Delta_{nn}^{-1}. \tag{6}$$

Alternatively,

$$|t_L|^2 = |t_R|^2 \text{ for } \left|\Delta_{nm}^{-1}\right| = \left|\Delta_{mn}^{-1}\right|. \tag{7}$$

The difference between the numerators of $|r_L|^2$ and $|r_R|^2$ is given by

$$\begin{aligned} &\left|\Delta_{mn}^{-1}\Delta_{nm}^{-1} - (J^{-1}e^{ik} + \Delta_{mm}^{-1})(J^{-1}e^{-ik} + \Delta_{nn}^{-1})\right|^2 - \left|\Delta_{mn}^{-1}\Delta_{nm}^{-1} - (J^{-1}e^{ik} + \Delta_{nn}^{-1})(J^{-1}e^{-ik} + \Delta_{mm}^{-1})\right|^2 \\ &= 2J^{-2} i \sin(2k)\left(\Delta_{nn}^{-1}\Delta_{mm}^{-1*} - \Delta_{mm}^{-1}\Delta_{nn}^{-1*}\right) \\ &\quad + 2J^{-1} i \sin k [\left(J^{-2} + \Delta_{mm}^{-1*}\Delta_{nn}^{-1*} - \Delta_{mn}^{-1*}\Delta_{nm}^{-1*}\right)\left(\Delta_{nn}^{-1} - \Delta_{mm}^{-1}\right) - \left(J^{-2} + \Delta_{mm}^{-1}\Delta_{nn}^{-1} - \Delta_{mn}^{-1}\Delta_{nm}^{-1}\right)\left(\Delta_{nn}^{-1*} - \Delta_{mm}^{-1*}\right)]. \end{aligned} \tag{8}$$

Thus, we have $|r_L|^2 = |r_R|^2$ for real Δ_{mm}^{-1}, Δ_{nn}^{-1}, and $\Delta_{mn}^{-1}\Delta_{nm}^{-1}$. These are the conclusions of Eqs. (9) and (10) in the main text. Notably, the accidental symmetric reflection $|r_L|^2 = |r_R|^2$ occurs if the conditions $J^{-2} + \Delta_{mm}^{-1*}\Delta_{nn}^{-1*} - \Delta_{mn}^{-1*}\Delta_{nm}^{-1*} = 0$ and $\Delta_{nn}^{-1}\Delta_{mm}^{-1*} = \Delta_{mm}^{-1}\Delta_{nn}^{-1*}$ are simultaneously satisfied.

Symmetry-protected scattering.—The mapping relations are

$$\mathbf{1} : U_{\mathbf{1}}[\cdots, |m\rangle_c, \cdots, |n\rangle_c, \cdots]^T = [\cdots, |m\rangle_c, \cdots, e^{i\alpha}|n\rangle_c, \cdots]^T, \tag{9}$$

$$\mathcal{I} : U_{\mathcal{I}}[\cdots, |m\rangle_c, \cdots, |n\rangle_c, \cdots]^T = [\cdots, |n\rangle_c, \cdots, e^{i\alpha}|m\rangle_c, \cdots]^T. \tag{10}$$

In the general situation, the phase factor $e^{i\alpha}$ is not necessarily to be 1. The four elements in the unitary operators that relate to the mapping between the connection sites $|m\rangle_c$ and $|n\rangle_c$ are

$$\mathbf{1} : \begin{pmatrix} U_{\mathbf{1},mm} & U_{\mathbf{1},nm} \\ U_{\mathbf{1},mn} & U_{\mathbf{1},nn} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}, \tag{11}$$

$$\mathcal{I} : \begin{pmatrix} U_{\mathcal{I},mm} & U_{\mathcal{I},nm} \\ U_{\mathcal{I},mn} & U_{\mathcal{I},nn} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ e^{i\alpha} & 0 \end{pmatrix}. \tag{12}$$

Other relevant elements in the mapping relation are $U_{\mathbf{1},mj}=0$ and $U_{\mathbf{1},jn}=0$ for $j\neq m,n$; $U_{\mathcal{I},mj}=0$ and $U_{\mathcal{I},jn}=0$ for $j\neq m,n$.

The scattering center with the even-parity C symmetry satisfies $H_c = cH_c^T c^{-1}$. From the definition $\Delta = H_c-(2J\cos k)\,\mathbf{1}$, we have the relation $\Delta = c\Delta^T c^{-1}$; therefore, we obtain

$$\Delta^{-1}=c\left(\Delta^{-1}\right)^T c^{-1}. \tag{13}$$

For the $C_{\mathbf{1}}$ symmetry, we have

$$\Delta_{nm}^{-1}=e^{i\alpha}\Delta_{mn}^{-1}, \tag{14}$$

thus, we obtain the symmetric transmission

$$|t_L|=|t_R|\,. \tag{15}$$

For the $C_{\mathcal{I}}$ symmetry, we have

$$\Delta_{mm}^{-1}=\Delta_{nn}^{-1}, \tag{16}$$

besides, the $C_{\mathcal{I}}$ symmetry also demands $e^{i\alpha}=1$ or $\Delta_{mn}^{-1}=\Delta_{nm}^{-1}=0$. Thus, in general case we only have the symmetric reflection

$$r_L=r_R. \tag{17}$$

The scattering center with the even-parity K symmetry satisfies $H_c=\Bbbk H_c^*\Bbbk^{-1}$. Therefore, we obtain the relation $\Delta=\Bbbk\Delta^*\Bbbk^{-1}$; and consequently,

$$\Delta^{-1}=\Bbbk\left(\Delta^{-1}\right)^*\Bbbk^{-1}. \tag{18}$$

For the $K_{\mathbf{1}}$ symmetry, we obtain

$$\Delta_{mm}^{-1}=(\Delta_{mm}^{-1})^*,\Delta_{nn}^{-1}=(\Delta_{nn}^{-1})^*;\Delta_{nm}^{-1}=e^{i\alpha}\left(\Delta_{nm}^{-1}\right)^*,\Delta_{mn}^{-1}=e^{-i\alpha}(\Delta_{mn}^{-1})^*, \tag{19}$$

thus, Δ_{mm}^{-1}, Δ_{nn}^{-1}, and $\Delta_{mn}^{-1}\Delta_{nm}^{-1}$ are all real numbers; and we have symmetric reflection

$$|r_L|=|r_R|\,. \tag{20}$$

For the $K_{\mathcal{I}}$ symmetry, we obtain

$$\Delta_{nm}^{-1}=e^{i\alpha}\left(\Delta_{mn}^{-1}\right)^*, \tag{21}$$

thus, we have symmetric transmission

$$|t_L|=|t_R|\,. \tag{22}$$

we also have $\Delta_{mm}^{-1}=\left(\Delta_{nn}^{-1}\right)^*$, which does not lead to a symmetric relation on the scattering coefficients.

The scattering center with the even-parity Q symmetry satisfies $H_c=qH_c^\dagger q^{-1}$. Therefore, we obtain $\Delta=q\Delta^\dagger q^{-1}$; and consequently

$$\Delta^{-1}=q\left(\Delta^{-1}\right)^\dagger q^{-1}. \tag{23}$$

For the $Q_{\mathbf{1}}$ symmetry, we have

$$\Delta_{mm}^{-1}=(\Delta_{mm}^{-1})^*,\Delta_{nn}^{-1}=(\Delta_{nn}^{-1})^*;\Delta_{nm}^{-1}=e^{i\alpha}\left(\Delta_{mn}^{-1}\right)^*,\Delta_{mn}^{-1}=e^{-i\alpha}(\Delta_{nm}^{-1})^*, \tag{24}$$

the $Q_{\mathbf{1}}$ symmetry also requires $e^{2i\alpha}=1$. From $\Delta_{nm}^{-1}=e^{i\alpha}\left(\Delta_{mn}^{-1}\right)^{*}$, we obtain $|t_L|=|t_R|$. Notice that Δ_{mm}^{-1}, Δ_{nn}^{-1}, and $\Delta_{mn}^{-1}\Delta_{nm}^{-1}$ are all real numbers; thus, we have both the symmetric transmission and reflection

$$|t_L|=|t_R|\,,|r_L|=|r_R|\,. \tag{25}$$

For the $Q_{\mathcal{I}}$ symmetry, we have

$$\Delta_{mm}^{-1}=\left(\Delta_{nn}^{-1}\right)^{*},\Delta_{nm}^{-1}=e^{i\alpha}\left(\Delta_{nm}^{-1}\right)^{*},\Delta_{mn}^{-1}=e^{-i\alpha}\left(\Delta_{mn}^{-1}\right)^{*}. \tag{26}$$

These constraints are insufficient to result in the symmetric transmission as well as the symmetric reflection.

The scattering center with the even-parity P symmetry satisfies $H_c=pH_cp^{-1}$; therefore, we obtain $\Delta=p\Delta p^{-1}$ and

$$\Delta^{-1}=p\left(\Delta^{-1}\right)p^{-1}. \tag{27}$$

For the $P_{\mathbf{1}}$ symmetry, we have

$$\Delta_{nm}^{-1}=e^{i\alpha}\Delta_{nm}^{-1},\Delta_{mn}^{-1}=e^{-i\alpha}\Delta_{mn}^{-1}. \tag{28}$$

These constraints are insufficient to result in the symmetric transmission as well as the symmetric reflection. For the $P_{\mathcal{I}}$ symmetry, we have

$$\Delta_{mm}^{-1}=\Delta_{nn}^{-1},\Delta_{nm}^{-1}=e^{i\alpha}\Delta_{mn}^{-1}. \tag{29}$$

Thus, we have both the symmetric transmission and reflection

$$|t_L|=|t_R|\,,r_L=r_R. \tag{30}$$

These conclusions are summarized in Supplemental Table I and give the symmetric constraints on the scattering coefficients listed in Table I of the main text.

Supplemental Table I. Symmetry-protected constraints on the transmission and reflection coefficients.

Symmetry	$C_{\mathbf{1}}$	$C_{\mathcal{I}}$	$K_{\mathbf{1}}$	$K_{\mathcal{I}}$	$Q_{\mathbf{1}}$	$P_{\mathcal{I}}$	$Q_{\mathcal{I}}, P_{\mathbf{1}}$
Constraint	$\|t_L\|=\|t_R\|$	$r_L=r_R$	$\|r_L\|=\|r_R\|$	$\|t_L\|=\|t_R\|$	$\|t_L\|=\|t_R\|,\|r_L\|=\|r_R\|$	$\|t_L\|=\|t_R\|,r_L=r_R$	None

These conclusions are straightforwardly applicable to explain the many intriguing symmetric/asymmetric scattering phenomena reported in the literatures. For example, the $C_{\mathbf{1}}$ symmetry protected symmetric transmission [1, 2]; the $C_{\mathcal{I}}$ symmetry protected symmetric reflection [3–9]; the $K_{\mathbf{1}}$ symmetry protected symmetric reflection [10, 11]; the $K_{\mathcal{I}}$ symmetry protected symmetric transmission [11–61]; the $Q_{\mathbf{1}}$ symmetry protected symmetric transmission and reflection [62, 63]; and the $P_{\mathcal{I}}$ symmetry protected symmetric transmission and reflection [64–71]. Without the protection of these symmetries, both the transmission and reflection are asymmetric [8, 72, 73].

B: Details for the light scattering in the three-coupled-resonator scattering center

The scattering center of the three coupled resonators is schematically illustrated in the main text Fig. 2(a). The scattering center encloses a synthetic magnetic flux ϕ induced by the Peierls phase factor in the coupling. We now analyze the properties of the scattering center through the two-port scattering dynamics. The leads 1 and 3 are under consideration; the lead 2 is disconnected from the scattering center.

For the case $\{V_1, V_2, V_3\} = \{i\gamma, -i\gamma, 0\}$, the Hamiltonian H_c is not symmetry-protected. The scattering coefficients are obtained in the form of

$$t_L = \frac{J(e^{2ik}-1)(Je^{i\phi}+i\gamma+2J\cos k)}{(2-e^{-2ik}+e^{2ik}+e^{i(k-\phi)}+e^{i(k+\phi)})J^2+iJ\gamma e^{ik}-\gamma^2}, \tag{31}$$

$$r_L = -\frac{(1+e^{2ik}+e^{i(k-\phi)}+e^{i(k+\phi)})J^2+iJ\gamma e^{-ik}-\gamma^2}{(2-e^{-2ik}+e^{2ik}+e^{i(k-\phi)}+e^{i(k+\phi)})J^2+iJ\gamma e^{ik}-\gamma^2}, \tag{32}$$

$$t_R = \frac{Je^{-i\phi}(e^{2ik}-1)(J+i\gamma e^{i\phi}+2Je^{i\phi}\cos k)}{(2-e^{-2ik}+e^{2ik}+e^{i(k-\phi)}+e^{i(k+\phi)})J^2+iJ\gamma e^{ik}-\gamma^2}, \tag{33}$$

$$r_R = -\frac{(1+e^{2ik}+e^{i(k-\phi)}+e^{i(k+\phi)})J^2+iJ\gamma e^{3ik}-\gamma^2 e^{2ik}}{(2-e^{-2ik}+e^{2ik}+e^{i(k-\phi)}+e^{i(k+\phi)})J^2+iJ\gamma e^{ik}-\gamma^2}. \tag{34}$$

The scattering coefficients are depicted in Figs. 2(b) and 2(c) of the main text; and they diverge at $J = \gamma/2$ and $\phi = \pm\pi/2$ for the resonant inputs at the momentum $k = -\pi/2$. In the general case, the transmission and reflection are asymmetric; these are reflected from the four elements of the inverse matrix Δ^{-1}

$$\begin{pmatrix} \Delta^{-1}_{mm} & \Delta^{-1}_{mn} \\ \Delta^{-1}_{nm} & \Delta^{-1}_{nn} \end{pmatrix} = \frac{J^2}{\det\Delta}\begin{pmatrix} 1+e^{2ik}+e^{-2ik}+i(\gamma/J)\left(e^{ik}+e^{-ik}\right) & e^{ik}+e^{-ik}+e^{-i\phi}+i(\gamma/J) \\ e^{ik}+e^{-ik}+e^{i\phi}+i(\gamma/J) & 1+e^{2ik}+e^{-2ik}+(\gamma/J)^2 \end{pmatrix}. \tag{35}$$

where $\Delta = H_c - (2J\cos k)\,\mathbf{1}$. The determinant is $\det\Delta = J^3\left[e^{-i\phi}+e^{i\phi}-e^{-3ik}-e^{3ik}-\left(\gamma^2/J^2\right)\left(e^{-ik}+e^{ik}\right)\right]$. Then, we obtain the contrast between the transmission and reflection of the inputs from the forward and backward directions for the resonant incidence with the momentum $k = -\pi/2$

$$\frac{t_R}{t_L} = \frac{e^{-i\phi}+i\gamma/J}{e^{i\phi}+i\gamma/J}, \frac{r_R}{r_L} = \frac{2\cos\phi - i\gamma/J + i\gamma^2/J^2}{2\cos\phi - i\gamma/J - i\gamma^2/J^2}. \tag{36}$$

At $\gamma = J$, the special cases of $\phi = -\pi/2$ for a unidirectional transmissionless $t_L = 0$, $r_L = 1$; $t_R = -2i$, $r_R = 0$ and $\phi = \pi/2$ for a unidirectional absorption $t_L = -2i$, $r_L = 1$; $t_R = 0$, $r_R = 0$ are shown in the main text Fig. 3.

C: Non-Hermitian three-coupled-resonator scattering center with dissipative coupling

In the coupled mode theory, the equations of motion for the three-site scattering center with dissipative coupling $-i\kappa$ are given by

$$i\dot{\phi}_c^k(1) = \omega_0\phi_c^k(1) - i\kappa\phi_c^k(2) + J\phi_c^k(3) + J\phi_1^k(-1), \tag{37}$$

$$i\dot{\phi}_c^k(2) = \omega_0\phi_c^k(2) - i\kappa\phi_c^k(1) + Je^{-i\phi}\phi_c^k(3), \tag{38}$$

$$i\dot{\phi}_c^k(3) = \omega_0\phi_c^k(3) + J\phi_c^k(1) + Je^{i\phi}\phi_c^k(2) + J\phi_3^k(1), \tag{39}$$

where $\phi_1^k(-1)$ and $\phi_3^k(1)$ are the wavefunctions of the sites on the leads 1 and 3 that are connected with the scattering center sites 1 and 3, respectively. The dissipative coupling is reciprocal and is directly induced by the dissipation in the link resonator between the primary resonators 1 and 2 [74]. The link resonator and the primary resonators are on resonant. The effective dissipative coupling strength $\kappa = \kappa_0^2/\gamma$ is inversely proportional to the link resonator dissipation γ and quadratically proportional to the coupling strength between the link resonator and the primary resonators κ_0 [75, 76]. The dissipative coupling has also been experimentally realized in many anti-parity-time symmetric systems [77–80].

The scattering center is described by the Hamiltonian

$$H_c = \begin{pmatrix} 0 & -i\kappa & J \\ -i\kappa & 0 & Je^{-i\phi} \\ J & Je^{i\phi} & 0 \end{pmatrix}, \tag{40}$$

the scattering properties of which are not symmetry-protected.

The contrast between the transmission and reflection from opposite incident directions are calculated as follows. We obtain the four elements of the inverse matrix Δ^{-1} in the form of

$$\begin{pmatrix} \Delta^{-1}_{mm} & \Delta^{-1}_{mn} \\ \Delta^{-1}_{nm} & \Delta^{-1}_{nn} \end{pmatrix} = \frac{J^2}{\det \Delta} \begin{pmatrix} e^{2ik}+1+e^{-2ik} & e^{ik}+e^{-ik}-i\left(\kappa/J\right)e^{-i\phi} \\ e^{ik}+e^{-ik}-i\left(\kappa/J\right)e^{i\phi} & e^{2ik}+2+e^{-2ik}+\left(\kappa/J\right)^2 \end{pmatrix}. \tag{41}$$

In general case, the transmission and reflection are asymmetric.

However, for the resonant incidence with the momentum $k=-\pi/2$, the transmission is symmetric $|t_L|=|t_R|$ because of $\left|\Delta^{-1}_{nm}\right|=\left|\Delta^{-1}_{mn}\right|$; but the reflection is asymmetric. To break the reciprocity at $k=-\pi/2$, the on-site terms $\{V_1,V_2,V_3\}$ of the scattering center are helpful

$$H_c = \begin{pmatrix} V_1 & -i\kappa & J \\ -i\kappa & V_2 & Je^{-i\phi} \\ J & Je^{i\phi} & V_3 \end{pmatrix}. \tag{42}$$

For the resonant incidence $k=-\pi/2$, the four elements of the inverse matrix Δ^{-1} are in the form of

$$\begin{pmatrix} \Delta^{-1}_{mm} & \Delta^{-1}_{mn} \\ \Delta^{-1}_{nm} & \Delta^{-1}_{nn} \end{pmatrix} = \frac{1}{\det \Delta} \begin{pmatrix} V_2V_3-J^2 & -JV_2-iJ\kappa e^{-i\phi} \\ -JV_2-iJ\kappa e^{i\phi} & V_1V_2+\kappa^2 \end{pmatrix}, \tag{43}$$

where the determinant $D\equiv\det\Delta=V_1V_2V_3-J^2V_1-J^2V_2+\kappa^2V_3-iJ^2\kappa\left(e^{-i\phi}+e^{i\phi}\right)$.

At $\phi=\pm\pi/2$, the transmission is asymmetric for the real V_2; the transmission is symmetric for the imaginary V_2. For the real $V_{1,2,3}$, the reflection is symmetric; otherwise, the reflection is asymmetric.

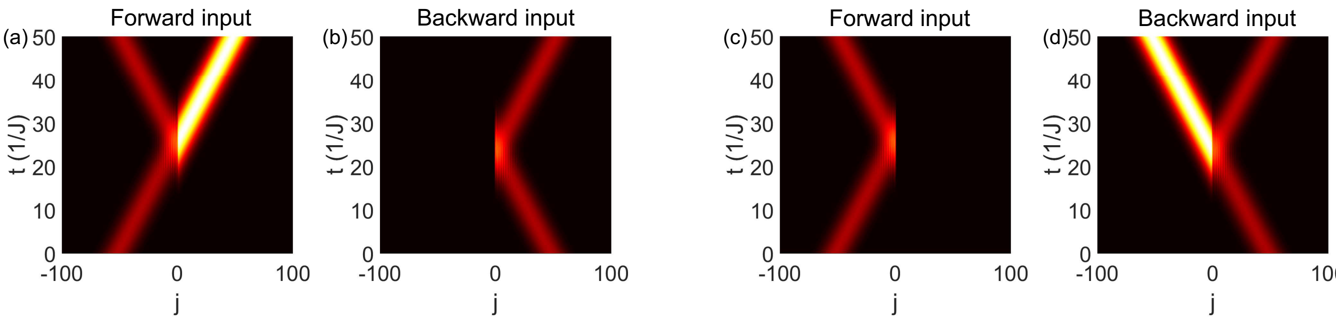
We take $V_1=V_3=0$ as an illustration. The scattering coefficients for the momentum $k=-\pi/2$ are obtained in the form of

$$t_L = \frac{J(e^{2ik}-1)(-iV_2+\kappa e^{i\phi}+2iJ\cos k)}{iJ^2e^{-2ik}(e^{4ik}+e^{2ik}-1)+J(iV_2e^{-ik}-iV_2e^{ik}+\kappa e^{i(k-\phi)}+\kappa e^{i(k+\phi)})-i\kappa^2}, \tag{44}$$

$$r_L = -\frac{e^{-i\phi}(e^{i(k+\phi)}J-i\kappa)(iJe^{ik}+\kappa e^{i\phi})}{iJ^2e^{-2ik}(e^{4ik}+e^{2ik}-1)+J(iV_2e^{-ik}-iV_2e^{ik}+\kappa e^{i(k-\phi)}+\kappa e^{i(k+\phi)})-i\kappa^2}, \tag{45}$$

$$t_R = \frac{iJe^{-i\phi}(e^{2ik}-1)(-V_2e^{i\phi}-i\kappa+2Je^{i\phi}\cos k)}{iJ^2e^{-2ik}(e^{4ik}+e^{2ik}-1)+J(iV_2e^{-ik}-iV_2e^{ik}+\kappa e^{i(k-\phi)}+\kappa e^{i(k+\phi)})-i\kappa^2}, \tag{46}$$

$$r_R = -\frac{e^{-i\phi}(e^{i(k+\phi)}\kappa+iJ)(Je^{i\phi}-i\kappa e^{ik})}{iJ^2e^{-2ik}(e^{4ik}+e^{2ik}-1)+J(iV_2e^{-ik}-iV_2e^{ik}+\kappa e^{i(k-\phi)}+\kappa e^{i(k+\phi)})-i\kappa^2}. \tag{47}$$



Supplemental Figure 1. Numerical simulation of the asymmetric transmission in the three-site center with dissipative coupling [Eq. (42)]. The incidences are Gaussian wave packet with the momentum $k=-\pi/2$ in the simulations. The excitation intensity is depicted. (a) Forward and (b) backward incidence for $\phi=-\pi/2$. $|t_L|^2=4$, $|r_L|^2=1$; $|t_R|^2=0$, $|r_R|^2=1$. (c) Forward and (d) backward incidence for $\phi=\pi/2$. $|t_L|^2=0$, $|r_L|^2=1$; $|t_R|^2=4$, $|r_R|^2=1$. The dissipative coupling is $\kappa=J=1$ and the on-site terms are $\{V_1,V_2,V_3\}=\{0,J,0\}$. The incident frequency is ω_0.

Both the scattering coefficients t and r diverge at $\phi = \pm \arccos\left(J^2 - \kappa^2\right)\left(2J\kappa\right)^{-1}$ and $V_2 = 0$ when the resonant momentum $k = -\pi/2$. For the real $V_2 = J$, we have the scattering coefficients $t_L = -2i, r_L = -i; t_R = 0, r_R = i$ at $\phi = -\pi/2$; and we have the scattering coefficients $t_L = 0, r_L = -i; t_R = -2i, r_R = i$ at $\phi = \pi/2$. The numerical simulations are shown in Supplementary Figure 1.

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