Supplementary Material

Distinct three-level spin-orbit control associated with electrically controlled band swapping

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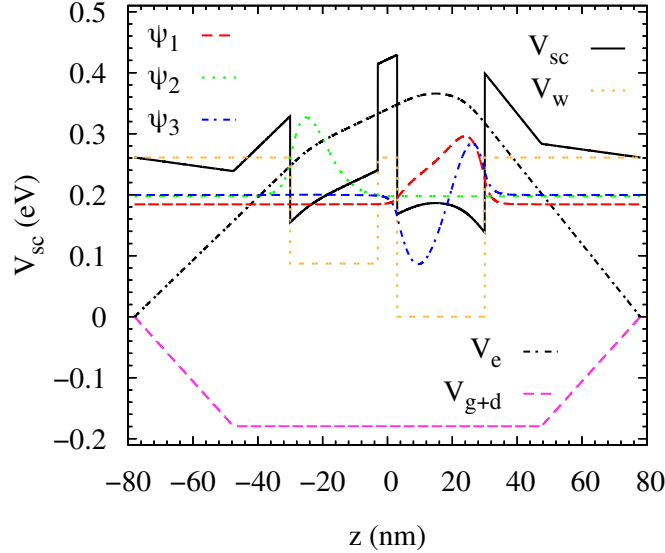
I. ZERO BIAS SELF-CONSISTENT SOLUTIONS

Figure S1. Self-consistent potential V_{sc} including its several constituent contributions as well as the corresponding wave function profiles ψ_1, ψ_2 and ψ_3 of three subbands for our GaAs/AlGaAs double well at $V_{\mathrm{g}} = 0$. These constituent potential contributions contain the quantum well band-offset potential V_{w}, the electron Hartree potential V_{e}, and the gate plus doping potential $V_{\mathrm{g+d}}$. The three subband energy levels are $\mathcal{E}_1 = 184.32$ meV, $\mathcal{E}_2 = 197.62$ meV, and $\mathcal{E}_3 = 199.59$ meV. The electron density is held fixed at $n_e = 8.0 \cdot 10^{11} \mathrm{cm}^{-2}$. The resulting Fermi energy is $\mathcal{E}_{\mathrm{F}} = 200.83$ meV. The fourth subband is about 27 meV above $\mathcal{E}_{\mathrm{F}}$, indicating it is unoccupied.

Figure S1 shows the self-consistent potential and the corresponding wave function profiles of three subbands for our GaAs/AlGaAs double well at $V_{\mathrm{g}} = 0$. In the parameter range considered, at zero bias, electrons of the first, second, and third subbands are largely localized in the right, left, and right wells, respectively, cf. Figs. S1 and Fig. (2) of the main text. Note that gate plus doping potential $V_{\mathrm{g+d}}$ maintains flat across the whole region having electron distributions (i.e., -40 nm $< z < 40$ nm), because of the symmetric doping condition we adopted, see the main text.

II. CONSITUENT CONTRIBUTIONS TO RASHBA SO COUPLINGS

We show several distinct contributions to the Rashba SO couplings. Below we consider the intraband Rashba terms in Sec. II A and the interband ones in Sec. II B.
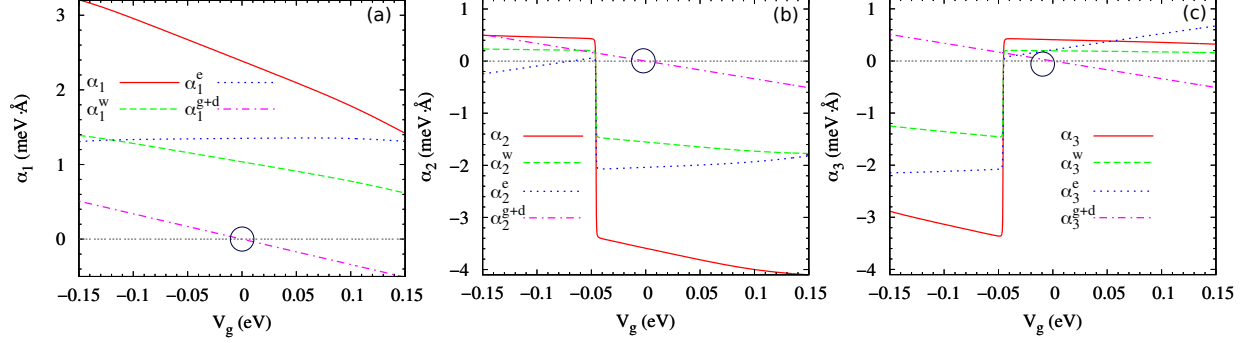
Figure S2. Distinct contributions to Rashba strengths of the first α_1 (a), second α_2 (b), and third α_3 (c) subbands as functions of V_g. These include the gate plus doping contribution α_ν^{g+d}, the electron Hartree contribution α_ν^e, and the structural contribution α_ν^w, with $\alpha_\nu = \alpha_\nu^e + \alpha_\nu^{g+d} + \alpha_\nu^w$. The black circles indicate that α_ν^{g+d} for all three subbands identically vanishes at $V_g = 0$.

A. Intraband Rashba terms

In Fig. S2, we show the intraband Rashba coefficients α_ν of the three subbands and the corresponding constituent contributions as functions of V_g. For the electron Hartree contribution α_ν^e, the first subband α_1^e maintains essentially constant as V_g varies [Fig. S2(a)], since the first-subband electrons are kept in the right well in the whole gate ranges (see Fig. 2 of the main text). In contrast, for the second [Fig. S2(b)] and third [Fig. S2(c)] subbands, α_2^e and α_3^e basically interchange the values near $V_g = -0.0457$ eV, at which the band swapping of the two subbands occurs. Concerning the structural contribution α_ν^w, it is found that α_1^w displays basically the linear behavior with V_g, while α_2^w and α_3^w interchange the values when we adjust V_g, similar to the electron Hartree contributions α_2^e and α_3^e. As for the gate plus doping contribution α_ν^{g+d}, it exhibits linear gate dependence for all three subbands, due to the linear characteristic of the gate potential we adopted [1–5].

Note that, at zero bias $V_g = 0$, even though the overall α_ν is nonzero because of the *intrinsic* structual inversion asymmetry between the right and left wells of the system (Fig. 1 of the main text), α_ν^{g+d} identically vanishes for all three subbands, as indicated by the black circles. This straightforwardly follows from the fact that the gate plus doping potential V_{g+d} is basically z independent ("flat" characteristic aforementioned) across the region where there are electron distributions, see Fig. S1 and Sec. I. A uniform V_{g+d} near the well regions leads to the zero force field of the gate plus doping potential, i.e., $\partial_z V_{g+d} = 0$, and so the vanishing Rashba contribution α_ν^{g+d}, see Eq. (3) of the main text.

B. Interband Rashba terms

Figure S3 shows the gate dependence of interband Rashba coefficients and the corresponding constituent contributions. For the electron Hartree contribution $\eta_{\mu\nu}^{\mathrm{e}}$ and the structural contribution $\eta_{\mu\nu}^{\mathrm{w}}$, we find that near $V_{\mathrm{g}} = -0.0457$ eV, $\eta_{12}^{\mathrm{e,w}}$ and $\eta_{13}^{\mathrm{e,w}}$ basically interchange the values while $\eta_{23}^{\mathrm{e,w}}$ exhibits a resonance, follows from the band swapping between the second and third subbands. We should emphasize that $\eta_{\mu\nu}^{\mathrm{g+d}}$ maintains zero in the whole gate ranges because of the orthogonality condition between distinct subbands μ and ν, cf. Figs. S3(a)-S3(c).
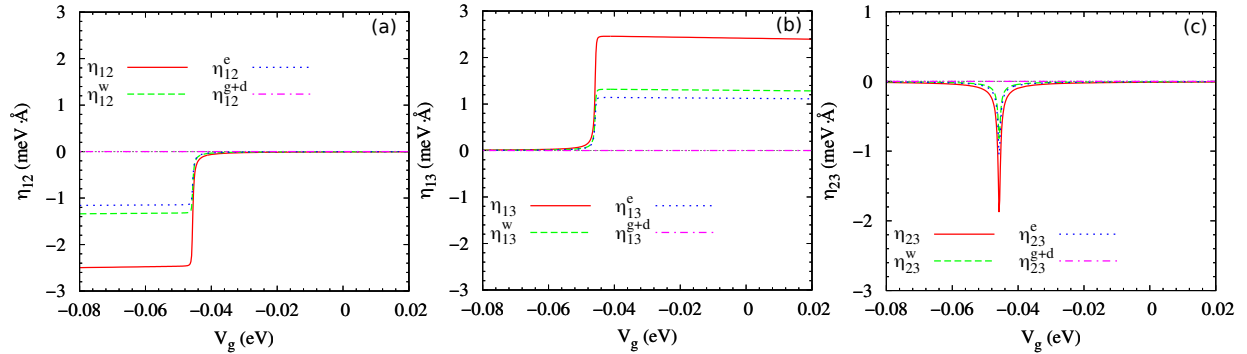

Figure S3. Interband Rashba coefficients $\eta_{\mu\nu}$ between subbands μ and ν ($\mu \neq \nu$) and its several constituent contributions: the gate plus doping $\eta_{\mu\nu}^{\mathrm{g+d}}$, the electron Hartree $\eta_{\mu\nu}^{\mathrm{e}}$, and the structural $\eta_{\mu\nu}^{\mathrm{w}}$ contributions with $\eta_{\mu\nu} = \eta_{\mu\nu}^{\mathrm{e}} + \eta_{\mu\nu}^{\mathrm{g+d}} + \eta_{\mu\nu}^{\mathrm{w}}$ as functions of V_{g}. (a) η_{12}, (b) η_{13}, and (c) η_{23}.

[1] E. Bernardes, J. Schliemann, M. Lee, J. C. Egues, and D. Loss, Spin-orbit interaction in symmetric wells with two subbands, Phys. Rev. Lett. **99**, 076603 (2007).

[2] R. S. Calsaverini, E. Bernardes, J. C. Egues, and D. Loss, Intersubband-induced spin-orbit interaction in quantum wells, Phys. Rev. B **78**, 155313 (2008).

[3] J. Y. Fu and J. C. Egues, Spin-orbit interaction in GaAs wells: From one to two subbands, Phys. Rev. B **91**, 075408 (2015).

[4] J. Y. Fu, P. H. Penteado, M. O. Hachiya, D. Loss, and J. C. Egues, Persistent skyrmion lattice of noninteracting electrons with spin-orbit coupling, Phys. Rev. Lett. **117**, 226401 (2016).

[5] F. Dettwiler, J. Y. Fu, S. Mack, P. J. Weigele, J. C. Egues, D. D. Awschalom, and D. M. Zumbühl, Stretchable persistent spin helices in GaAs quantum wells, Phys. Rev. X **7**, 031010 (2017).