CHIN. PHYS. LETT. Vol. 32, No. 5 (2015) 050303

An Ideal Experiment to Determine the 'Past of a Particle' in the Nested Mach–Zehnder Interferometer *

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(Received 22 December 2014)

An ideal experiment is designed to determine the past of a particle in the nested Mach–Zehnder interferometer (MZI) by using standard quantum mechanics with quantum non-demolition measurements. We find that when the photon reaches the detector, it only follows one arm of the outer interferometer and leaves no trace in the inner MZI. When it goes through the inner MZI, it cannot reach the detector. Our result obtained from the standard quantum mechanics contradicts the statement based on two-state vector formulism, 'the photon did not enter the (inner) interferometer, the photon never left the interferometer, but it was there'. Therefore, the statement and also the overlapping claim are incorrect.

PACS: 03.65.Ta, 03.65.Ca, 42.25.Hz DOI: 10.1088/0256-307X/32/5/050303

One obstacle to describe the past of a quantum particle is the inability to verify any prediction about the past of the particle, as any measurement to observe the particle's path information would cause the wavefunction to collapse. Early discussions on path information in quantum mechanics relied on the concept of the duality,[1−5] which tells us that the price to pay for acquiring the path information is a loss of interference. This old problem of describing the past of a quantum particle has recently resurfaced,[6,7] due to the development of weak measurements[8−13] that do not cause the complete collapse of the wavefunction, and due to counterfactual predications on the particle path, such as interaction-free measurement[14,15] and counterfactual communication,[16] which may find applications in technologies in the near future.

In the discussion on the path, an approach called the two-state vector formalism (TSVF) to study the quantum systems between two strong measurements, was proposed.[17,18] The TSVF makes use of the forward and backward evolving wavefunctions, starting at the time of the pre-selection and at the time of the postselection, respectively. The researchers who put forward the TSVF[6,19,20] claimed in Ref. [19] that the particle was in the overlap of the forward and backward evolving quantum states. Based on this claim, they stated that for the nested Mach–Zanhder interferometer (MZI), we can state: the photon did not enter the (inner) interferometer, the photon never left the interferometer, but it was there.[6] This statement raised serious controversies and led to considerable debate.[21,22] Recently, an experiment[7] was reported where the authors said that the experimental results have a simple explanation in the framework of the TSVF of quantum theory, which means that the experiment supports the claim and statement. However, their experiment itself is controversial,[23−25] and Ref. [25] clearly showed that the experiment did not prove that the statement was correct. The main point of contention is that the weak measurement will destroy the destructive interference at the dark port and cause a leakage through the dark port. That leakage contributes the photon's trace in the inner interferometer revealed at detector D. Consequently the weak measurement cannot resolve the controversy. For the delicate interference involved, the assumption in TSVF in which weak measurements do not significantly disturb the wavefunctions is not generally correct. The false assumption in the formalism leads to the predictions in contradiction with the standard quantum mechanics. In this Letter, an experiment, which uses quantum non-demolition (QND) measurements in the nested MZI to reveal whether the photon is presented in the inner MZI if the detector D clicks, is proposed. The novelty of our scheme is that we extract the path information without disturbing the quantum interference (the dark port remaining dark). Our results based on the standard quantum mechanics, while not on the TSVF, show that the statement in Ref. [7] is incorrect, which means that the TSVF itself should be re-examined.

The nested interferometer consists of an outer

*Supported by the National Basic Research Program of China under Grant Nos 2011CB922203 and 2012CB921603, and the National Natural Science Foundation of China under Grant Nos 1174026 and U1330203.

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larger interferometer with two beam splitters BS_1, and an inner interferometer, comprising of the beam splitters BS_2 along one arm of the outer interferometer. The beam splitters BS_1 have reflectivity r and transmissivity t, whereas the beam splitters BS_2 are 50:50. This arrangement means the output port of the inner interferometer is towards the detector D in a dark port, and the wavefunction from inside the inner interferometer cannot reach the detector. In this setup, we have three quantum paths which are indicated by red solid, blue dashed and green dotted lines in Fig. 1(a). We use column vectors $(n_1\ n_2\ n_3)^\dagger$ to describe the state of the system, where n_1, n_2, and n_3 are the number of photons along the modes $(1\ 0\ 0)^\dagger$ (red), $(0\ 1\ 0)^\dagger$ (blue), and $(0\ 0\ 1)^\dagger$ (green), respectively. We also define the corresponding photon creation and annihilation operators $\hat{a}_i^\dagger$ and $\hat{a}_i$ $(i = 1, 2, 3)$, respectively. The lines L1, L2, and L3 in Fig 1 (b) represent different stages during the evolution. Stage L1 is between the first BS_1 and the first BS_2, L2 between the two BS_2, and L3 is between the second BS_2 and the second BS_1.

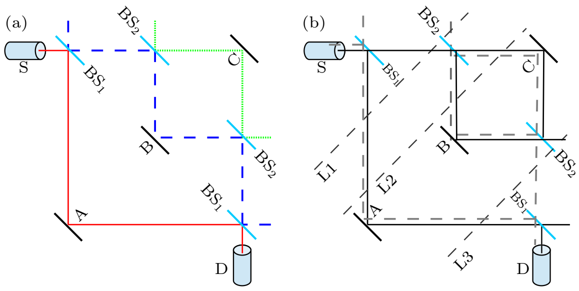

Fig. 1. The nested Mach–Zehnder interferometer. The modes $(1\ 0\ 0)^\dagger$ (red),$(0\ 1\ 0)^\dagger$ (blue), and $(0\ 0\ 1)^\dagger$ (green), respectively. The lines L1, L2, and L3 in Fig. 1(b) represent different stages during the evolution.

If a single photon coming from the source, S, has been detected by D, the pre-selection is $|\psi\rangle = (1\ 0\ 0)^\dagger$, and the post-selected state is $\langle\phi| = (1\ 0\ 0)$, in the TSVF. The pre-selected wavefunction evolves forward in time through different beam splitters following the black solid line, see Fig. 1(b). At stage L1, the wavefunction is $|\psi_{L1}\rangle = (-ir\ t\ 0)^\dagger$. At stage L2, the photon wave-packet is present along both the arm, A, of the outer interferometer and inside the inner interferometer, $|\psi_{\rm L2}\rangle = (-ir\ \ -it/\sqrt{2}\ t/\sqrt{2})^\dagger$. Due to the dark port of the inner interferometer along the second mode $(0\ 1\ 0)^\dagger$ (see the blue dashed line in Fig. 1(a)), the wavefunction at stage L3 is $|\psi_{L3}\rangle = (-ir\ 0\ it)^\dagger$. This shows that the particle, which was inside the inner interferometer, leaves the system along the mode $(0\ 0\ 1)^\dagger$ and cannot contribute to the post-selection at D.

The backward evolving wavefunction (the post-selected state) created at the detector after the successful photon detection evolves backward following the grey dashed line. This backward evolution can be through arm A and the inner interferometer. However, the portion passing through the inner interferometer will leave the system at stage L1 and cannot reach the source (due to the dark port), see the grey dashed line in Fig. 1(b). Based on the TSVF, the photon in its past should be present at the places where the two wave-functions overlap, which includes arm A of the outer interferometer and the inner interferometer, while not the paths leading to and coming out of the inner interferometer.[6,22]

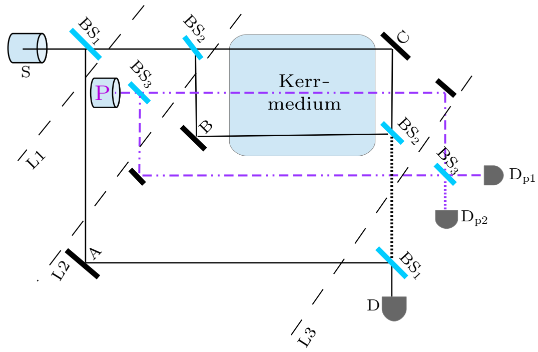

Fig. 2. Non-demolition (QND) measurement that can reveal the presence of the photon inside the inner interferometer without disturbing its interference.

To test the statement that the photon detected by D was present in the inner interferometer, we design an experiment with QND measurement.[26,27] The novelty of the setup is that we can probe the presence of the photon's trace without disturbing the destructive interference on the dark port. A third interferometer and a coherent field are added as a probe to reveal the photon trace in the inner interferometer. The coherent state $|\sqrt{2}\alpha\rangle$ is split by a 50:50 beam splitter (BS_3) into two coherent fields, $|\alpha\rangle_1$ and $|i\alpha\rangle_2$, which enter the two arms of the third interferometer, respectively. A Kerr medium is placed along the two paths of the inner interferometer and one arm of the third interferometer. This arm of the third interferometer is placed in the middle of two paths of the inner interferometer, see Fig. 2. The output of the third interferometer ($D_{\rm p1}$ and $D_{\rm p2}$) will give us a fringe pattern due to the interference between two coherent fields. The fringe pattern detected by $D_{\rm p1}$ (or $D_{\rm p2}$) in the case of the photon passing through the inner interferometer (due to the interaction with the coherent fields) is different from that in the case of the photon not passing through the inner interferometer. Thus from the fringe pattern, we can determine whether the photon passes through the inner interferometer.

The arm of the third interferometer that passes through the Kerr medium carries the field ($|\alpha\rangle_1$). Inside the Kerr medium, the interaction between the coherent beam and the photon inside the inner interferometer reads the Hamiltonian[27,28]

$$H = \varepsilon \sum_{i=2,3} \hat{a}_i^\dagger \hat{a}_i \hat{a}_{\rm p1}^\dagger \hat{a}_{\rm p1} + \eta \hat{a}_2^\dagger \hat{a}_2 \hat{a}_3^\dagger \hat{a}_3, \tag{1}$$

where $\hat{a}_{\rm p1}^{\dagger}$ and $\hat{a}_{\rm p1}$ are the creation and annihilation operators for the photons of $|\alpha\rangle_1$, and η is the interaction strength between the two paths of the inner interferometer in the Kerr medium. As the two paths of the inner interferometer are symmetric with respect to the middle coherent state $|\alpha\rangle_1$, the measurement interaction strength between the two paths of the inner interferometer with the coherent field in the Kerr medium are the same, noted with ϵ.

With the pre-selection $|\psi_i\rangle = (1\ 0\ 0)^{\dagger}$, the joint state at the stage L2 is

$$|\psi_{\rm L2}\rangle = \begin{pmatrix} ir \\ 0 \\ 0 \end{pmatrix} |\alpha\rangle_1 |i\alpha\rangle_2 + \begin{pmatrix} 0 \\ it/\sqrt{2} \\ t/\sqrt{2} \end{pmatrix} |\alpha\rangle_1 |i\alpha\rangle_2. \quad (2)$$

After the interaction with the Kerr medium,

$$|\psi'_{\rm L2}\rangle = \begin{pmatrix} ir \\ 0 \\ 0 \end{pmatrix} |\alpha\rangle_1 |i\alpha\rangle_2 + \begin{pmatrix} 0 \\ it/\sqrt{2} \\ t/\sqrt{2} \end{pmatrix} |\alpha e^{-i\epsilon\tau_0}\rangle_1 |i\alpha\rangle_2, \quad (3)$$

where we have set that the passing times of the two arms through the Kerr medium are the same, $\tau_1 = \tau_2 = \tau_0$. The state at the stage L3 is given as

$$|\psi_{L3}\rangle = \begin{pmatrix} ir \\ 0 \\ 0 \end{pmatrix} |\alpha\rangle_1 |i\alpha\rangle_2 + \begin{pmatrix} 0 \\ 0 \\ it \end{pmatrix} |\alpha e^{-i\varepsilon\tau_0}\rangle_1 |i\alpha\rangle_2. \quad (4)$$

Note that the second mode $(0\ 1\ 0)^{\dagger}$ is still empty. This is the consequence of not exploring the 'which-path' information in the inner interferometer. It is essential that $\tau_1 = \tau_2 = \tau_0$, thus the dark port of the inner MZI remained dark after the Kerr medium was added, which can be achieved by paralleling the left and right edges of the Kerr medium (see Fig. 2). Paralleling the two edges can be realized in experiment by using the current technology.

After the second BS_3 and before the second BS_1, the joint state is

$$|\psi'_{L3}\rangle = \begin{pmatrix} ir \\ 0 \\ 0 \end{pmatrix} |i\sqrt{2}\alpha\rangle_1 |\text{vacuum}\rangle_2 + \begin{pmatrix} 0 \\ 0 \\ it \end{pmatrix} \Big| i\alpha \frac{1+e^{-i\epsilon\tau_0}}{\sqrt{2}} \Big\rangle_1 \Big| \alpha \frac{1-e^{-i\epsilon\tau_0}}{\sqrt{2}} \Big\rangle_2. \quad (5)$$

Although we do not know the 'which-path' information of the photon in the inner interferometer, we can still determine whether the photon passed through the inner interferometer. Only the first term in Eq. (5) contains the system photon mode that can reach the detector D. However, it is clear from this term that the photon wavefunction reaching detector D has not interacted with the probe coherent field and has left no trace on the fringes in the detectors $\mathrm{D_{p1}}$ and $\mathrm{D_{p2}}$. The second term in Eq. (5) describes the portion of the wavefunction that has interacted with the photon inside the inner interferometer and has left a trace (a shift) on the fringes, while this portion of the photon wavefunction leaves the system along the mode $(0\ 0\ 1)^{\dagger}$ at the stage L3, and never reaches the detector D. It clearly proves that the photon detected at detector D was following only arm A of the outer interferometer. It was not inside the inner interferometer and has not left any trace inside the inner interferometer. This straight forward quantum mechanical reasoning is in clear contradiction with the prediction of TSVF that associates the past of the photon with the overlap of the forward and backward evolving waves. In the standard quantum mechanics, a forward evolving wavefunction is enough to describe the whole evolution of the system.

Let us tentatively use the backward evolving wavefunction of TSVF. Suppose that the back evolution state including the coherent state is

$$\langle\phi_f| = (\,1 \quad 0 \quad 0\,)\,\langle -i\sqrt{2}\alpha|_1 \langle\text{vacuum}|_2. \quad (6)$$

We can derive from the standard quantum mechanics the back evolution states at different stages,

$$\langle\phi_{L3}| = (\,ir \quad t \quad 0\,)\,\langle\alpha|_1\langle -i\alpha|_2, \quad (7)$$

$$\langle\phi_{\rm L2}| = (\,ir \quad 0 \quad 0\,)\,\langle\alpha|_1\langle -i\alpha|_2 + (\,0 \quad it/\sqrt{2} \quad t/\sqrt{2}\,)\,\langle\alpha e^{-i\varepsilon\tau_0}|_1\langle -i\alpha|_2, \quad (8)$$

$$\langle\phi_{L1}| = (\,ir \quad 0 \quad 0\,)\,\langle\sqrt{2}\alpha|_1\langle\text{vacuum}|_2 + (\,0 \quad 0 \quad it\,)\Big\langle \frac{1+e^{-i\varepsilon\tau_0}}{\sqrt{2}}\alpha\Big|_1 \cdot \Big\langle -i\frac{1-e^{-i\varepsilon\tau_0}}{\sqrt{2}}\alpha\Big|_2. \quad (9)$$

From the first term in Eq. (9), we can clearly see that if the single photon evolves back to the pre-selection, so does the coherent state, which means that the single photon leaves no trace in the inner interferometer (and on the measurement device). The second term tells us that part of the coherent state does not evolve to the pre-selection $\langle\sqrt{2}\alpha|_1$, and the measurement device gives us the information about the system. The second term is the result of the corresponding single photon that leaves a trace in the inner interferometer and then goes away, and will not reach the source S. Hence, the backward evolving wavefunction tells the same story as the forward evolving wavefunction. A particle going back from the detector D to the source S cannot leave a trace inside the inner interferometer.

To realize our ideal experiment, there are two major difficulties. One difficulty is obtaining the single photon source. If it is possible to send out the multiphoton state from the source, such as two-photon state ($|2\rangle$), there is a probability that one photon goes to the detector D (mode $(1\ 0\ 0)^{\dagger}$) following path A, and the other photon following B and C paths leaves the system along mode $(0\ 0\ 1)^{\dagger}$, which will result in the fringes in the detectors D_{p1} and D_{p2}. Consequently, the D lick and the fringes in detectors D_{p1} and D_{p2} occur at the same time, which demonstrates that the photon leaves the trace (the fringes) in the inner interferometer. However, note that this trace results from the photon that leaves the scheme following the mode $(0\ 0\ 1)^{\dagger}$, which cannot go to the detector. Thus a high quality single photon source is needed. Another difficulty is finding a material with a large $\chi(3)$. In our proposed experiment, the phase shift is proportional to the nonlinear coefficient, $\chi(3)$. A large amount of research work is focused on searching materials of large $\chi(3)$,[28−35] and controlling the shift from zero to π.[36] Although there is a measurable shift with a weak probe field,[37−39] even at the singe photon level,[30] was experimentally observed in a cold and dense Rydberg atomic gas, it is much more difficult to find a solid material that has a large $\chi(3)$. To set the passing times of the two arms through the Kerr medium to be the same ($\tau_1 = \tau_2 = \tau_0$), the best candidate nonlinear optical material is a solid material. Thus it is a challenge to find a nonlinear optical solid material which can provide the phase shift in the single photon level. However, we hope that in the near future, the proposed experiment would be realized with the development of experiment technology.

In summary, we have proposed an ideal experiment for the nested MZI system. In the experiment, the QND measurements are used to reveal the past of the quantum particle without disturbing the interference of the system (keeping the dark port still dark), which are different from the weak measurement in Refs. [6,7] that disturbs the interference (leading to a leakage to the dark port). Our derivation, based on the standard quantum, shows that the photon was only following path A and leaves no trace in the inner interferometer when detector D (post-selection) has a click. Contrarily, when the photon passes through the inner interferometer, detector D has no click. This conclusion is contradicted with the statement and the overlapping claim from the TSVF. Note that the overlapping claim of the TSVF is not derived from or a result of the standard quantum mechanics. Therefore, the contradictory between the overlapping claim of the TSVF and the standard quantum mechanics means that the overlapping claim is incorrect. Our conclusion only removes doubts or settles the argument on the counterfactual communication,[17,18,40,41] while it also alerts us to the fact that we should pay attention to the weakness and limitations of the weak measurement and TSVF when we use them.

References

[1] Scully M, Englert B and Walter H 1991 *Nature* **351** 111
[2] Herzog T J, Kwiat P G, Weinfurter H and Zeilinger A 1995 *Phys. Rev. Lett.* **75** 3034
[3] Jacques V, Wul E, Grosshans F, Treussart F, Grangier P, Aspect A and Roch J F 2007 *Science* **315** 966
[4] Jia A A, Huang J H, Feng W and Zhu S Y 2014 *Chin. Phys.* B **23** 030307
[5] Han Y, Wu C W, Wu W, Chen P X and Li C Z 2009 *Chin. Phys. Lett.* **26** 040303
[6] Vaidman L 2007 *Phys. Rev. Lett.* **98** 160403
[7] Danan A, Farfurnik D, Bar-Ad S and Vaidman L 2013 *Phys. Rev. Lett.* **111** 240402
[8] Liao X P, Fang M F, Fang J S and Zhu Q Q 2014 *Chin. Phys.* B **23** 20304
[9] Du S J, Xia Y J, Duan D Y, Zhang L and Gao Q 2015 *Chin. Phys.* B **24** 044205
[10] He Z and Yao C M 2014 *Chin. Phys.* B **23** 110601
[11] Hosten O and Kwiat P 2008 *Science* **319** 787
[12] Lundeen J S, Sutherland B, Patel A, Stewart C and Bamber C 2011 *Nature* **474** 188
[13] Kocsis S, Braverman B, Ravets S, Stevens M, Mirin R P, Shalm L K and Steinberg A M 2011 *Science* **332** 1170
[14] Hosten O, Rakher M T, Barreiro J T, Peters N A and Wiat P G 2006 *Nature* **439** 949
[15] Kwiat P, Weinfurter H, Herzog T and Zeilinge A 1995 *Phys. Rev. Lett.* **74** 4763
[16] Salih H, Li Z H, Al-Amri M and Zubairy M S 2013 *Phys. Rev. Lett.* **110** 170502
[17] Aharonov Y and Vaidman L 1990 *Phys. Rev.* A **41** 11
[18] Aharonov Y and Vaidman L 1991 *J. Phys. A: Math. Gen.* **24** 2315
[19] Vaidman L 2014 *Phys. Rev.* A **89** 024102
[20] Vaidman L 2013 *Phys. Rev.* A **87** 052104
[21] Li Z H, Al-Amri M and Zubairy M S 2013 *Phys. Rev.* A **88** 046102
[22] Vaidman L 2013 *Phys. Rev.* A **88** 046103
[23] Salih H 2014 arXiv:1401.4888v2
[24] Svensson B E Y 2014 arXiv:1402.4315
[25] Saldanha P L 2014 *Phys. Rev.* A **89** 033825
[26] Imoto N, Haus H A and Yamamoto Y 1985 *Phys. Rev.* A **32** 2287
[27] Feizpour A, Xing X X and Steinberg A M 2011 *Phys. Rev. Lett.* **107** 133603
[28] Zubairy M S, Matsko A B and Scully M O 2002 *Phys. Rev.* A **65** 043804
[29] Ottaviani C, Vitali D, Artoni M, Cataliotti F and Tombesi P 2003 *Phys. Rev. Lett.* **90** 197902
[30] Peyronel T, Firstenberg O, Liang Q Y, Hofferberth S, Gorshkov A V et al 2012 *Nature* **488** 57
[31] Zhang X Q, Wang C S, Lu G Y and He T C 2010 *Chin. Phys. Lett.* **27** 074201
[32] Fu G, Kazuo K, Hiroaki O and Shunsuke T 2005 *Chin. Phys. Lett.* **22** 1687
[33] Zhang Y N 2013 *Chin. Phys.* B **22** 014214
[34] Sun X, Chen Y S and Xu L 1995 *Chin. Phys. Lett.* **12** 541
[35] Wu S P, Zhang L J and Li G X 2008 *Chin. Phys.* B **17** 185
[36] Li R B, Deng L and Hagley E W 2013 *Phys. Rev. Lett.* **110** 113902
[37] Kang H and Zhu Y f 2003 *Phys. Rev. Lett.* **91** 093601
[38] Lo H Y, Chen Y C et al 2011 *Phys. Rev.* A **83** 041804
[39] Lo H Y, Su P C and Chen Y F 2010 *Phys. Rev.* A **81** 053829
[40] Vaidman L 2014 *Phys. Rev. Lett.* **112** 208901
[41] Salih H, Li Z H, Al-Amri M and Zubairy M S 2014 *Phys. Rev. Lett.* **112** 208902