Supplementary Materials for
Exciton vortices in two-dimensional hybrid perovskite monolayers

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SECTION S1: SOLUTION OF THE QUANTUM CONFINED EXCITON UNDER ELECTRIC FIELD

The single particle $H_{ez}(H_{hz})$ denotes the Hamiltonian of the electron(hole) in the inorganic layer under the electric field F, i.e.,

$$\begin{cases} H_{ez}(z_e) = -\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial z_e^2} + V_{\text{conf}}(z_e) + eFz_e \\ H_{hz}(z_h) = -\frac{\hbar^2}{2m_h}\frac{\partial^2}{\partial z_h^2} + V_{\text{conf}}(z_h) - eFz_h \end{cases}, \tag{S1}$$

where V_{conf} is the infinite-potential-barrier for the electron and the hole, since they are confined in the single inorganic layer. Eigenstates of Eq.(S1) are represented by the Airy functions Ai and Bi,

$$\zeta_{l_e(l_h)}(z) = a_{l_e(l_h)}\text{Ai}\left(Z_{l_e(l_h)}(z)\right) + b_{l_e(l_h)}\text{Bi}\left(Z_{l_e(l_h)}(z)\right), \tag{S2}$$

where $l_e(l_h) = 1, 2, \ldots$ is the subband index of the electron(hole), and

$$\begin{cases} Z_{l_e} = -[2m_e/(e\hbar F)^2]^{\frac{1}{3}}(E_{l_e}^{(e)} - eFz) \\ Z_{l_h} = -[2m_h/(e\hbar F)^2]^{\frac{1}{3}}(E_{l_h}^{(h)} + eFz) \end{cases}. \tag{S3}$$

Here $E_{l_e}^{(e)}(E_{l_h}^{(h)})$ is the l_e-th electron (l_h-th hole) subband energy. The parameters are normalized as

$$\begin{cases} a_l = [1 + \text{Ai}^2(Z_{l\pm})/\text{Bi}^2(Z_{l\pm})]^{-1/2} \\ b_l = -a_l\text{Ai}(Z_{l\pm})/\text{Bi}(Z_{l\pm}) \end{cases} \tag{S4}$$

, with $Z_{l\pm} = Z_l(z = \pm L_w/2)$. The eigenenergies E_l are determined by the $(l-1)$-zeros of

$$S(E_l) = \text{Ai}(Z_{l+})\,\text{Bi}(Z_{l-}) - \text{Bi}(Z_{l+})\,\text{Ai}(Z_{l-}). \tag{S5}$$

SECTION S2: TREATMENT OF THE PUMPING AND DECAYING TERM

Here we deal with the time step containing the pumping and decaying term $i\partial_t\psi = i\left(\hat{R} - \Gamma|\psi|^2\right)\psi$, which can be written as

$$\partial_t\psi = \left(\hat{R} - \Gamma|\psi|^2\right)\psi. \tag{S6}$$

Since the Eq.(S6) is real, we have

$$\partial_t\rho = 2\left(\hat{R} - \Gamma\rho\right)\rho. \tag{S7}$$

Next we solve $\rho(t)$ from Eq.(S7), and obtain

$$\rho(t) = \frac{\hat{R}}{\Gamma}\frac{1}{1 + Ce^{-2\hat{R}t}}, \tag{S8}$$

with

$$C = \frac{\hat{R}}{\Gamma}\frac{1}{\rho_0} - 1. \tag{S9}$$

Recalling Eq.(S6),

$$\psi_1' = e^{\int_0^{\Delta t}\left(\hat{R}-\Gamma\rho(s)\right)ds}\psi_1, \rho(0) = \psi_1^2. \tag{S10}$$

Putting Eq.(S8) into the integral in Eq.(S10), we have

$$\int_0^{\Delta t}\left(\hat{R} - \Gamma\rho(s)\right)ds = \hat{R}\Delta t + \frac{1}{2}\ln\left(\frac{\hat{R}}{\hat{R} + \Gamma\rho_0\left(e^{2\hat{R}\Delta t} - 1\right)}\right) \tag{S11}$$

The second-order time splitting can be written as

$$\psi(t + \Delta t) = e^{-i\frac{T}{2}\Delta t}e^{-i\frac{V}{2}\Delta t}e^{\int_0^{\Delta t}\left(\hat{R}-\Gamma\rho(s)\right)ds}e^{-i\frac{V}{2}\Delta t}e^{-i\frac{T}{2}\Delta t}\psi(t). \tag{S12}$$

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