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Opinion Dynamics on Complex Networks with Communities *

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The Ising or Potts models of ferromagnetism have been widely used to describe locally interacting social or economic systems. We consider a related model, introduced by Sznajd to describe the evolution of consensus in the scale-free networks with the tunable strength (noted by Q) of community structure. In the Sznajd model, the opinion or state of any spins can only be changed by the influence of neighbouring pairs of similar connection spins. Such pairs can polarize their neighbours. Using asynchronous updating, it is found that the smaller the community strength Q, the larger the slope of the exponential relaxation time distribution. Then the effect of the initial up-spin concentration p as a function of the final all up probability E is investigated by taking different initialization strategies, the random node-chosen initialization strategy has no difference under different community strengths, while the strategies of community node-chosen initialization and hub node-chosen initialization are different in final probability under different Q, and the latter one is more effective in reaching final state.

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In recent years, a large class of interdisciplinary problems has been successfully studied with methods of statistical physics, in particular those related to the characterization of the collective social behaviour of individuals, such as opinion formation,[1−5] the spreading of rumor or disease,[6,7] the language dynamics,[8] etc. The study of opinion dynamics has become a main stream of research in physics. Processes of opinion formation are usually modelled as simple collective dynamics in which the agents update their opinions following local majority[3] or imitation.[5] Of the many recently simulated models of opinion dynamics,[1−5] the Sznajd model[9] was studied particularly often; seen in reviews[10] and recent examples.[11] It is a successful Ising spin model describing a simple mechanism of making up decisions in a closed community. As the Sznajd model acquires much attention for the marketing,[10] finance[10] and politics application[10] and can reach good explanation in the vote distribution in elections,[12] it has rapidly gained importance in the new field of computational socio-physics. Different from other opinion models, such as the Deffuant model,[5] Krause–Hegselmann model,[5] voter model,[2] and Galam majority rule,[3] the Sznajd model is a model which opinion influence flows outward a given group to the nearest individuals, while the other models are all in opposite situation that opinion influence flows inward.[13]

The Sznajd model was simulated by both asynchronous and synchronous updating[14] on a one-dimensional lattice, on a square lattice, on a triangular lattice, the dilute, on a three-dimensional cubic lattice and even on some networks, such as small-world networks, deterministic pseudo-fractal networks and Barabasi networks. In particular, Slanina and Lavicka made a mean field theory and obtained analytical results.[15] The effect of dimension, number of different opinion choices, the clustering coefficient, the noise and advertising influence on networks have extensively investigated.[10]

Besides the small-world[16] and scale-free property,[17] it is well-known that social networks exhibit modular structure of weakly coupled clusters, namely, the community structure,[18,19] i.e., nodes of the same community are highly connected, while there are few links between the nodes of different communities. It is therefore of particular interest to understand how the opinion models behave on these realistic geometries. Only recently, the opinion model of Majority Rule has been theoretically analysed on networks with communities which are constructed by coupled fully connected networks[20] and coupled random networks.[21] In this Letter, we provide analysis on how community structure affects the outward opinion influence of the Sznajd opinion model.

The Sznajd opinion rule describes the influence of a given pair on the decision of its nearest neighbours. When members of a pair have the same opinion, then their nearest neighbours agree with them. On the system network, each node i ($i = 1, 2, \cdots, N$) carries a spin s_i, which has two possible directions: $s_i = +1$ (up) or $s_i = -1$ (down). It can be considered to be an individual that takes one of the two possible attitudes such as approval or disapproval. Then we use asynchronous updating, and the rules are as follows: (1) In each time a pair of spins $s_i(t)$ and $s_j(t)$ is chosen to

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change their nearest neighbours, i.e. the spins $s_{\Gamma(i)}(t)$ and $s_{\Gamma(j)}(t)$, where $\Gamma(i)$ is the set of node i's neighbour nodes. (2) If $s_i(t)s_j(t) = +1$ then $s_{k\in\Gamma(i)}(t) = s_i(t)$ and $s_{l\in\Gamma(j)}(t) = s_j(t)$ (social validation). (3) If $s_i(t)s_j(t) = -1$ then $s_{k\in\Gamma(i)}(t) = s_{k\in\Gamma(i)}(t)$ and $s_{l\in\Gamma(j)}(t) = s_{l\in\Gamma(j)}(t)$ (nothing happens).

This process continues until each of the N nodes is selected once as the first member of the neighbour pair and it constitutes one time step t.

Inspired by two ingredients of the Barabasi–Albert model[22], i.e., growth and preferential attachment, some scale-free networks with community or hierarchical structure models were proposed.[23−27] Here, we investigate the community structure of scale-free network model as suggested in Ref. [27], which is a growth model to create a network with a tunable parameter Q denoting the strength of community structure. It goes as follows: Starting with c communities denoted by $U_1, U_2, \cdots, U_{c-1}, U_c$, and each community started with a small number m_0 of fully connected nodes. At one time step, we add into each community a new node with m ($< m_0$) edges that link the new node to n ($n < m$) different nodes within this community and $m - n$ different nodes within other $c - 1$ communities already existing in the system. When adding a new node into community U_i, first choose n different nodes in community U_i with preferential attachment rule $\Pi(k_i) = k_i/\Sigma k_j$, which means that the probability Π that the new node will be connected to node i depends on the degree k_i of node i. Then for each one of the other $m - n$ edges of the new node, choose a community U_l ($U_l \neq U_i$) randomly and connect the new node to one node in U_l according to the preferential attachment rule described above.

The scaling behaviour of the degree distribution can be calculated by using several approaches.[28−30] The result shows that the equilibrium status of systems is stability and the scaling behaviour is independent of its initial topology (the number of nodes m_0) and the variation of m and n values. The degree distribution $p(k)$ of nodes of the local network (inside each community), and the global network are power law with exponent 3, that is, $P(k) \propto k^{-3}$(see Fig. 1).

Based on the previous measure of assortative mixing, Newman and Girvan[31] proposed a measure of the strength of community, which later modified by Kashtan and Alon:[32]

$$Q = \sum_c \left[\frac{l_s}{L} - (\frac{d_s}{2L})^2\right], \tag{1}$$

where c is the number of communities, L is the number of edges in the network, l_s is the number of edges between nodes in community U_s, and d_s is the sum of the degrees of the nodes in community U_s. Briefly, the Newman and Girvan algorithm finds the division of the nodes into modules that maximizes a measure Q. This measure is defined by the fraction of the edges in the network that connect between nodes in a module minus the expected value of the same quantity in a network with the same assignment of nodes into modules but random connections between the nodes. For simple analytic process, we can get the Q as the function of m_0, c, m, n, N, here the values of m and n are not necessary integers to obtain the different Q. In this network model, we can adjust n and keep the other parameters fixed in order to reach the networks with different strengths Q of community structure. If the number of inter-community edges is no better than random, we will take $Q = 0$. Values approaching $Q = 1$, which is the maximum, indicate strong community structure.

Once the network with the value of community strength Q has been completely constructed, we perform standard Monte Carlo simulations using random sequential updating.[33] Beginning with the totally random initial spin, $s_i = +1$ (up) with concentration $p = 1/2$, and $s_i = -1$ (down) with concentration $1-p = 1/2$, the system always reaches consensus state with one of two fixed points all $s_i = +1$ or $s_i = -1$. For different community strengths Q ranging in $[0, 1]$ and each Q averaged over 1000 samples, the Sznajd dynamics will end up with half of the samples having $s_i = -1\forall_i$ and the remaining half having $s_i = +1\forall_i$, which obtain the same result as in Ref. [15].

Figure 2 shows the distribution $p(\mu)$ of the relaxation time μ needed to reach the consensus state on networks with different community strengths Q. It decays exponentially and the larger Q costs longer consensus time, that is, smaller exponential slope. Specifically, for $Q = 0.85$, 0.75 and 0.7, the slope exponent τ are 0.17, 0.64 and 0.83. This result shows that the less of the communication between the different communities, the harder to reach the consensus, so the longer time needed to reach the fixed points. Thus, the community structure plays an important role in the slow-moving opinions. For example, when Q=0.3, the relaxation time is almost one time step.

$$p(\mu) = 10^{-\tau\mu+b}, \tag{2}$$

To future exploring the impact of the network structure on the evolution of the Sznajd opinion dynamics, the effect of the initial configuration is studied. We take the initial up concentration p as a function of the final all up probability E, averaged 1000 samples. In these scale-free networks with communities, we take three strategies for selecting the initial concentration. The first initialization strategy is: (i) Random node-chosen concentration p_R nodes in the network with the spin of $s_i = +1$. The results of different initial concentration p_R by random node-chosen strategy have the same final up probability E no mat-

ter what Q are, which shows in Fig. 3(a). The same result is also obtained in Ref. [34], which was simulated on a one-dimensional chain with different system sizes.

The second initialization strategy is: (ii) Community node-chosen concentration p_C of communities in the network in which all the nodes in the chosen community are in the spin of $s_i = +1$. Since the whole network is composed of c communities with each having equal number of nodes and edges, the selecting of one community nodes has the same number of nodes as random selecting N/c nodes in the network. For example, in this study, we set the community number $c = 10$, when $p_C = 0.2$, the number of $\dfrac{N}{c} * p_C * c = 0.2 * N$ nodes in randomly chosen $c * p_C = 2$ communities are initially in the spin of $s_i = +1$. The result of this strategy shows in Fig. 3(b), which displays that the larger Q the larger probability of finally $s_i = +1$ state. The strategy (ii) is evidently different from strategy (i) of random node-chosen. For the strategy (i) of randomly chosen nodes, the result will always the same regardless the network structure, including the one-dimension chain, the community strength and the system size.[34] However, the strategy (ii) investigate the role of the community, it demonstrates that the highly densely inner-connected community results in the larger Q, and the larger influence of consensus spin persuasion on the other nodes leads to larger final all up probability.

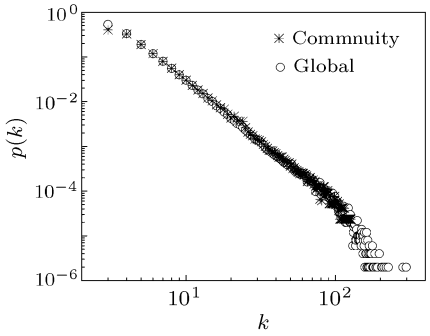


Fig. 1. Global and local degree distribution of the network with $N = 10^4$, $c = 10$, $m_0 = m = 4$ and $n = 3.0$, where $Q = 0.673$. The result are averaged for 100 samples.

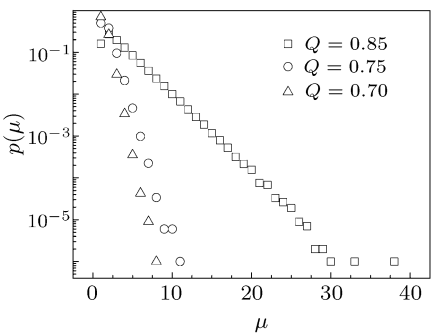


Fig. 2. Distribution of consensus time on networks with different community strengths Q. The network is $N = 3200$, $c = 10$, $m = m_0 = 4$ and the result are averaged over 1000000 samples.

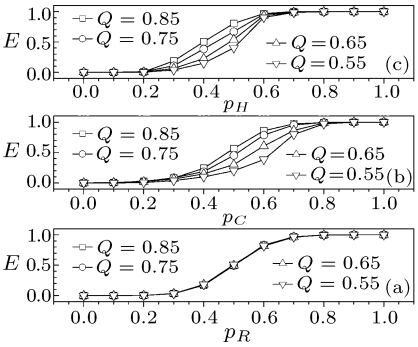


Fig. 3. (a) Using strategy of randomly choose nodes, (b) using strategy of choose community nodes, (c) using strategy of choose hub nodes. The network is $N = 3200$, $c = 10$, $m = m_0 = 4$ and the results are averaged over 1000 samples.

For the 'richer-get-richer' construction of scale-free network, there are a lot of 'hub nodes' which are the highly connected nodes.[33] However, these hub nodes are sensitive to the network attack and have many interesting properties.[33] For the scale-free degree distribution of both the global and local network, these hub nodes are distributed in each community. So, the third initialization strategy is: (iii) Hub node-chosen concentration p_H of the most connected nodes instead of random choosing in the network. Come to speak concretely, if $p_H = 0.1$, take the top number of $N * p_H = 0.1 * N$ nodes which have the largest degree with the spin of $s_i = +1$. The result of this strategy is quite different from the strategy (i) as showed in Fig. 3(a) and have the same trend as strategy (ii) showed in Fig. 3(b). It also exhibits that the larger Q the larger probability of finally all $s_i = +1$ state, similar to the result of strategy (ii). The result is easy to understand, because the hub nodes are located in different communities, the stronger of community, the highly connected hub nodes in one community would be in up spin, and the larger probability to find a pair of spins having the same spin, which leads to easier getting final up consensus.

From Fig. 3, we find that both of the strategy (ii) and strategy (iii) can get the same result, which declares that the larger the community strength, the larger the probability of final all up state. For comparison of the two strategies, Fig. 4 illustrates the difference. We choose the result of strategies (ii) and (iii) on networks with the first two largest Q, 0.85 and 0.75 in comparison for short. For the same Q, the strategy (iii) is more efficient in reaching final all up state, and both of the two strategies result in smaller E as the community strength decrease. This result indicates that the hub node-chosen strategy is more effective than community node-chosen strategy. However, in strategy (ii), the initial up spin nodes are located in the same community, which is different from strategy (iii) that the up spin nodes are the hub nodes that discretely distributed in all the communities, thus the probability to find a pair of up spin persuading a lot of connected nodes are larger for the latter one. This is the reason why the strategy (iii) is more effective

than strategy (ii) in getting final all up probability. It is proven that the decentralized persuasion is more powerful than centralized persuasion.

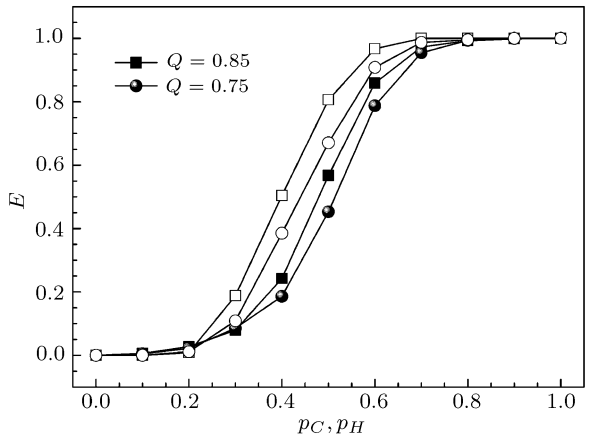


Fig. 4. Initial up concentration p_H (empty symbols) and p_C (filled symbols) as a function of the final all up probability E on networks with different initial strategies and different community strength. The network is $N = 3200$ and the results are averaged over 1000 samples.

In summary, we have investigated the opinion dynamics on networks with communities. The relaxation time decays exponentially, and the exponent decreases with increasing Q. When taking different initial configuration into consideration, we use three strategies for the initial up nodes chosen, which are random node-chosen, community node-chosen and hub node-chosen, the results show that hub node-chosen strategy is the most effective method to get higher probability of final up state than that of community node-chosen strategy dose, and both of the two strategies result in different E for different community strengths, while the randomly node-chosen strategy has no change whatever the community strength Q is.

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