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# Higher Dimensional Strange Quark Matter Coupled to the String Cloud with Electromagnetic Field Admitting One Parameter Group of Conformal Motion

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*We solve Einstein's field equations in higher-dimensional spherically symmetric spacetime with strange quark matter attached to the string cloud, assuming one parameter group of conformal motions. The solutions match with the higher-dimensional Reissner–Nordström metric on the boundary at $r = r_0$. The features of the solutions are also discussed in the framework of higher-dimensional spacetime.*

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The exact physical modelled situation at very early stages of the formation of our universe have provoked great interest of researchers and it is still a challenging problem. It is generally assumed that during the phase transition, the symmetry of the universe is broken spontaneously. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories.[1−4] The general treatment of strings was initiated by Letelier[5,6] and Stachel.[7] Of all these cosmological structures, cosmic strings have provoked the most interest. The present day configurations of the universe are not contradicted by the large scale network of strings in the early universe. Moreover, they may act as gravitational lenses and may give rise to density fluctuations leading to the formations of galaxies.[4,8] Strings possess stress energy are coupled to the gravitational field.

Higher-dimensional spacetime is an active research in its attempt to unify gravity with other forces in nature. This idea is particularly important in the field of cosmology since one knows that our universe was much smaller in its early stage than that today. In this connection a number of attempts have been made to study the role of gravity with other fundamental forces in nature. The most famous five-dimensional theory proposed by Kaluza[9] and Klein[10] was the first theory, in which gravitation and electromagnetism could be unified in a single geometrical structure. A number of authors (e.g. Refs. [11–17] and references therein) have studied the physics of the universe in higher-dimensional spacetime.

A great deal of attention has recently been paid to strange quark matter. In this Letter, we attach strange quark matter to the string cloud. It is plausible to attach strange quark matter to the string cloud. This is because one of such transitions during the phase transitions of the universe could be quark gluon plasma (QGP) → hadron gas (called the quark–hadron phase transition) when cosmic temperature is $T \sim 200\,\mathrm{MeV}$. Recently some authors[18−21] have studied strange quark matter in different contexts.

Typically, strange quark matter is modelled with an equation of state (EQS) bases on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are thought of as degenerate Fermi gases, which exist only in the region of space endowed with a vacuum energy density $B_c$ (called the bag constant). Also, in the framework of this model, the quark matter is composed of massless $u$ and $d$ quarks, massive $s$ quarks and electrons. In the simplified version of the bag model, assuming quarks are massless and non-interacting, we then have quark pressure $p_q = \rho_q/3$ ($\rho_q$ is the quark energy density); the total energy density is $\rho = \rho_q + B_c$ but total pressure is $p = p_q - B_c$.

In this study we examine charged strange quark matter attached to the string cloud in the higher-dimensional spherical symmetric spacetime admitting one parameter group of conformal motions. For this purpose, we solve Einstein's field equations for spherically symmetric spacetime with charged strange quark matter attached to the string cloud via conformal motions. This work is the generalization of the work obtained earlier by Yavuz *et al.*[18]

We consider the higher-dimensional spherically symmetric line element in the form

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2, \tag{1}$$

with

$$d\Omega^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 + \cdots + \prod_{i=1}^{n-1} \sin^2\theta_i d\theta_n^2,$$

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$$x^{1,2,3,\cdots,(n+1),(n+2)} = r, \theta_1, \theta_2, \cdots, \theta_{(n+1)}, t.$$

The total energy-momentum tensor $T_{ab}$ is assumed to be the sum of the two parts, $T^S_{ab}$ and $T^E_{ab}$, for string cloud and electromagnetic contributions, respectively, i.e.

$$T_{ab} = T^S_{ab} + T^E_{ab}. \tag{2}$$

The energy momentum tensor for string cloud given by Letelier[5] reads

$$T^S_{ab} = \rho U_a U_b - \rho_s X_a X_b, \tag{3}$$

where $\rho$ is the rest energy density for the cloud of strings with particles attached to them and $\rho_s$ is the tension density of the strings. Here

$$\rho = \rho_p + \rho_s, \tag{4}$$

with $\rho_p$ being the particle energy density.

In this study we take the string quark matter energy density instead of particle energy density in the string cloud. Hence Eq. (4) leads to

$$\rho = \rho_q + \rho_s + B_c. \tag{5}$$

With the help of Eqs. (3) and (5), we have the energy-momentum tensor for string quark attached to the string cloud,

$$T^S_{ab} = (\rho_q + \rho_s + B_c) U_a U_b - \rho_s X_a X_b, \tag{6}$$

where $U^a$ is the $(n+2)$ velocity $U^a = \delta^a_{(n+2)} e^{-\nu/2}$, $X^a$ is the unit space-like vector in the radial direction $X^a = \delta^a_1 e^{-\lambda/2}$, which represents the strings directions in the cloud, i.e. the direction of anisotropy. The energy-momentum tensor for electromagnetic field reads

$$T^E_{ab} = -\frac{1}{4\pi}\Big(F^c_a F_{bc} - \frac{1}{4} g_{ab} F_{\alpha\beta} F^{\alpha\beta}\Big), \tag{7}$$

where $F_{ab}$ is the electromagnetic field tensor in terms of the $(n+2)$-potential $A_a$ as

$$F_{ab} = A_{b;a} - A_{a;b}.$$

For the electromagnetic field we consider the gauge

$$A_a(0, 0, 0, 0, \cdots (n+1) 0, \phi(r)).$$

Einstein–Maxwell equations can be expressed as

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}, \tag{8}$$

$$F_{ab;c} + F_{bc;a} + F_{ca;b} = 0, \tag{9}$$

$$F^{ab}_{;b} = -4\pi J^a, \tag{10}$$

where $J^a$ is the $(n+2)$-current density that becomes $J^a = \bar{\rho}_e U^a$, $\bar{\rho}_e$ being the proper charge density.

In such a background the field Eqs. (6)–(10) for the line-element (1) lead to the following system of equations

$$8\pi\rho + E^2 = -e^{-\lambda}\Big[\frac{n(n-1)}{2r^2} - \frac{n\lambda'}{2r}\Big] + \frac{n(n-1)}{2r}, \tag{11}$$

$$-8\pi\rho_s + E^2 = -e^{-\lambda}\Big[\frac{n\nu'}{2r} + \frac{n(n-1)}{2r^2}\Big] + \frac{n(n-1)}{2r^2}, \tag{12}$$

$$E^2 = \frac{e^{-\lambda}}{2}\Big[\nu'' - \frac{\nu'\lambda'}{2} + \frac{\nu'^2}{2} - \frac{(n-1)(\lambda'-\nu')}{r} + \frac{(n-1)(n-2)}{r^2}\Big] - \frac{(n-1)(n-2)}{2r^2}, \tag{13}$$

$$[r^n E(r)]' = 4\pi\rho_e r^n, \tag{14}$$

where the primes denote the differentiation with respect to $r$, and $E$ is the usual electric field intensity defined as

$$F_{(n+2)1} F^{(n+2)1} = -E^2,$$
$$E(r) = -e^{-(\nu+\lambda)/2}\phi'(r),$$
$$\phi'(r) = F_{1(n+2)} = -F_{(n+2)1}. \tag{15}$$

The charge density $\rho_e$ defined in Eq. (14) is related to the proper charge density $\bar{\rho}_e$ by

$$\rho_e = \bar{\rho}_e e^{\lambda/2}. \tag{16}$$

General relativity provides a rich arena to use symmetries in order to understand the natural relation between geometry and matter furnished by Einstein equations. Symmetries of geometrical/physical relevant quantities of this theory are known as collineations. The most useful collineations are conformal killing vectors which provide a deeper insight into the spacetime geometry and facilitate generation of exact solutions to the field equations. Yavuz and Yilmaz[19] and Yilmaz *et al.*[20] have considered the inheriting conformal and special conformal Killing vectors, as well as the curvature inheritance symmetry in the string cosmology (string cloud and string fluid). Baysal *et al.*[21] have studied the conformal collineation in the string cosmology. The conformal collineation is usually defined by

$$\pounds_\xi g_{ab} = 2\psi g_{ab}, \quad \psi = \psi(x^a), \tag{17}$$

where $\pounds_\xi$ signifies the Lie derivative along $\xi^a$ and $\psi(x^a)$ is the conformal factor. In particular, $\xi$ is a special conformal Killing vector (SCKV) if $\psi_{;ab} = 0$ and $\psi_{,a} \neq 0$. Other sub-cases are homothetic vector (HV) if $\psi_{,a} = 0$ and $\psi \neq 0$, and Killing vector (KV) if $\psi = 0$. Here the subscripts of semicolon and comma denote the covariant and ordinary derivatives, respectively.

To obtain the deterministic solution, we assume that the spacetime admits a one-parameter group of conformal motions, i.e.

$$\pounds_{\xi} g_{ab} = \xi_{a;b} + \xi_{b;a} = 2\psi g_{ab}, \tag{18}$$

where $\psi$ is an arbitrary function of $r$. From Eqs. (1) and (18) and by virtue of spherical symmetry, we can obtain

$$\xi^1 \nu' = \psi, \quad \xi^1 = \frac{\psi r}{2}, \tag{19}$$

$$\lambda' \xi' + 2\xi^1_{,1} = \psi, \quad \xi^{n+2} = C_1 = \text{constant}, \tag{20}$$

where the subscript of comma denotes the partial derivative with respective $r$. From Eqs. (19) and (20), we obtain

$$e^{\nu} = C_2^2 r^2, \quad e^{\lambda} = \left(\frac{C_3^2}{\psi}\right)^2, \tag{21}$$

$$\xi^a = \frac{\psi r}{2}\delta^a_1 + C_1 \delta^a_{n+1}, \tag{22}$$

where $C_2$ and $C_3$ are the constants of integration. Expressions (21) and (22) contain all the implications derived from the existence of the conformal collineation.

Substituting Eq. (21) into Eqs. (11)–(13) yields

$$\rho + E^2 = \frac{n(n-1)}{2r^2}\left[1 - \frac{\psi^2}{C_3^2}\right] - \frac{n\psi\psi'}{C_3^2 r}, \tag{23}$$

$$\rho_s + E^2 = -\frac{n\psi^2}{C_3^2 r^2}\left[1 + \frac{(n-1)}{2}\right] + \frac{n(n-1)}{2r^2}, \tag{24}$$

$$E^2 = \frac{n\psi\psi'}{C_3^2 r} + \frac{n(n-1)\psi^2}{2C_3^2 r^2} - \frac{(n-1)(n-2)}{2r^2}. \tag{25}$$

Here we have used the geometrized unit so that $8\pi G = c = 1$. From Eqs. (5) and (23)–(25), we obtain

$$\rho = -\frac{2n\psi\psi'}{C_3^2 r} - \frac{n(n-1)\psi^2}{C_3^2 r^2} + \frac{(n-1)^2}{r^2}, \tag{26}$$

$$\rho_s = -\frac{n\psi\psi'}{C_3^2 r} + \frac{(n-1)^2}{r^2} - \frac{n^2\psi^2}{C_3^2 r^2}, \tag{27}$$

$$\rho_p = \rho_q + B_c = \rho - \rho_s = \frac{n}{r}\left(\frac{\psi^2}{C_3^2 r} - \frac{\psi\psi'}{C_3^2}\right). \tag{28}$$

Using Eq. (21), the line-element (1) reduces to

$$ds^2 = C_2^2 r^2 dt^2 - \frac{C_3^2}{\psi^2} dr^2 - r^2 d\Omega^2. \tag{29}$$

If the function $\psi$ and an equation of state for the stresses are specified *a priori*, the problem will be fully determined.

If $\psi = C_4 r$, then from Eqs. (25)–(28) we obtain

$$\rho = \rho_s = \frac{(n-1)^2}{r^2} - n(n+1)\left(\frac{C_4}{C_3}\right)^2, \quad \rho_p = 0, \tag{30}$$

$$E^2 = \frac{n(n+1)}{2}\left(\frac{C_4}{C_3}\right)^2 - \frac{(n-1)(n-2)}{2r^2}, \tag{31}$$

where $C_4$ is an integrating constant.

Let us now consider that the charged sphere extends to radius $r_0$. Then the solution of Einstein–Maxwell equations for $r > r_0$ is given by the higher-dimensional Reissner–Nordstöm metric as

$$ds^2 = \left[1 - \frac{2m}{r^{(n-1)}} + \frac{2q^2}{n(n-1)r^{2(n-1)}}\right]dt^2 - \left[1 - \frac{2m}{r^{(n-1)}} + \frac{2q^2}{n(n-1)r^{2(n-1)}}\right]^{-1}dr^2 - r d\Omega^2, \tag{32}$$

and the radial electric field in higher dimensions is

$$E = \frac{q(r)}{r^n}, \tag{33}$$

where $m$ and $q$ are the total mass and charge, respectively.

To match the line-element (29) with the Reisoner–Nordstöm metric (33) across the boundary $r = r_0$ we require continuity of gravitational potential $g_{ab}$ at $r = r_0$,

$$(C_2 r_0)^2 = \left(\frac{\psi}{C_3}\right)^2 = 1 - \frac{2m}{r_0^{(n-1)}} + \frac{2q^2}{n(n-1)r_0^{2(n-1)}}, \tag{34}$$

and also we require the continuity of the electric field, which leads to

$$E(r_0) = \frac{q}{r_0^n}. \tag{35}$$

From Eqs. (31) and (35) we obtain

$$\frac{q}{r_0^{2n}} = \frac{n(n+1)}{2}\left(\frac{C_4}{C_3}\right)^2 - \frac{(n-1)(n-2)}{2r_0^2}. \tag{36}$$

Putting this expression back into Eq. (34) we obtain

$$\frac{m}{r_0^{(n-1)}} = \frac{1}{2} + \frac{1}{(n-1)}\left(\frac{C_4}{C_3}\right)^2 r_0^2 - \frac{(n-2)}{2n}. \tag{37}$$

Also, from Eqs. (31) and (35) we obtain

$$m = \frac{(2n-1)}{n(n+1)} r_0^{(n-1)} + \frac{2q^2}{n(n-1)(n+1)r_0^2}. \tag{38}$$

In summary, we have studied charged strange quark matter attached to the string cloud in the spherically symmetric spacetime admitting one parameter group of conformal motion in the context of higher dimensional spacetime. It is observed that the difference is significant at least in the principle to the analogous situation in four-dimensional spacetime. The solutions reduce to the four-dimensional form when $n = 2$.

We have observed that $e^{\nu}$ and $e^{\lambda}$ are positive, continuous and non-singular for $r < r_0$. For a particular case $\psi = C_4 r$, we have matched our solutions with higher-dimensional Reissner–Nordström metric at $r = r_0$. For this case we have obtained charged geometric string solutions and charged black string solutions (Eqs. (30) and (38)). Also, if $q = 0$ we can obtain total mass for non-charged black string, i.e. higher-dimensional Schwarzschild-like string. It is also observed that if we consider the case $\rho_s = 0$, i.e. $\psi = \frac{1}{n}\sqrt{(n-1)^2C_3^2 + \frac{n^2C_5}{r^{2n}}}$, we will obtain non-charged strange quark matter attached to the string cloud. For $E^2 = 0$, i.e. $\psi = \sqrt{\frac{(n-2)}{n}C_3^2 + \frac{1}{r^{(n-1)}}}$, it is observed that the strange quark matter decreases the energy of the string in the context of higher-dimensional spacetime.

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