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# Dynamics of Two Dark Solitons in a Polariton Condensate

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We theoretically investigate dynamics of two dark solitons in a polariton condensate under nonresonant pumping, based on driven dissipative Gross–Pitaevskii equations coupled to the rate equation. The equation of motion of the relative center position of two-dark soliton is obtained analytically by using the Lagrangian approach. In particular, the analytical expression of the effective potential between two dark solitons is given. The resulting equation of motion captures how the open-dissipative character of a polariton Bose–Einstein condensate affects properties of dynamics of two-dark soliton, i.e., two-dark soliton relax by blending with the background at a finite time. We further simulate the relative motion of two dark solitons numerically with the emphasis on how two-soliton motion is manipulated by the initial velocity, in excellent agreement with the analytical results. The prediction of this work is sufficient for the experimental observations within current facilities.

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To date, there have been significant research interests in exciton-polariton Bose–Einstein condensates (BEC) in semiconductor microcavities as a novel platform for realization and investigation of nonlinear physics.[1−4] On the one hand, at the mean-field level, the static and dynamic properties of a polariton BEC can be well described by the nonlinear Schrödinger equation or Gross–Pitaevskii equation (GPE), which has been a paradigm of theoretical and experimental studies of coherent nonlinear dynamics.[5−10] On the other hand, a polariton condensate is intrinsically non-equilibrium, with coherent and dissipative dynamics occurring on an equal footing. This has provided a new stage for practical applications of the GPE. Up to date, the non-equilibrium nature of the polariton has resulted in numerous intriguing nonlinear phenomena in polariton condensates.[11,12]

In the quest for novel scenarios that display combined effects of dissipation and nonlinearity on the nonlinear phenomena, the study of soliton in polariton condensates is among the hottest topics, with an emphasis on capturing the non-equilibrium nature of the soliton with no analog in the static counterpart.[13−22] Generally speaking, soliton is a kind of self-reinforcing solitary wave, which is formed by the cancelation of nonlinear effect and dispersion effect in mediums and it can maintain its shape during propagation.[23−27] Dark or bright solitons can exist provided that the interaction is repulsive or attractive.[28,29] In a polariton condensate, the nonlinearity of the polariton condensate arises from strongly and repulsively interacting excitons, where the interaction can be controlled by Feshbach resonance.[30,31] A series of experiments have demonstrated existence of the oblique dark solitons and vortices,[32−36] or bright spatial and temporal solitons.[37] For example, in condensates created spontaneously under incoherently pumping, formation and properties of dark solitons have been investigated theoretically in Ref. [38], and the existence of stable dark soliton trains has been reported in the non-resonantly driven spinor polariton BEC at one dimension.[39] However, to our best knowledge, previous studies on solitons in a polariton condensate were limited to the one-soliton problem, whereas the two-soliton problem in non-equilibrium polariton BEC has remained to be unexplored so far. It is the purpose of the present work to investigate how the interplay of nonlinearity, dispersion and dissipation affect existence and properties of two-dark soliton in a polariton BEC.

In this Letter, we present the first analytical result on the two-dark soliton problem in the context of a polariton BEC formed under non-resonant pumping by solving the dissipative GPE. First, we use the variational approach and analytically derive the time evolution equations for the soliton parameters, i.e., the relative distance between solitons. We compare this analytical result with the numerical solutions for the trajectory of two solitons directly obtained from the dissipative GPE, finding a remarkable agreement between both of them. Our results open a new route to observe stable two-solitons in non-equilibrium polariton BEC within current experimental facilities.

*Model.* We are interested in an exciton-polariton BEC under nonresonant pumping created in a wire-shaped microcavity similar to the one implemented

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in Ref. [40], which bounds the polaritons to a quasi-one-dimensional (1D) channel. Within the framework of the mean field theory, the polariton field described by the condensate order parameter of $\psi(x,t)$ evolves along an effectively 1D driven-dissipative GPE coupled to a rate equation for the density $n_{\rm R}(x,t)$ of reservoir polaritons as follows:[41]

$$i\hbar\frac{\partial\psi}{\partial t}=\Big[-\frac{\hbar^2}{2\,m}\frac{\partial^2}{\partial x^2}+g_{\rm C}|\psi|^2+g_{\rm R}n_{\rm R}+\frac{i\hbar}{2}(Rn_{\rm R}-\gamma_{\rm C})\Big]\psi, \tag{1}$$

$$\frac{\partial n_{\rm R}}{\partial t}=P-(\gamma_{\rm R}+R|\psi|^2)n_{\rm R}, \tag{2}$$

where $m$ is the effective mass of lower polaritons; $P$ is the rate of an off-resonant continuous-wave pumping; $\gamma_{\rm C}$ and $\gamma_{\rm R}$ describe the lifetime of the condensate and reservoir polaritons, respectively; and $R$ is the stimulated scattering rate of reservoir polaritons into the condensate; $g_{\rm C}$ and $g_{\rm R}$ characterize the strength of nonlinear interaction of polaritons and the interaction strength between reservoir and polariton, respectively. Note that the parameters of $g_{\rm C}$, $g_{\rm R}$, and $R$ have been rescaled into the one-dimensional case by the width $d$ of the nanowire thickness as that ($g_{\rm C}\to g_{\rm C}/\sqrt{2\pi}d$, $g_{\rm R}\to g_{\rm R}/\sqrt{2\pi}d$, $R\to R/\sqrt{2\pi}d$).

The emphasis and value of this work are to take account of the intrinsic dissipation and to search for the possibility of the existence and dynamics of two-dark soliton in a polariton condensate. It is well known that the dark soliton is characterized with a localized dip in the condensate density with an associated phase gradient. Hence, we first need to determine the steady state of the model system based on Eqs. (1) and (2), which serve as the density background of the dark soliton's propagation. Directly following Ref. [41], the stable condensate appears under the condition of the pumping $P$ in Eq. (2) that is larger than a critical value of $P_{\rm th}=\gamma_{\rm R}\gamma_{\rm C}/R$. Then, the steady homogeneous condensate and reservoir densities are expressed as follows: $n^0_{\rm C}=(P-P_{\rm th})/\gamma_{\rm C}$ and $m^0_{\rm R}=\gamma_{\rm C}/R$.

For convenience, we proceed to rescale Eqs. (1) and (2) into the dimensionless form. In more details, we rescale $\psi\to\psi/\sqrt{n^0_{\rm C}}$ and introduce $m_{\rm R}=n_{\rm R}-n^0_{\rm R}$; as a result, Eqs. (1) and (2) can be rewritten as

$$i\frac{\partial\psi}{\partial t}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}-(|\psi|^2-1)\psi=\Big(\bar g_{\rm R}m_{\rm R}+\frac{i}{2}\bar Rm_{\rm R}\Big)\psi, \tag{3}$$

$$\frac{\partial m_{\rm R}}{\partial t}=\bar\gamma_{\rm C}(1-|\psi|^2)-\bar\gamma_{\rm R}m_{\rm R}-\bar R|\psi|^2m_{\rm R}. \tag{4}$$

Here, the dimensionless parameters are given by $\bar g_{\rm R}=g_{\rm R}/g_{\rm C}$, $\bar\gamma_{\rm C}=\gamma_{\rm C}\bar\gamma_{\rm R}/\gamma_{\rm R}$ and $\bar R=\hbar R/g_{\rm C}n^0_{\rm C}$. Moreover, the time $t$ and the space coordinate $x$ are measured in units of $\tau_0=\hbar gn^0_{\rm C}$ and $x_{\rm h}=\sqrt{\hbar^2/mgn^0_{\rm C}}$. Note that Eqs. (3) and (4) are the starting point of investigating the two-dark soliton problem in the context of the polariton BEC. The non-equilibrium nature of the model system is captured by the parameters of $\bar R$ on the right hand of Eq. (3). In the following, we are theoretically interested in how the nonequilibrium nature affects the dynamics of two-dark soliton.

*Two Dark Solitons.* Before investigating effects of non-equilibrium nature of the model system characterized by $\bar R$ on the two-dark-soliton solution, we first briefly review some important features of the normal GPE, corresponding to Eq. (3) with $\bar g_{\rm R}=\bar R=0$. Under the boundary condition of $\psi\to 1$ as $|x|\to\infty$, the two-dark-soliton solution can be written as[42]

$$\psi=(B\tanh x_+-iA)(B\tanh x_-+iA), \tag{5}$$

with $A^2+B^2=1$. Here $x_\pm=B(x\pm x_0)$ is referred as to the center position of two dark solitons and $2x_0$ can be treated as the relative center position between two solitons. In general, we can obtain $x_0=v_{\rm s}t$, $A=v_{\rm s}$ and $B=\sqrt{1-v_{\rm s}^2}$ with $v_{\rm s}$ being the velocity of the dark soliton. In order to better understand the trial wavefunction of two dark solitons in Eq. (5), we plot the density profile with the parameter of $v_{\rm s}=0$ in Fig. 1(a), characterizing that the phase of the two-dark-soliton solution $\psi(x,t)$ has $\pi$ phase jump profile [see Fig. 1(b)].

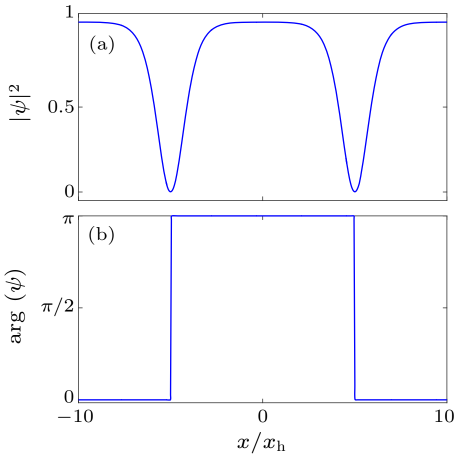


**Fig. 1.** (a) Schematics of the condensate density of $|\psi|^2$ and (b) phase of the condensate order parameter of arg($\psi$) corresponding to a stationary one-dimensional two-dark-soliton solution in Eq. (5) with the parameter of $v_{\rm s}=0$.

Adding the open-dissipative effects as captured by $\bar R$ introduces an external perturbation of the two dark solitons in Eq. (5), which leads to two immediate consequences: Firstly, all the parameters of two-dark-soliton solution in Eq. (5) become slow functions of time, i.e., $A\to A(t)$, $B\to B(t)$, and $x_0\to x_0(t)$, while the functional form of the soliton remains unchanged, which is at heart of the Lagrange approach of quantum dynamics of two dark solitons in the presence of perturbation. Secondly, it is supposed that the two dark solitons will relax by blending with the background at a finite time. Thus, we will rely on the Lagrange approach of the dark soliton perturbation theory which allows for the analytical treatment of effects of the right hand in Eq. (3).

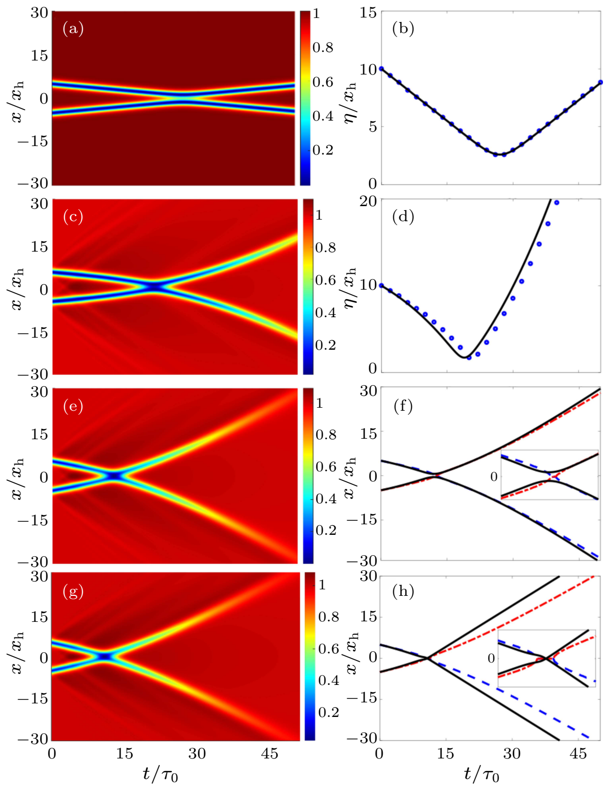


**Fig. 2.** Dynamics of 1D two dark solitons in a polariton BEC with different initial velocities. Left: the contour plots of the two dark solitons of $|\psi|^2$. Right: the equations of motion of relative center mass of two dark solitons corresponding to the analytical results (solid curves) in Eq. (10) and the numerical results (dotted curved) by solving Eqs. (3) and (4). The solid black lines are calculated using the analytical results of the relative motion of two dark solitons, $\eta = 2x_0$, in Eq. (10) in (b), (d) and (f). For the parameters: (a) and (b) $v_{\rm s} = 0.15$, $\bar{g}_{\rm R} = \bar{\gamma}_{\rm C} = \bar{\gamma}_{\rm R} = \bar{R} = 0$. In other plots, we have chosen $\bar{g}_{\rm R} = 2$, $\bar{\gamma}_{\rm C} = 3$, $\bar{\gamma}_{\rm R} = 15$ and $\bar{R} = 1.5$; (c) and (d) $v_{\rm s} = 0.15$; (e) and (f) $v_{\rm s} = 0.32$; (g) and (h) $v_{\rm s} = 0.4$.

We focus on the relative center mass position of two-dark-soliton solution corresponding to the time variation of the parameter of $x_0(t)$. As such, we can obtain the equation of motion of $x_0$ by employing the Lagrangian approach for the perturbation theory of soliton as Refs. [42–44], reading

$$\frac{d}{dt}\Big(\frac{\partial L}{\partial \dot{x}_0}\Big) - \frac{\partial L}{\partial x_0} = -2{\rm Re}\Big(\int_{-\infty}^{+\infty} R^* \psi \frac{\partial \psi^*}{\partial x_0} dx\Big). \quad (6)$$

Here, the Lagrangian $L$ has the following form $L = \int_{-\infty}^{\infty} [\frac{i}{2}(\psi^* \frac{\partial}{\partial t}\psi - \psi \frac{\partial}{\partial t}\psi^*)(1 - \frac{1}{|\psi|^2}) - \frac{1}{2}|\frac{\partial \psi}{\partial x}|^2 - \frac{1}{2}(|\psi|^2 - 1)^2] dx$. The term of $R^* = (\bar{g}_{\rm R} + i\bar{R}/2) m_{\rm R}$ is referred as to dissipative effects induced by the reservoir. Directly substituting Eq. (5) into Eq. (6), the Lagrangian $L$ has the following specific form

$$L = 2L_0 + 4\frac{dB}{dt} AB[\tanh(2Bx_0)]^{-1} + 16B^5 e^{-4Bx_0}, \quad (7)$$

with $L_0 = 2\frac{dx_0}{dt}[-AB + \tan^{-1}\big(\frac{B}{A}\big)] - \frac{4}{3}B^3$ being the one soliton's Lagrangian.

The next calculations of the right-hand side of Eq. (6) require knowledge of the reservoir density $m_{\rm R}$. In this work, we have limited the calculations in the fast reservoir limit, under which the reservoir density can be simplified to the following form

$$m_{\rm R}^0 = \frac{\bar{\gamma}_{\rm C}}{\bar{\gamma}_{\rm R}}(1 - |\psi|^2). \quad (8)$$

Substituting Eqs. (5) and (8) into Eq. (6), we can routinely calculate the right-hand side of Eq. (6).

Next, directly following Ref. [43], we employ the variational method by substituting Eqs. (5) and (8) into Eq. (6) and obtain the equation of motion of the two-soliton parameters as follows:

$$\frac{dx_0}{dt} = A, \qquad \frac{dA}{dt} = 8B^5 \exp(-4x_0 B) + F_{\rm eff}. \quad (9)$$

Here, the velocity of the center mass of soliton of $dx_0/dt$ is only connected with the parameter of $A$. Such an approximation has been firstly adopted by Ref. [43], which is further checked to be valid by comparing the numerical results with the analytical ones as is expected.

By eliminating the variable $A$ in Eq. (9) under the condition of $A^2 + B^2 = 1$, we can obtain the equation of motion of the relative center motion $x_0$ of the two-soliton as follows:

$$\frac{d^2 x_0}{dt^2} = -\frac{\partial V(x_0)}{\partial x_0} + F_{\rm eff}, \quad (10)$$

with $V(x_0) = 2B^4 \exp(-4x_0 B)$ being the effective potential of the relative center motion of the two solitons and the dissipation-induced force $F_{\rm eff}$, reading

$$F_{\rm eff} = \frac{1}{3}\bar{R}\frac{\bar{\gamma}_{\rm C}}{\bar{\gamma}_{\rm R}} AB^2 - \frac{4}{3}B^3 \bar{g}_{\rm R}\frac{\bar{\gamma}_{\rm C}}{\bar{\gamma}_{\rm R}}(8B^2 - 12Bx_0 + 9)e^{-4Bx_0} + \frac{32}{3}B^7 \bar{g}_{\rm R}\frac{\bar{\gamma}_{\rm C}}{\bar{\gamma}_{\rm R}}(24Bx_0 - 25)\exp(-8Bx_0). \quad (11)$$

Equation (10) is the key result of this work, which allows us to interpret the dynamics of the two-dark soliton in terms of the motion of a classical particle trapped in an effective potential $V(x_0)$ subjected to an external force $F_{\rm eff}$. Equation (10) can recover the corresponding result in Ref. [43] as is expected. From Eq. (10), it is clear that the key physics is that the effective potential of $V(x_0)$ is repulsive; as a result, the two dark solitons is supposed to repel each other when they move close to each other. Based on Eq. (10), we conclude the effective potential $V(x_0) = 2B^4 \exp(-4x_0 B)$, which decays exponentially with the $x_0$ dominating in the case of $x_0 \ll 1$, i.e., two dark solitons are relative closer to each other. This can be understood as follows: the effective potential physically originates from the overlapping between the wave functions of two solitons and $V(x_0)$ is supposed to only play an important role under $x_0 \ll 1$. In contrast, in the case of $x_0 \gg 1$ under which the effective potential $V(x_0)$ decays exponentially with the $x_0$, the dissipation effect, in particular the first term of $F_{\rm eff}$ in Eq. (11), which is independent of $x_0$, plays a more important role in such a case, resulting in two-dark-soliton relax by blending with the background at a finite time.

So far, we have developed an analytically physical picture of two-dark-soliton solution based on Eq. (10) and predicted features of the dissipation affecting the dynamics of two-dark soliton compared to the coherent case. In what follows, we plan to investigate dynamics of two-dark-soliton solution in a more broader parameter regimes by numerically solving Eqs. (3) and (4) with the initial wave function given by Eq. (5). We focus on the interplay of nonlinearity, dispersion and dissipation affecting the existence and properties of two dark solitons in a polariton BEC.

We first briefly review some important features of a two-dark-soliton solution in the coherent case, corresponding to the $m_{\rm R} = 0$ in Eq. (3). As a benchmark for later analysis, Eq. (10) can be simplified to $d^2x_0/dt^2 = -\partial V(x_0)/\partial x_0$. As a double check of whether our analytical and numerical results are correct, we compare the analytical results (solid curves) based on Eq. (10) with the numerical ones (circled curves) in Fig. 2(b). As is expected, the analytical and numerical simulations are in agreement with each other very well, showing that the two dark solitons repel each other because the effective potential $V(x_0)$ in Eq. (10) between two dark solitons is repulsive.

Then, we consider how the non-equilibrium nature of model affects the dynamics of two-dark-soliton solution when the dissipation parameters are turned on. In this end, we devise two scenarios: First, we choose a small initial velocity of $v_{\rm s}$ and the two dark solitons will never penetrate but repel each other governed by the effective potential of $V(x_0)$ in Eq. (11) when they are closer each other. Then, when the initial velocity of $v_{\rm s}$ is larger than a critical value, the two dark solitons will overcome the effectively repulsive potential and penetrate. In the first scenario, we have chosen the initial soliton's velocity with $v_{\rm s} = 0.15$. As shown in Figs. 2(c) and 2(d), the relative motion's minimum value is positive due to the reduction of their repulsive force between the solitons rooted into the interaction between atoms. Moreover, the numerical results (black solid curves) based on Eq. (10) find remarkable agreement with the analytically ones (blue dotted curves). Compared with Fig. 2(b) without dissipation, the results with the introduction of dissipation in Fig. 2(d) show that dark solitons are rebounded into farther positions, suggesting that the dissipation increases the repulsive effective potential. In the second scenario, the initial velocity of $v_{\rm s}$ is chosen to be large enough to penetrate each other as shown in Figs. 2(g) and 2(h). Note that our analytical results in Eq. (10) are valid under the condition that the relative distance of two solitons should be larger than the width of soliton. Therefore, our analytical results in Eq. (10) are supposed to be invalid when penetrating each other. In contrast, before and after collision of two dark solitons corresponding to the relative distance of two solitons being larger than the width of soliton, our analytical results in Eq. (10) are found to be consistent with the numerically ones as shown in Figs. 2(e) and 2(f). Lastly, we can also determine the critical velocity of the two dark solitons just penetrating each other both numerically and analytically. As shown in Figs. 2(e) and 2(f), the critical velocity is numerically calculated to be 0.32 for the parameters used in this work by directly solving Eqs. (3) and (4). If the initial velocity of the dark soliton is above 0.32, the two dark solitons will penetrate each other, otherwise, repel each other. Such a critical value of the initial velocity 0.32 can be analytically understood by taking the right hand of Eq. (10) as zero. As such, the analytical critical value of the initial velocity is calculated to be 0.34, which agrees well with the above numerical value of 0.32.

*Conclusion and Outlook.* In summary, we have investigated the dynamics of two dark solitons appearing in polariton BECs under nonresonant pumping. We derive analytically the evolution equations for the solitons' parameters. Within the framework of Lagrangian approach, our analytical results capture the essential physics about how the open-dissipative effects affect the relative motion of two solitons at a finite time. We also solve the dissipative equation by the initial wave function of two dark solitons in a numerically exact fashion. The numerical results are in remarkable agreement with the analytically ones. We have also studied the collision of two solitons in polariton BECs under nonresonant pumping. By manipulating the initial velocity, the two solitons can repel or penetrate each other. Although the model system studied in this work is limited to a polariton condensate under nonresonant pumping, the variational method used in this work can be directly extended to the case of a polariton condensate under the resonant pumping.

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