Teleported State and its Fidelity in Quantum Teleportation of Continuous Variables *

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(Received 13 August 2002)

When given an unknown quantum state which may be either a pure or a mixed state in the coherent state representation, we show that explicit expressions for the teleported state and its fidelity in the teleportation process (S. L. Braunstein and H. J. Kimble 1998 Phys. Rev. Lett. **80** *869) can be obtained without explicit expansions for the two-mode squeezed vacuum state and the Bell basis in a specified representation.*

PACS: 03. 67. Hk, 42. 50. −p

In the rapidly developing field of quantum communication, one of the crucial problems is how to send a quantum state from one place to another since information is encoded in quantum states. This transmission has two essential points. Firstly, only a quantum state itself is transported and a carrier of the state is kept at the original location. Secondly, no information about a quantum state to be sent is given to the sender prior to the transmission. Thus, in the transmission, it would seem that an unknown quantum state disappears at one place and later emerges at another one. This process is termed teleportation of an unknown quantum state. Bennet *et al.*[1] first proposed a protocol for teleporting an unknown quantum state of two-state systems such as spin-1/2 particles via a classical information channel and a quantum channel built from nonlocal quantum correlation between the sender and the receiver which share a quantum entangled state. This protocol has been extended for various cases.[2] Several experiments have demonstrated the protocol.[3,4] In addition, teleportation of continuous variables corresponding to transferring quantum states in an infinite-dimensional Hilbert space was suggested by Vaidman, employing the perfect correlation in both position and momentum of two particles in the Einstein–Podolsky–Rosen state.[5] It is noted that two quadrature-phase components of a single mode optical field are analogous to position and momentum of a particle. Braunstein and Kimble[6] employed quantum nonlocal correlation between quadrature-phase components of optical fields in a two-mode squeezed vacuum state as a quantum information channel and proposed a quantum optical version of teleportation of continuous variables. Based on this protocol, Furusawa *et al.*[7] experimentally demonstrated quantum teleportation of a coherent state of a single-mode optical field.

Either Bennet's protocol or Braunstein & Kimble's one for quantum teleportation can theoretically be described in terms of Wigner functions.[8] In this formulation, one has to know the Wigner functions for an entangled state which is used to build a quantum information channel and an input state which is to be teleported. Moreover, the connection of this description to the original protocol for teleporting a finite-dimensional quantum state does not become entirely clear. In a teleportation process, four kinds of quantum states are involved, these are an entangled state, an input state, the Bell basis and the teleported state. In a direct approach to the description of quantum teleportation, one expands the former three states in terms of a complete set of basic states and then finds the teleported state in the chosen representation by performing a Bell-state projection. Along this line, Braunstein and Kimble's protocol for quantum teleportation of continuous variables has been studied in the representations of truncated photon number states,[9] displaced photon number states,[10] coherent states,[11,12] and eigenfunctions of quadrapure-phase amplitude operators.[13] In fact, those basic states provide us only with a theoretical working state-space and do not involve any physical information on the teleportation process. Moreover, a concrete representation for the Bell states and the entangled state may make the presentation of the teleportation protocol complicated.[9] In this Letter, we show that the Bell states in Braunstein and Kimble's protocol naturally provide us with a complete set and one can work out the teleported state and its fidelity when only given an input state in the representation of coherent states but without concrete representations for the Bell states and the entangled state.

We suppose that two modes A and B of an optical field are prepared in a squeezed vacuum state

$$|S\rangle_{AB} = \cosh^{-1} r \exp[-a^\dagger b^\dagger \tanh r]|0\rangle, \qquad (1)$$

where $a^\dagger(a)$ and $b^\dagger(b)$ are the bosonic creation and annihilation operators for modes A and B, respectively. When $r \neq 0$, modes A and B are entangled. Here we consider the entanglement between quadratrue-phase components in modes A and B as a quantum information channel for teleportation. If we give mode A to the sender (Alice) and simultaneously mode B to the recipient (Bob), i.e., both sender and recipient share the squeezed state, then a quantum channel is built between Alice and Bob no matter how far away they are or where they are in space because of the existence of the nonlocal quantum correlation between the two modes on their hands. Now let us hand over an arbitrary quantum state $|\varphi_i\rangle$ to Alice. Although no information about this state is given to her, we ask her to send this state to Bob. According to the Braun-

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* Supported by the National Natural Science Foundation of China under Grant Nos 19974032 and 10274058.

stein and Kimble protocol,[6] the task can be fulfilled by the following steps. First, Alice performs a joint Bell-state measurement on the subsystem composed of mode A and the input mode. Then, she sends the measured results to Bob via a classical information channel. Based on Alice's measurements, Bob performs an appropriate unitary transformation on mode B. When the squeezing degree of Eq. (1) is infinite, the output state from Bob can be completely the same as the input state.[6]

Before the teleportation process starts, the whole system consisting of the two entangled modes and the input mode is in the direct product state

$$|\Psi_t\rangle = |\varphi_i\rangle \otimes |S\rangle_{AB}. \tag{2}$$

The local Bell-state measurement performed by Alice on the entangled mode A and the input mode theoretically means that one expands the whole state (2) in terms of the Bell states and projects it on to one of them. In the teleportation protocol,[6] the Bell-state measurement consists of a 50/50 beamsplitter and two homodyne detectors. Imposing the entangled mode A with the input mode on two input ports of the beamsplitter, we obtain two mixed modes which result from both the transmitting part of the entangled mode and the reflecting part of the input mode or vice versa at two output ports of the beamsplitter. The action of the beamsplitter to mode A and the input mode can be represented by the unitary transformation $\hat{U} = \exp[(a^\dagger a_i - a_i^\dagger a)\pi/4]$ with $a_i^\dagger(a_i)$ being creation (annihination) operators for the input mode. In the Heisenberg picture, creation operators for the mixed modes at the two output ports are given by

$$c_1^\dagger = \hat{U}^\dagger a^\dagger \hat{U} = \frac{1}{\sqrt{2}}(a^\dagger + a_i^\dagger),$$
$$c_2^\dagger = \hat{U}^\dagger a_i^\dagger \hat{U} = \frac{1}{\sqrt{2}}(a^\dagger - a_i^\dagger). \tag{3}$$

In the Schrödinger picture, after being transformed by the beamsplitter, the whole state vector (2) becomes $|\Psi_t'\rangle = \hat{U}|\Psi_t\rangle$.

Let us introduce two quadrature-phase operators

$$\hat{x}_j = \frac{1}{2}(c_j + c_j^\dagger), \quad \hat{p}_j = \frac{1}{2\mathrm{i}}(c_j - c_j^\dagger), \quad j = 1, 2. \tag{4}$$

In the next joint homodyne measurement, one of the homodyne detectors is arranged to measure eigenvalues of the quadrature-phase operator $\hat{x}_1$ and the other to measure eigenvalues of the quadrature-phase operator $\hat{p}_2$. The measurement projects the entangled mode A and the input mode into one of simultaneous eigenstates of the commutative operators $\hat{x}_1$ and $\hat{p}_2$, depending on which of the eigenvalues are measured. Suppose that $|x_j\rangle$ and $|p_j\rangle$ are eigenstates of the quadrature-phase operators $\hat{x}_j$ and $\hat{p}_j$ with eigenvalues x_j and p_j, respectively. According to the commutator $[\hat{x}_j, \hat{p}_j] = \mathrm{i}/2$, we can show that these eigenstates have the following mutual expansion properties

$$|x_j\rangle = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{d}p_j \mathrm{e}^{-\mathrm{i}2p_j x_j} |p_j\rangle,$$
$$|p_j\rangle = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{d}x_j \mathrm{e}^{\mathrm{i}2p_j x_j} |x_j\rangle, \tag{5}$$

and satisfy the completeness conditions

$$\int_{-\infty}^{\infty} \mathrm{d}x_j |x_j\rangle\langle x_j| = \hat{I}, \quad \int_{-\infty}^{\infty} \mathrm{d}p_j |p_j\rangle\langle p_j| = \hat{I}, \tag{6}$$

and hold the orthogonal conditions $\langle x_j|x_j'\rangle = \delta(x_j - x_j')$ and $\langle p_j|p_j'\rangle = \delta(p_j - p_j')$. When the two homodyne detectors have outputs x_1 and p_2, the entangled mode A and the input mode are projected into the eigenstate $|x_1, p_2\rangle = |x_1\rangle \otimes |p_2\rangle$, and meanwhile the entangled mode B is placed into the state $|\psi(x_1, p_2)\rangle = \langle x_1, p_2|\Psi_t\rangle$. In the Heisenberg picture, simultaneous eigenstates $|x_1, p_2\rangle$ are obviously just the Bell states in the teleportation protocol under consideration. In the Shrödinger picture, the joint homodyne measurement makes $|\Psi_t'\rangle$ jump to $\langle x_i, p_a|\hat{U}|\Psi_t\rangle$, where $|x_i, p_a\rangle$ is one of simultaneous eigenstates of the quadrature phase operators $\hat{x}_i$ of the input mode and $\hat{p}_a$ of the entangled mode A. Thus, in the Schrödinger picture, the Bell states are $\hat{U}^\dagger|x_i, p_a\rangle$. By use of Eqs. (3), it is easily shown that $\hat{U}^\dagger|x_i, p_a\rangle$ are just simultaneous eigenstates of the quadrature phase operators $\hat{x}_1$ and $\hat{p}_2$. It should be pointed out that the joint homodyne measurement cannot distinguish the projective states $|x_i, p_a\rangle$ from $|x_1, p_2\rangle$ because both have the same eigenvalues. Choosing a complete set of states, one can find out the corresponding representation for the Bell states. Previous studies have obtained the representations for the Bell basis in the truncated photon number state space,[9] the displaced photon number state space,[10] the coherent state space,[11] and the space of eigenwavefunctions of the quadrature-phase amplitude operators.[13] In the following discussions, we will see that, to obtain explicit expressions for the teleported state in mode B and its fidelity, one must not work in a concrete representation for the Bell basis.

In the representation of coherent states, an input state can be expanded as

$$|\varphi_i\rangle = \int \mathrm{d}^2\alpha P(\alpha) \mathrm{e}^{-\frac{1}{2}|\alpha|^2} \mathrm{e}^{\alpha a_i^\dagger} |0\rangle, \tag{7}$$

where the complex amplitude $P(\alpha) = \langle\alpha|\varphi_i\rangle/\pi$. Substituting the inverse transformations of Eq. (3) into Eq. (2), we have

$$\begin{aligned}|\Psi_t\rangle =& \cosh^{-1} r \int \mathrm{d}^2\alpha P(\alpha) \mathrm{e}^{-\frac{1}{2}|\alpha|^2} \\ &\cdot \exp\Big\{\frac{1}{\sqrt{2}}\big[(\alpha - \tanh r b^\dagger)(\hat{x}_1 - \mathrm{i}\hat{p}_1) \\ &- (\alpha + \tanh r b^\dagger)(\hat{x}_2 - \mathrm{i}\hat{p}_2)\big]\Big\}|0\rangle.\end{aligned} \tag{8}$$

By using the completeness conditions (6), the expansion relations (5), and the operator theorem $\mathrm{e}^{c\hat{x}_j + \mathrm{d}\hat{p}_j} = \mathrm{e}^{c\hat{x}_j}\mathrm{e}^{\mathrm{d}\hat{p}_j}\mathrm{e}^{-\mathrm{i}cd/4} = \mathrm{e}^{\mathrm{d}\hat{p}_j}\mathrm{e}^{c\hat{x}_j}\mathrm{e}^{\mathrm{i}cd/4}$ with c-numbers c and d, we can rewrite state (8) as

$$|\Psi_t\rangle = \int \mathrm{d}x_1 \mathrm{d}p_2 |\psi(x_1, p_2)\rangle \otimes |x_1, p_2\rangle, \tag{9}$$

where $|\psi(x_1, p_2)\rangle$ is the unnormalized state of mode B and is given by

$$|\psi(x_1, p_2)\rangle = \sqrt{\frac{2}{\pi}} \cosh^{-1} r \exp(-|z|^2) \int \mathrm{d}^2\alpha P(\alpha)$$

$$\cdot \exp\Big[-\frac{1}{2}|\alpha|^2+\sqrt{2}z^*\alpha\Big]\exp[(\alpha-\sqrt{2}z)b^\dagger\tanh r]|0\rangle \tag{10}$$

with $z=x_1-\mathrm{i}p_2$. In Eq. (9), we have expanded the state of the whole system in terms of the Bell states. After the Bell-state measurement, mode B is forced into state (10).

In the limit $r\to\infty$, the entanglement between modes A and B becomes perfect and the state vector (10) must be entirely the same as the input state (7) except a normalization factor. In order to satisfy this requirement, compared Eq. (10) with Eq. (7) under the limitation, we note that the unitary transformation $D(\beta)=\exp(\beta b^\dagger-\beta^* b)$ with the amplitude $\beta=-\sqrt{2}(x_1-\mathrm{i}p_2)$ has to be applied to mode B. After performing the unitary transformation $D^+(\beta)$ on Eq. (10), we obtain the unnormalized state for mode B

$$|\phi(x_1,p_2)\rangle=\sqrt{\frac{2}{\pi}}\cosh^{-1}r\exp[-2(1-\tanh r)|z|^2]$$
$$\cdot\int \mathrm{d}^2\alpha P(\alpha)\exp\Big[-\frac{1}{2}|\alpha|^2+\sqrt{2}(1-\tanh r)z^*\alpha\Big]$$
$$\cdot\exp\{[\alpha\tanh r+\sqrt{2}(1-\tanh r)z]b^\dagger\}|0\rangle. \tag{11}$$

Since this unitary transformation is dependent on Alice's measurement results and is performed on the side of Bob, to accomplish the transformation, Bob has to know Alice's measurement outputs (x_1,p_2). Because the measurement results are classical quantities, Alice can send them to Bob via a classical information channel.

After the processes mentioned above, mode B can be placed in the normalized state $|\Phi(x_1,p_2)\rangle = w^{-1/2}(x_1,p_2)|\phi(x_1,p_2)\rangle$, where $w(x_1,p_2)$ is the probability density for outputs (x_1,p_2) of the joint homodyne measurement and is given by

$$w(x_1,p_2)=\frac{2}{\pi}\cosh^{-2}r\int \mathrm{d}^2\alpha \mathrm{d}^2\beta P(\alpha)P^*(\beta)\langle\beta|\alpha\rangle$$
$$\cdot\exp\Big\{-2\cosh^{-2}r\Big[\Big(x_1-\frac{1}{2\sqrt{2}}(\alpha+\beta^*)\Big)^2$$
$$+\Big(p_2-\frac{\mathrm{i}}{2\sqrt{2}}(\alpha-\beta^*)\Big)^2\Big]\Big\}. \tag{12}$$

It is clear that $|\Phi(x_1,p_2)\rangle$ approaches the input state (7) when $r\to\infty$. This means that in this limitation, an unknown quantum state can exactly be teleported from Alice to Bob. If the squeezing degree is finite, however, we can see that the complex amplitudes of coherent states in Eq. (11) are not exactly the same as that in Eq. (7). Therefore, statistical properties of the original state are distorted in the teleportation process. When r is sufficient large, $|\Phi(x_1,p_2)\rangle$ may be viewed as an approximate copy of Eq. (7).

From the above discussion, it is obvious that the teleportation is a process in which an unknown quantum state is disintegrated into a projective state of the whole system on one of the Bell states, which is quantum information and can be instantaneously transferred from a sender to a remote receiver via the quantum channel, and the classical information that exhibits which of the Bell states the entangled mode A and the input mode are projected on. When the projective state is in a unitary transformation different from the input one, based on the classical information, Bob can reconstruct the input state on his side.

If eigenvalues (x_1,p_2) of $\hat{x}_1$ and $\hat{p}_2$ were discrete, the Bell basis would be discrete and the process mentioned above would be just the version of teleportation of an unknown quantum state, which was first proposed by Bennet *et al.*[1] There exist only four discrete Bell states in the original teleportation, each of which can be measured with a 1/4 probability, respectively. By use of only four Bell-state detectors in the protocol, one can definitely realize the teleportation in a single joint measurement. In the present case, however, (x_1,p_2) are continuous variables and are measured with the probability density $w(x_1,p_2)$, and the number of Bell states $|x_1,p_2\rangle$ is infinite. Therefore, only when the two homodyne detectors are definitely locked at a pair of outputs (x_1,p_2), the mode B could be probably placed in $|\Phi(x_1,p_2)\rangle$ in the single joint homodyne measurements. Therefore, $|\Phi(x_1,p_2)\rangle$ is a conditional output state. The fidelity of the conditional teleportation can be measured by the squared modulo of the overlap between the teleported state and the input state, which is given by $F(x_1,p_2)=|\langle\varphi_i|\Phi(x_1,p_2)\rangle|^2$. If a series of the entirely same input states are in sequence given to Alice and the two homodyne detectors are able to respond to all of eigenvalues of the quadrature-phase operators in the teleportation process, regardless of which state appears on Bob's side, the teleported field is into a mixed state, which is described by the density matrix

$$\hat{\rho}=\int \mathrm{d}x_1\mathrm{d}p_2 w(x_1,p_2)|\Phi(x_1,p_2)\rangle\langle\Phi(x_1,p_2)|$$
$$=\int \mathrm{d}x_1\mathrm{d}p_2|\phi(x_1,p_2)\rangle\langle\phi(x_1,p_2)|. \tag{13}$$

Since (x_1,p_2) is continuous and measured probably, in this case, an averaged fidelity is appreciate for the teleportation process and is given by

$$F_a=\int \mathrm{d}x_1\mathrm{d}p_2 w(x_1,p_2)F(x_1,p_2)$$
$$=\frac{1}{2}(1+\tanh r)\int \mathrm{d}^2\alpha\mathrm{d}^2\beta\mathrm{d}^2\xi\mathrm{d}^2\eta$$
$$\cdot P(\alpha)P(\xi)P(\beta)^*P(\eta)^*$$
$$\cdot\exp\Big[-\frac{1}{2}(|\alpha|^2+|\xi|^2+|\eta|^2+|\beta|^2)\Big]$$
$$\cdot\exp\Big\{\frac{1}{2}[(\alpha+\xi)(\eta^*+\beta^*)$$
$$+\tanh r(\alpha-\xi)(\eta^*-\beta^*)]\Big\}. \tag{14}$$

Using this general and explicit expression, we can easily calculate the fidelity of the teleportation for various quantum states, especially for the discrete linear superposition of coherent states. For example, the averaged fidelity for a linear superposition of two coherent states $c_1|\alpha_1\rangle+c_2|\alpha_2\rangle$ is given by

$$F_a=\frac{1}{2}(1+\tanh r)\Big\{1-2|c_1c_2|^2\Big[1+\mathrm{e}^{4\mathrm{Re}(\alpha\alpha^*)}$$
$$-\mathrm{e}^{-\frac{1}{2}(1+\tanh r)(|\alpha_1|^2+|\alpha_2|^2-2\mathrm{Re}(\alpha_1\alpha_2^*))}$$

$$- \mathrm{e}^{-\frac{1}{2}(1-\tanh r)(|\alpha_1|^2+|\alpha_2|^2-2\mathrm{Re}(\alpha_1\alpha_2^*))}\Big]$$
$$\cdot \Big[|c_1|+|c_2|^2+2\mathrm{Re}(c_1c_2^*\mathrm{e}^{\alpha_1\alpha_2^*})$$
$$\cdot \mathrm{e}^{\frac{1}{2}(|\alpha_1|^2+|\alpha_2|^2)}\Big]^{-2}\Big\}. \quad (15)$$

For a coherent state, Eq. (15) reduces to $F_a = (1+\tanh r)/2$. When $\alpha_1 = -\alpha_2$ and $c_1 = 1$ and $c_2 = \exp(\mathrm{i}\varphi)$, Eq. (15) recovers the result given in Ref. [6]. In the two limits $r = 0$ and $r \to \infty$, $F_a = 0.5$ and 1.0, respectively. This means that when F_a is beyond 0.5, the quantum nonlocal correlation between the modes A and B of the squeezed vacuum state definitely plays a role in the teleportation process.

The above result can easily be generalized to an input mixed state. In Glauber's representation for a density matrix, an input state $\hat{\rho}_i$ can be written as $\hat{\rho}_i = \int \mathrm{d}^2\alpha \mathrm{d}^2\beta R(\alpha,\beta)|\alpha\rangle\langle\beta|$, where $R(\alpha,\beta) = \langle\alpha|\hat{\rho}_i|\beta\rangle/\pi^2$. Following the same procedure as stated above, we have the average normalized density matrix for mode B: $\hat{\rho}_a = \int \mathrm{d}x_1\mathrm{d}p_2 w(x_1,p_2)\hat{\rho}(x_1,p_2)$, where the conditional normalized matrix density $\hat{\rho}(x_1,p_2)$ is given by

$$\begin{aligned}\hat{\rho}(x_1,p_2) =& \frac{2}{\pi}\cosh^{-2} r \exp[-4(1-\tanh r)|z|^2] \\ &\cdot \int \mathrm{d}^2\alpha\mathrm{d}^2\beta R(\alpha,\beta)\exp\Big[-\frac{1}{2}(|\alpha|^2+|\beta|^2)\Big] \\ &\cdot \exp[\sqrt{2}(1-\tanh r)(z^*\alpha+z\beta^*)] \\ &\cdot \exp\{[\alpha\tanh r+\sqrt{2}(1-\tanh r)z]b^\dagger\}|0\rangle \\ &\otimes \langle 0|\exp\{[\beta^*\tanh r+\sqrt{2}(1-\tanh r)z^*]b\},\end{aligned} \quad (16)$$

and the probability density for measuring results (x_1,p_2) is given by

$$\begin{aligned}w(x_1,p_2) =& \frac{2}{\pi}\cosh^{-2} r\int \mathrm{d}^2\alpha\mathrm{d}^2\beta R(\alpha,\beta)\langle\beta|\alpha\rangle \\ &\cdot \exp\Big\{-2\cosh^{-2} r\Big[\Big(x_1-\frac{1}{2\sqrt{2}}(\alpha+\beta^*)\Big)^2 \\ &+\Big(p_2-\frac{\mathrm{i}}{2\sqrt{2}}(\alpha-\beta^*)\Big)^2\Big]\Big\}.\end{aligned} \quad (17)$$

The averaged fidelity is

$$\begin{aligned}F_a =& \frac{\mathrm{tr}(\hat{\rho}_i\otimes\hat{\rho}_a)}{\mathrm{tr}(\hat{\rho}_i\otimes\hat{\rho}_i)} \\ =&\frac{1}{2}(1+\tanh r)\int \mathrm{d}^2\alpha\mathrm{d}^2\beta\mathrm{d}^2\xi\mathrm{d}^2\eta R(\alpha,\beta)R(\xi,\eta) \\ &\cdot \exp\Big[-\frac{1}{2}(|\alpha|^2+|\xi|^2+|\eta|^2+|\beta|^2)\Big] \\ &\cdot \exp\Big\{\frac{1}{2}[(\alpha+\xi)(\eta^*+\beta^*)+\tanh r(\alpha-\xi)(\eta^*-\beta^*)]\Big\} \\ &\cdot \Big[\int \mathrm{d}^2\alpha\mathrm{d}^2\beta\mathrm{d}^2\xi\mathrm{d}^2\eta R(\alpha,\beta)R(\xi,\eta)\langle\beta|\xi\rangle\langle\eta|\alpha\rangle\Big]^{-1}.\end{aligned} \quad (18)$$

In order to make $F_a = 1$ when $r \to \infty$, we have introduced a normalized factor $\mathrm{tr}(\hat{\rho}_i\otimes\hat{\rho}_i)$ in Eq. (18) because $\mathrm{tr}(\hat{\rho}_i\otimes\hat{\rho}_i) < 1$ for a mixed state.

Besides the above general coherent state representation for an input state, a density matrix may have a diagonal representation in terms of coherent states:[15,16] $\hat{\rho}_i = \int \mathrm{d}^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$. In this representation, we can find the conditional density matrix for mode B

$$\begin{aligned}\hat{\rho}(x_1,p_2) =& \frac{2}{\pi}\cosh^{-2} r\exp[-4(1-\tanh r)|z|^2] \\ &\cdot \int \mathrm{d}^2\alpha P(\alpha)\exp[-|\alpha|^2] \\ &\cdot \exp[\sqrt{2}(1-\tanh r)(z^*\alpha+z\alpha^*)] \\ &\cdot \exp\{[\alpha\tanh r+\sqrt{2}(1-\tanh r)z]b^\dagger\}|0\rangle \\ &\otimes \langle 0|\exp\{[\alpha^*\tanh r+\sqrt{2}(1-\tanh r)z^*]b\},\end{aligned} \quad (19)$$

and the probability density for measuring eigenvalues x_1 and p_2

$$\begin{aligned}w(x_1,p_2) =& \frac{2}{\pi}\cosh^{-2} r\int \mathrm{d}^2\alpha P(\alpha) \\ &\cdot \exp\Big\{-2\cosh^{-2} r\Big[\Big(x_1-\frac{1}{2\sqrt{2}}(\alpha+\alpha^*)\Big)^2 \\ &+\Big(p_2-\frac{\mathrm{i}}{2\sqrt{2}}(\alpha-\alpha^*)\Big)^2\Big]\Big\}.\end{aligned} \quad (20)$$

The averaged fidelity is given by

$$\begin{aligned}F_a =& \frac{1}{2}(1+\tanh r)\Big\{\int \mathrm{d}^2\alpha\mathrm{d}^2\beta P(\alpha)P(\beta) \\ &\cdot \exp\Big[-\frac{1}{2}(1+\tanh r)(|\alpha|^2+|\beta|^2-2\mathrm{Re}(\alpha\beta^*))\Big]\Big\} \\ &\cdot \Big\{\int \mathrm{d}^2\alpha\mathrm{d}^2\beta P(\alpha)P(\beta) \\ &\cdot \exp\Big[-\frac{1}{2}(|\alpha|^2+|\beta|^2-2\mathrm{Re}(\alpha\beta^*))\Big]\Big\}^{-1}.\end{aligned} \quad (21)$$

From the above discussions, we can see that Braunstein and Kimble's protocol can also be used to teleport unknown mixed quantum states in completely the same way as it is used for pure states since the teleportation process is, in principle, a linear one.

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