Supplementary Material for "Influence of Device Geometry on Transport Properties of Topological Insulator Microflakes"

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I. Magnetoconductivity tensor

Our simulation is based on Boltzmann diffusive transport theory, which is valid when the mean free path l_e is much smaller than the size of the system L. In the linear response approximation, the Boltzmann equation in zero magnetic field leads to Ohm's law,

$$\boldsymbol{J} = \sigma_0 \boldsymbol{E}, \tag{1}$$

where $\boldsymbol{J}$, σ_0 and $\boldsymbol{E}$ are the electric field, conductivity, and current density, respectively. The conductivity is given by $\sigma_0 = nq^2\tau/m^*$, with n, q, τ and m^* being the carrier density, carrier charge, momentum relaxation time, and effective mass, respectively. This result is an upgraded version of the classical Drude formula, and hence can be easily generalized to the case of finite magnetic fields. In this case, the current density satisfies

$$\boldsymbol{J} = \sigma_0(\boldsymbol{E} + \boldsymbol{v}_\mathrm{d} \times \boldsymbol{B}), \tag{2}$$

with $\boldsymbol{v}_\mathrm{d} = \frac{q\tau}{m^*}\mathbf{E}$ being the drift velocity. Suppose magnetic field $\boldsymbol{B}$ is applied in the z direction, i.e. $\boldsymbol{B} = (0,0,B)$, we have

$$J_x = \sigma_0 E_x + \mu_H B J_y, \tag{3. a}$$

$$J_y = \sigma_0 E_y - \mu_H B J_x, \tag{3. b}$$

$$J_z = \sigma_0 E_z. \tag{3. c}$$

Here μ_H of $q\tau/m^*$ is the Hall mobility. Solving Eq. (3. a), (3. b), and (3. c) gives $\boldsymbol{J} = \boldsymbol{\sigma}\boldsymbol{E}$ with

$$\boldsymbol{\sigma}(\boldsymbol{B}) = \frac{\sigma_0}{1+(\mu_\mathrm{H} B)^2}\begin{pmatrix} 1 & R_\mathrm{H} B\sigma_0 & 0 \\ -R_\mathrm{H} B\sigma_0 & 1 & 0 \\ 0 & 0 & 1+(\mu_\mathrm{H} B)^2 \end{pmatrix}, \tag{4}$$

where the Hall coefficient is given by $R_\mathrm{H} = 1/nq$.

II. Four-point resistances of standard Hall bars

A sketch of a standard Hall bar is shown in Fig. S1. In such a device geometry, the current distribution is barely influenced by the voltage probes. The four-point resistances in a conventional 3D thin film can be straightforwardly evaluated with $R_{xx} = \frac{V_1 - V_2}{I} = \rho_{xx}\frac{l}{wd}$, and $R_{yx} = \frac{V_3 - V_1}{I} = \frac{\rho_{yx}}{d}$, where l is the distance between the two neighboring voltage probes, w is the width, and d is the thickness of the Hall bar.

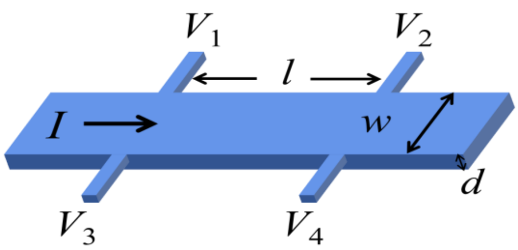

Fig. S1. Schematic illustration of a standard Hall bar.

For a Hall bar based on a 3D topological insulator (TI) flake or thin film, the relations between the resistivities and the four-point resistances are not given in the main text. For the longitudinal transport, the contributions of all conduction components, the top and bottom surfaces, the front and back side surfaces, and the bulk layer need to be included. If the resistivity is identical for all surfaces, the Hall resistance $R_{xx,\mathrm{H}}$ satisfies

$$\frac{1}{R_{xx,\mathrm{H}}} \equiv \frac{I}{V_1 - V_2} = 2\frac{1}{\rho_{xx,\mathrm{surf}}}\frac{w}{l} + 2\frac{1}{\rho_{xx,\mathrm{surf}}}\frac{d}{l} + \frac{1}{\rho_{xx,\mathrm{bulk}}}\frac{wd}{l}. \qquad (5)$$

Here, $\rho_{xx,\mathrm{surf}}$ and $\rho_{xx,\mathrm{bulk}}$ are the longitudinal resistivities of the surface and the bulk, respectively. As for the transverse resistance, there is no perpendicular magnetic field component, so that the two side surfaces do not contribute. For the bulk resistivities of interest to this work, $\frac{\rho_{xx,\mathrm{bulk}}}{d} \gg \rho_{xx,\mathrm{surf}}$, most of the current distributes on the surface, and according to the two-band model, the contribution of the bulk layer to the Hall resistance can also be neglected for weak magnetic fields. In case of symmetric top and bottom surfaces, we then have

$$R_{yx,\mathrm{H}} \equiv \frac{V_3 - V_1}{I} = \rho_{yx,\mathrm{surf}}/2. \qquad (6)$$

in which the surface Hall resistivity is give by $\rho_{yx,\mathrm{surf}} = R_{\mathrm{H,surf}} B$, where $R_{\mathrm{H,surf}}$ is the Hall coefficient of the surface states.