Supplementary material: Prediction of an $\Omega_{bbb}\Omega_{bbb}$ dibaryon in the extended one-boson exchange model

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I. ONLY S-WAVE INTERACTION

To directly compare with the lattice QCD studies [1, 2], we show the results obtained with only S-wave interactions in the extended OBE model. Turning off the S-D mixing, but with the same cutoffs Λ and coupling ratios r as given in Tables III and IV of the main text, we repeat the OBE study of the $\Omega_{ccc}\Omega_{ccc}$, $\Omega\Omega$, and $\Omega_{bbb}\Omega_{bbb}$ systems and search for bound states. The results are shown in Table I. It is clear that without the S-D mixing, all the systems become less bound. Adding the Coulomb interaction, some of the dibaryons even become unbound. On the other hand, with a slight fine-tuning (increase) of the cutoff or coupling ratio, the systems can become bound as shown in Table 1. Therefore, as concluded in the main text, the extended OBE model supports the main results of the lattice QCD studies [1, 2].

TABLE S1. Binding energy B and RMS radius R of the $J^P = 0^+$ $\Omega_{ccc}\Omega_{ccc}$, $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ bound states obtained with different ratios r and cutoffs Λ. The numbers in the brackets represent the results obtained with the Coulomb interaction taken into account. The "−" denotes that there exists no bound state.

Molecule	J^P	Ratio r	Λ(GeV)	B (MeV)	R(fm)
$\Omega_{ccc}\Omega_{ccc}$	0^+	1	3.78	2.6(−)	1.4(−)
		1/2	4.60	1.9(−)	1.6(−)
$\Omega\Omega$	0^+	1	1.62	0.9(**0.2**)	4.0(**6.1**)
		1/2	2.83	0.3(−)	5.9(−)
$\Omega_{bbb}\Omega_{bbb}$	0^+	1	10.073	1.1(−)	1.2(−)
		1/2	10.718	0.5(−)	1.7(−)

TABLE S2. Binding energy B and RMS radius R of the $J^P = 0^+$ $\Omega_{ccc}\Omega_{ccc}$, $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ bound states obtained with different ratios r and cutoffs Λ. The numbers in the brackets represent the results obtained with the Coulomb interaction taken into account. The "−" denotes that there exists no bound state.

Molecule	J^P	Ratio r	Λ(GeV)	B (MeV)	R(fm)
$\Omega_{ccc}\Omega_{ccc}$	0^+	1	3.8075	5.7(−)	1.0(−)
		1/2	4.681	5.7(-)	1.0(-)
$\Omega\Omega$	0^+	1	1.64	1.6(**0.7**)	3.2(**4.1**)
		1/2	2.98	1.6(**0.6**)	3.0(**4.1**)
$\Omega_{bbb}\Omega_{bbb}$	0^+	1	10.10	5.3(**0.3**)	0.57(**1.3**)
		1/2	10.79	5.7(**0.5**)	0.55(**1.1**)

II. $J = 2$ CONFIGURATIONS

In the following, we discuss about the $J = 2$ configurations for the $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ systems. For $J = 2$, the following combinations of angular momentum and spin are allowed: $(L = 0, S = 2)$, $(L = 2, S = 0)$, and

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$(L = 2, S = 2)$. With these partial waves the relevant matrix elements of the spin-spin and tensor operators of the extended OBE potential are shown in Table III.

TABLE S3. Matrix elements of the spin-spin and tensor operators for the $\Omega_{ccc}\Omega_{ccc}$ system.

State	J^P	Partial wave	$\langle \vec{a}_1 \cdot \vec{a}_2 \rangle$	$S_{12}(\vec{a}_1, \vec{a}_2, \vec{r})$
$\Omega_{ccc}\Omega_{ccc}$	$J = 2$	${}^5S_2$-${}^1D_2$-${}^5D_2$	$\begin{pmatrix} -\frac{3}{4} & 0 & 0 \\ 0 & -\frac{15}{4} & 0 \\ 0 & 0 & -\frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} 0 & -\frac{3}{\sqrt{5}} & 3\sqrt{\frac{7}{10}} \\ -\frac{3}{\sqrt{5}} & 0 & 3\sqrt{\frac{2}{7}} \\ 3\sqrt{\frac{7}{10}} & 3\sqrt{\frac{2}{7}} & \frac{9}{14} \end{pmatrix}$

First, we notice that with the same parameters as those used for the $J = 0$ configurations studied in the main text, none of these systems bind, which indicates that in the extended OBE model, the $J = 0$ potential is more attractive than that of $J = 2$, in agreement with the quark model [3].

Next we investigate whether these systems can bind with reasonable cutoffs. For the $\Omega_{ccc}\Omega_{ccc}$ system, with a cutoff of 4.18 GeV and the coupling ratio $r = 1$, we obtain one bound state. If we change the coupling ratio to $1/2$, we obtain a bound state for a cutoff of 5.45 GeV. Both cutoffs are much larger than those for $J = 0$ by about 0.4-0.9 GeV, which do not seem very natural. With the same approach, we obtain the results for the $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ as shown in Table IV. Judging from the cutoffs needed to obtain bound states, we conclude that although a $J = 2$ $\Omega_{bbb}\Omega_{bbb}$ dibaryon is unlikely, a $J = 2$ $\Omega\Omega$ dibaryon cannot be ruled out. We show the binding energy of $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$ and $\Omega_{bbb}\Omega_{bbb}$ as a function of the cutoff with S-D mixing turned on for the cases of $r = 1$ and $r = 1/2$ in Figs. [1-6].

TABLE S4. Binding energy B and RMS radius R of the $J^P = 2^+$ $\Omega_{ccc}\Omega_{ccc}$, $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ bound states obtained with different ratios r and cutoffs Λ. The numbers in the brackets represent the results obtained with the Coulomb interaction taken into account. The " $-$" denotes that there exists no bound state.

Molecule J^P	Ratio r	Λ(GeV)	B (MeV)	R(fm)
$\Omega_{ccc}\Omega_{ccc}$ 2^+	1	4.18	7.6(−)	0.9(−)
	1/2	5.45	6.6(−)	0.9(−)
$\Omega\Omega$ 2^+	1	1.80	5.4(**3.9**)	1.8(**2.0**)
	1/2	3.83	2.5(**1.1**)	2.4(**3.1**)
$\Omega_{bbb}\Omega_{bbb}$ 2^+	1	10.393	3.5(−)	0.67(−)
	1/2	11.378	5.3(−)	0.55(−)

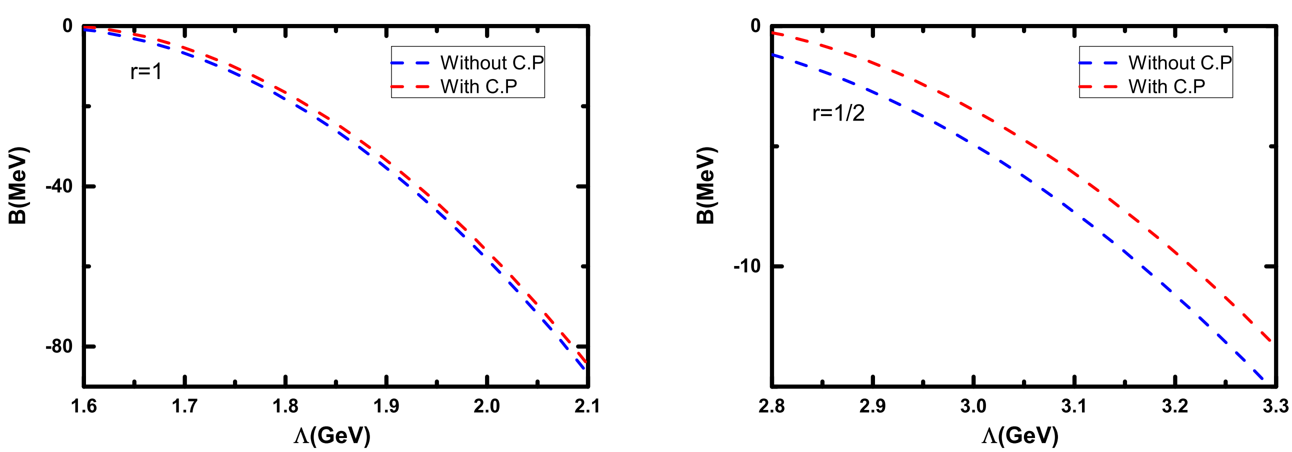

FIG S1. With and without the Coulomb interaction, the binding energy of the $J = 0$ $\Omega\Omega$ dibaryon as a function of the cutoff for the cases of $r = 1$ and $1/2$.

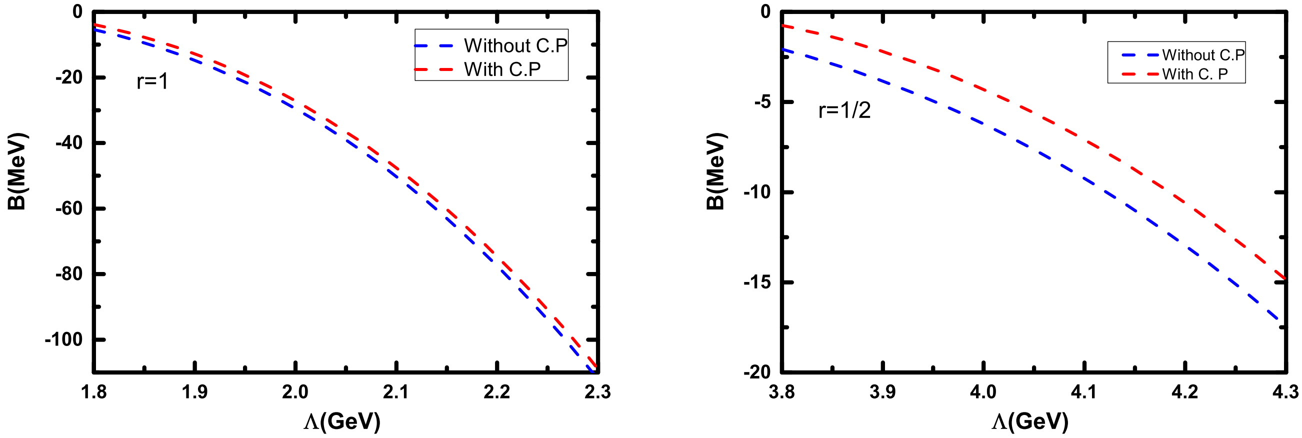

FIG S2. With and without the Coulomb interaction, the binding energy of the $J = 2$ $\Omega\Omega$ dibaryon as a function of the cutoff for the cases of $r = 1$ and $1/2$.

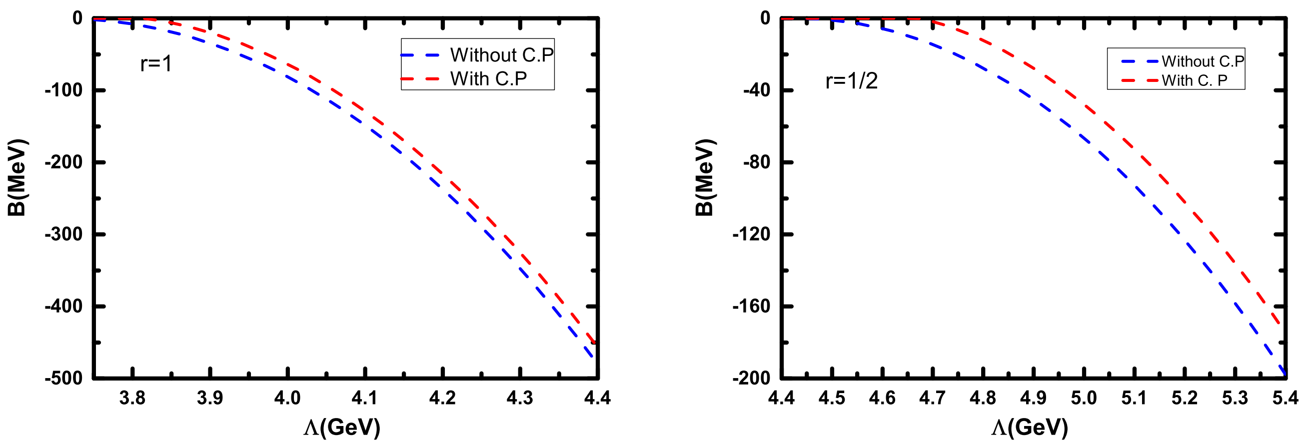

FIG S3. With and without the Coulomb potential, the binding energy of the $J = 0$ $\Omega_{ccc}\Omega_{ccc}$ dibaryon as a function of the cutoff for the case of r=1 and $1/2$.

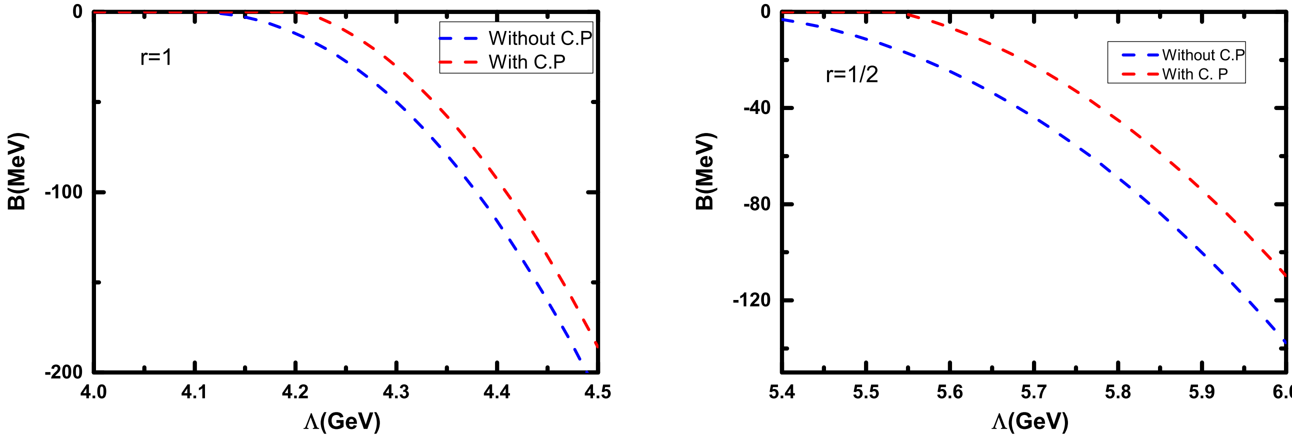

FIG S4. With and without the Coulomb potential, the binding energy of the $J = 2$ $\Omega_{ccc}\Omega_{ccc}$ dibaryon as a function of the cutoff for the case of r=1 and 1/2.

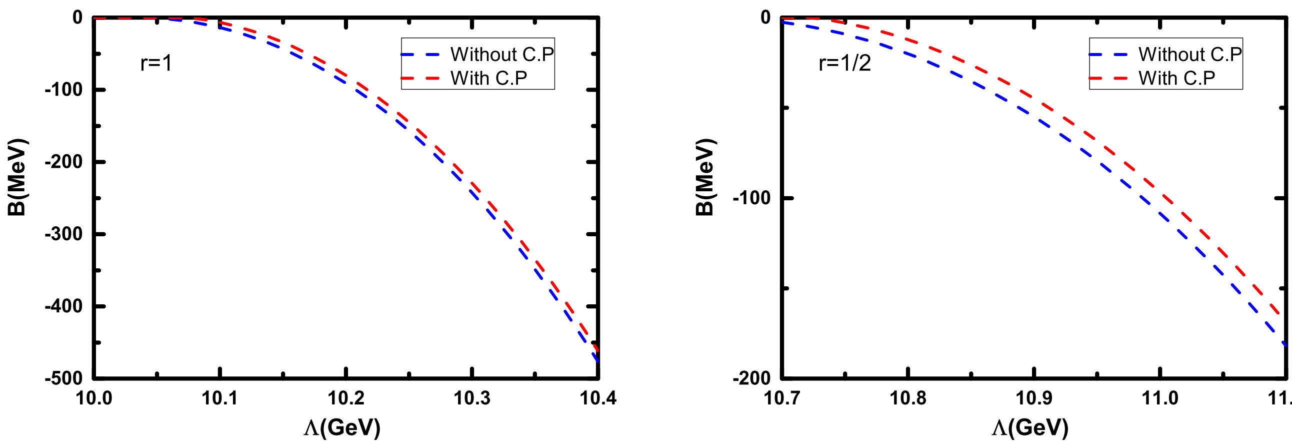

FIG S5. With and without the Coulomb potential, the binding energy of the $J = 0$ $\Omega_{bbb}\Omega_{bbb}$ dibaryon as a function of the cutoff for the case of r=1 and 1/2.

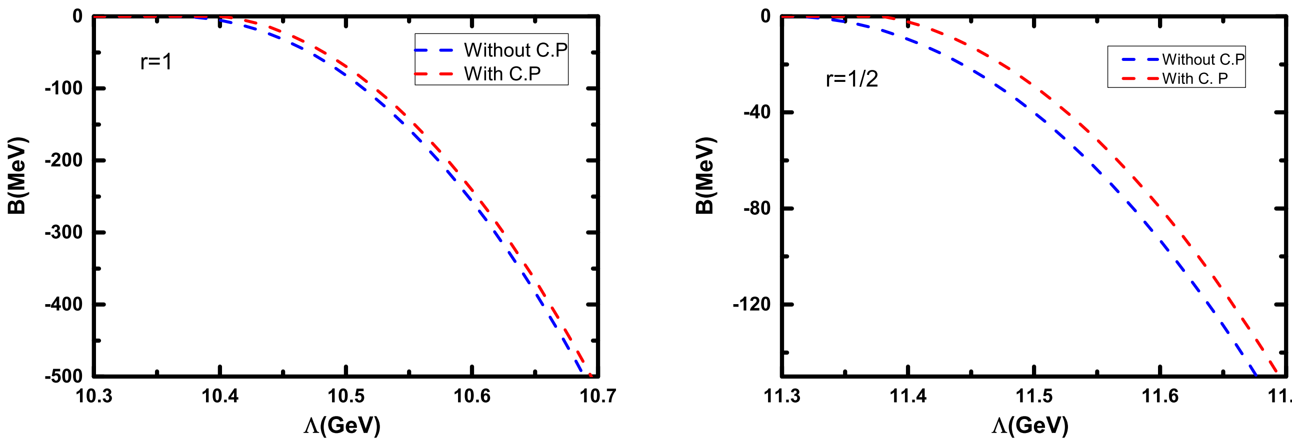

FIG S6. With and without the Coulomb potential, the binding energy of the $J = 2$ $\Omega_{bbb}\Omega_{bbb}$ dibaryon as a function of the cutoff for the case of r=1 and 1/2.

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[2] Y. Lyu, H. Tong, T. Sugiura, S. Aoki, T. Doi, T. Hatsuda, J. Meng, and T. Miyamoto, (2021), arXiv:2102.00181 [hep-lat].
[3] H. Huang, J. Ping, X. Zhu, and F. Wang, (2020), arXiv:2011.00513 [hep-ph].