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Continuous-Variable Entanglement Transfer from Cavity Field to Two Mesoscopic Josephson Junctions *

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We consider a composite system of two remote mesoscopic Josephson junctions interacting locally with a two-mode non-classical cavity field and investigate entanglement transfer from a bipartite continuous-variable (CV) system to a pair of localized mesoscopic Josephson junctions. We obtain analytically the time-dependent characteristic functions in the Wigner representation for the two CV subsystems, where two cases are considered for the zero and finite temperatures. Furthermore, we analyse the influences of the temperature on the period recovery of the entanglement.

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Entanglement is the main resource of quantum information processing[1] where it provides a quantum channel for teleportation,[2] superdense coding,[3] and distribution of cryptographic keys.[4] These protocols are initially proposed for discrete variables, but recently protocols utilizing continuous quantum variables have emerged as an interesting alternative. Remarkable theoretical and experimental efforts have been devoted to generating and quantifying CV entangled states. So far, much considerations deal with Gaussian entangled states due to both the simplicity of characterizing their entanglement properties[5−7] and the possibilities of their experimental realization.[8] It is now believed that the entanglement of a quantum composite system with continuous degree of freedom has important applications in CV quantum information processing.[9] It is well known that if two quantum systems are not entangled initially, they can become entangled through the exchange of quantum information. One important way of the exchange of quantum information is entanglement transfer from one subsystem to another. A few schemes have been suggested to transfer entanglement between qubits and CV system.[10−14] Experimentally, the entanglement transfer from free propagating light to atomic system has been achieved.[15] On the other hand, a number of systems have been proposed as potentially valuable qubit models. Among them, solid-state qubits are of particular interest because of their potential for scalability. At present the best candidates for the implementation of quantum gates based on solid state devices are superconducting small capacitance Josephson junction. Recently, the interactions of radiation field with the mesoscopic Josephson junction have been studied and some interesting results are investigated.[16−19] In this Letter, we would like to study the dynamics of a two-mode cavity field coupled to a pair of localized superconducting junctions and investigate the entanglement transfer process between the two CV subsystems where the cavity field is initially prepared in a two-mode squeezed vacuum state and the junctions initially prepared in a thermal state. We focus attention on the efficiency that how the amount of entanglement of a two-mode squeezed state is transferred to the junctions for the two cases of the zero temperature and finite temperature. In particular, the effect of temperature on efficiency of the CV entanglement transfer is discussed in detail.

We now consider two identical circular superconducting rings each with a mesoscopic Josephson junction are placed in two spatially separate but identical cavities, and the two-mode nonclassical state field is injected, respectively, into these two separate cavities. Each junction interacts with its own local cavity field.

We first analyse a single mesoscopic Josephson junction coupled to an external single-mode cavity field. The Hamiltonian describing both inductive and capacitive coupling is written as[20] (in units $\hbar = k_B = c = 1$)

$$H = \frac{1}{2C}(q-Q)^2 + E_J[1-\cos(2e\phi)] + \frac{1}{2L}(\phi-\Phi)^2, \quad (1)$$

where C is the capacitance of the Josephson junction, q the charge tunnelling through the junction, Q the external charge which depends on radiation field, E_J the Josephson coupling energy, ϕ the total magnetic flux through the ring, Φ the external flux and L the inductance of the ring. For the case of small inductance of the ring, the Josephson nonlinear term is small in comparison with the third term of Hamiltonian (1) and a truncated Taylor expansion of nonlinear term

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to quadratic term is taken. Based on the commutation relations $[\phi, q] = i$ and $[\Phi, Q] = i$ of quantum mechanical variables, we can express them in terms of the Boson operators of the field and the junction. Therefore, the Hamiltonian (1) can be simplified to[17]

$$H = \omega\Big[(a^\dagger a + \frac{1}{2}) + (1+2\lambda)(b^\dagger b + \frac{1}{2})\Big] + H_{\rm int}, \tag{2}$$

$$H_{\rm int} = \omega[(a^\dagger b + a b^\dagger) + \lambda(a^\dagger b^\dagger + ab)], \tag{3}$$

where ω is the angular frequency of the field and $\lambda = e^2 E_J L$. The above equations describe the Hamiltonian of a single-mode cavity field interacting with a mesoscopic Josephson junction. While for the system of the two-mode cavity field interacting with the two mesoscopic Josephson junctions, the interaction Hamiltonian of the system can be written as

$$H'_{\rm int} = \sum_{j=1,2} \omega[(a_j^\dagger b_j + a_j b_j^\dagger) + \lambda(a_j^\dagger b_j^\dagger + a_j b_j)], \tag{4}$$

where b_j and $b_j^\dagger$ are the operators of the jth junction, a_j and $a_j^\dagger$ are operators of the jth mode field. The time evolution of the system in the interaction picture is given by the following master equation:

$$\frac{d\rho}{dt} = -i[H'_{\rm int}, \rho]. \tag{5}$$

In order to solve analytically the master equation (5) by using the standard method[21] of the characteristic function, we introduce new boson operators $c_j = (b_j + \lambda b_j^\dagger)/\sqrt{1-\lambda^2}$ to rewrite the master equation (5) in the following form:

$$\frac{d\rho}{dt} = -i\Big[\sum_{j=1,2} \omega\sqrt{1-\lambda^2}(a_j^\dagger c_j + a_j c_j^\dagger), \rho\Big]. \tag{6}$$

The characteristic function of the density operator $\rho(t)$ in Wigner representation is defined as[21]

$$\begin{aligned}&\chi(\xi_{a_1}, \xi_{a_2}, \xi_{c_1}, \xi_{c_2}, t)\\ &= {\rm Tr}[\rho(t)\exp(-\xi_{a_1}^* a_1 + \xi_{a_1} a_1^\dagger - \xi_{a_2}^* a_2 + \xi_{a_2} a_2^\dagger\\ &\quad - \xi_{c_1}^* c_1 + \xi_{c_1} c_1^\dagger - \xi_{c_2}^* c_2 + \xi_{c_2} c_2^\dagger)],\end{aligned} \tag{7}$$

which obeys

$$\begin{aligned}&\chi(\xi_{a_1}, \xi_{a_2}, \xi_{c_1}, \xi_{c_2}, t)\\ &= \exp\Big[\sum_{j=1,2} -it\omega\sqrt{1-\lambda^2}\Big(\xi_{a_j}^* \frac{\partial}{\partial \xi_{c_j}^*}\\ &\quad - \xi_{a_j}\frac{\partial}{\partial \xi_{c_j}} + \xi_{c_j}^*\frac{\partial}{\partial \xi_{a_j}^*} - \xi_{c_j}\frac{\partial}{\partial \xi_{a_j}}\Big)\Big]\\ &\quad \times \chi(\xi_{a_1}, \xi_{a_2}, \xi_{c_1}, \xi_{c_2}, 0).\end{aligned} \tag{8}$$

We assume that two junctions are initially prepared in the separable thermal state $\rho_{th} = \rho_1 \otimes \rho_2$ with $\rho_i = \sum_{n=0}^\infty \bar{n}^n/(1+\bar{n})^{n+1}|n>_{i\ i}<n|$, $i = 1, 2$, at a certain temperature T with $\bar{n} = 1/[\exp(\omega/T)-1]$. The field is initially prepared in the two-mode squeezed vacuum state[22] $|r\rangle = \exp[r(a_1 a_2 - a_1^\dagger a_2^\dagger)]|0_{a_1}, 0_{a_2}\rangle$ with r being a squeezing parameter. By the transformation relation among the boson operators b_j and c_j, and the definition of the characteristic function in Eq. (7), we obtain the parameter relations $\xi_{c_j} = (\xi_{b_j} + \lambda\xi_{b_j}^*)/\sqrt{1-\lambda^2}$. Furthermore, with the help of the method of characteristic function,[21] after a direct calculation, we find the time-dependent characteristic function $\chi(\xi_{a_1}, \xi_{a_2}, \xi_{b_1}, \xi_{b_2}, t)$ of the field-junction interaction system. However, we are interested in studying the entanglement transfer between the field and the junctions and hence need to know the characteristic function of their subsystems. The time dependent characteristic function of the field subsystem and that of the junction subsystem can be obtained through the relation $\chi_f(\xi_{a_1}, \xi_{a_2}, t) = \chi(\xi_{a_1}, \xi_{a_2}, \xi_{b_1} = 0, \xi_{b_2} = 0, t)$ and $\chi_j(\xi_{b_1}, \xi_{b_2}, t) = \chi(\xi_{a_1} = 0, \xi_{a_2} = 0, \xi_{b_1}, \xi_{b_2}, t)$ respectively, and given by

$$\begin{aligned}\chi_f(\xi_{a_1}, \xi_{a_2}, t) = \exp\Big\{ &-\frac{1}{2}[f_{1a}(|\xi_{a_1}|^2 + |\xi_{a_2}|^2)\\ &+ f_{2a}(\xi_{a_1}^2 + \xi_{a_2}^2 + \xi_{a_1}^{*2} + \xi_{a_2}^{*2})\\ &+ f_{3a}(\xi_{a_1}\xi_{a_2} + \xi_{a_1}^*\xi_{a_2}^*)]\Big\},\end{aligned} \tag{9}$$

$$\begin{aligned}\chi_j(\xi_{b_1}, \xi_{b_2}, t) = \exp\Big\{ &-\frac{1}{2}[f_{1b}(|\xi_{b_1}|^2 + |\xi_{b_2}|^2)\\ &+ f_{2b}(\xi_{b_1}^2 + \xi_{b_2}^2 + \xi_{b_1}^{*2} + \xi_{b_2}^{*2})\\ &+ f_{3b}(\xi_{b_1}\xi_{b_2} + \xi_{b_1}^*\xi_{b_2}^*)\\ &+ f_{4b}(\xi_{b_1}\xi_{b_2}^* + \xi_{b_1}^*\xi_{b_2})]\Big\}\end{aligned} \tag{10}$$

where

$$\begin{aligned}&f_{1a} = \cos^2(gt)\cosh(2r) + n(1+\lambda^2)f,\\ &f_{2a} = n\lambda f, \quad f_{3a} = \cos^2(gt)\sinh(2r),\\ &f_{1b} = n\cos^2(gt) + (1+\lambda^2)f\cosh(2r),\\ &f_{2b} = \lambda\cosh(2r)f, \quad f_{3b} = -(1+\lambda^2)\sinh(2r)f\\ &f_{4b} = -2\lambda\sinh(2r)f, \quad f = \frac{\sin^2(gt)}{1-\lambda^2},\\ &g = \omega\sqrt{1-\lambda^2}, \quad n = 2\bar{n}+1.\end{aligned} \tag{11}$$

It is known that the state of a system is a Gaussian one if its Wigner characteristic function is a CV Gaussian function. One can see that the characteristic functions $\chi_f(\xi_{a_1}, \xi_{a_2}, t)$ and $\chi_j(\xi_{b_1}, \xi_{b_2}, t)$ in Eq. (9) and (10) are explicitly Gaussian-style. After having the characteristic function of the two subsystems, we can discuss the entanglement transfer from the field to the two junctions. For any two-mode Gaussian state, the characteristic function can be expressed in a compact form $\chi(\xi_1, \xi_2, t) = \exp(-\Lambda^\dagger M\Lambda/2)$, where M is a 4×4 matrix termed as the covariance matrix[5] and $\Lambda = (\xi_1^*, \xi_1, \xi_2^*, \xi_2)^\dagger$ is a four-dimensional vector. It

is convenient to express the covariance matrix in the block form

$$M = \begin{bmatrix} A & D \\ D^{\dagger} & B \end{bmatrix} \tag{12}$$

with 2×2 matrices A, B,D. For general two-mode Gaussian states, the entanglement can be measured by the logarithmic negativity (LN) introduced by Ref. [6] and given by

$$E_N = \max[-\log_2(2\tilde{c}_-), 0], \tag{13}$$

where the symplectic eigenvalue $\tilde{c}_-$ can be found in terms of a characteristic equation[6] with $\tilde{c}_- = \sqrt{|(K - \sqrt{K^2 - 4\det M})/2|}$ and $K = \det A + \det B - 2\det D$. Note that in the special instance of symmetric two-mode Gaussian state, the entanglement of formation[7] is also computable, but, being again a decreasing function of $\tilde{c}_-$, it provides the same characterization of entanglement and is thus equivalent to the LN.

We now consider the zero temperature case, i.e. $T \to 0$ and assume that the junctions are initially prepared in the vacuum state. The characteristic function of the field and the junctions can be obtained respectively from Eqs. (9) and (10) by letting $\bar{n} = 0$. In Fig. 1 we plot the LN of the junctions and the field as a function of scaled time gt/π and squeezing degree r for $\lambda = 0.1$ and $\omega = 10$. The numerical calculation shows that the entanglement of junctions oscillates periodically. It is seen that the maximal entanglement depends on the squeezing degree r of the initial field state. With the increasing of the squeezing degree, the entanglement increases monotonically. In comparison of Figs. 1(a) and 1(b), we can see that rise of the entanglement of the junctions is followed by the lower of entanglement of the field, and vice versa. This behaviour implies that entanglement transfer process takes place between the field and the junction subsystem, where the entanglement increases in one system at the expense of the loss of the other one through their interaction. In particular, at the time $gt = (n + 1/2)\pi$ $(n = 0, 1, 2\cdots)$, the entanglement of the junctions reaches its maximum (the entanglement of the initial field), while the entanglement of the field reaches its minimum (zero). This shows that the amount of the initial entanglement of the field subsystem is completely transferred to the junction subsystem, demonstrating that the initial field state evolves into a separable state and at the same time the initial vacuum state of the two junctions evolves into a two-mode entangled Gaussian state. Apparently, the perfect entanglement transfer from the two-mode field to the two junctions is realized in the field-junction interaction system.

We now turn to the case of $T \neq 0$ and study how the temperature of the junctions affects the efficiency of the entanglement transfer from the field to the junction subsystem. At finite temperature, the density matrix of the system is the summation of all states with its Boltzmann weight. Therefore, for very high temperature the density matrix consists of an almost uniform distribution in the state space, which leads to a vanishing of entanglement. In Fig. 2, we plot the LN of the junctions and the field as a function of the scaled time gt/π and squeezing parameter r for $\lambda = 0.1$, $\bar{n} = 0.1$ and $\omega = 10$. It can be seen that the entanglement decreases with the increase of the temperature except for the time points $gt = (n + 1/2)\pi$ $(n = 0, 1, 2\cdots)$. At these times, the entanglement of the junctions can always reach its maximum (the entanglement of the initial field) and the entanglement of the field reaches its minimum (zero). It implies that no matter how mixed the initial state of the junctions at a specific time the entanglement of the field can be completely transferred to junctions, and junctions evolve into a highly entangled state.

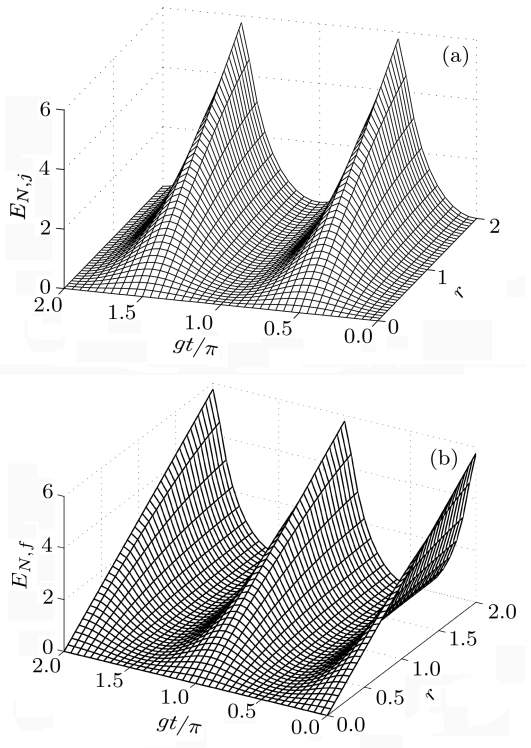

Fig. 1. Logarithmic negativity (LN) of junctions (a) and the field (b) as a function of scaled time gt/π and squeezing degree r for $\lambda = 0.1$, $\omega = 10$.

On the other hand, it can be seen that entanglement evolutions of field and junctions show the interesting phenomenon of entanglement sudden death where the entanglement can completely vanish for a period of time before it recovers. We find that the length of the time interval for the zero entanglement depends on the entanglement degree of the initial field as well as on the temperature. The death time of the

entanglement increases with the increase of the initial entanglement and decreases with the increase of the temperature. The ESD was first noted in the work of Yu and Eberly,[23] in which the disentanglement of two qubits via spontaneous emission was studied, and has been found in several contexts.[24] Recently, several works[25] are devoted to explaining the phenomenon from the viewpoint of energy transfer.

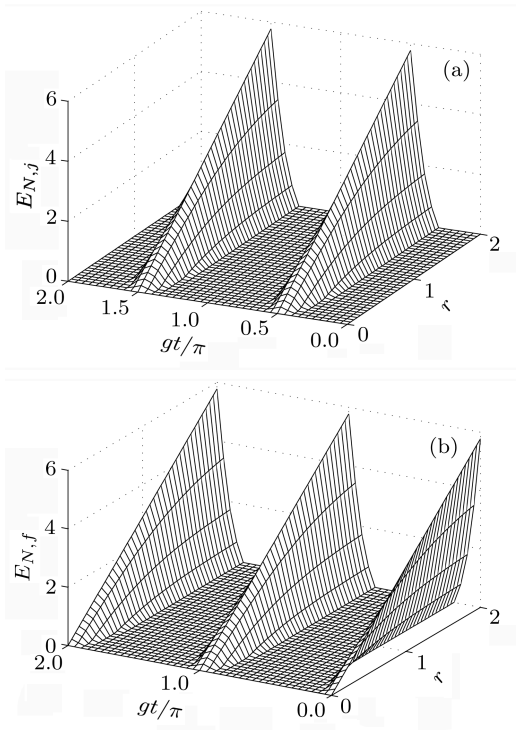

Fig. 2. LN of the junctions (a) and the field (b) as a function of scaled time gt/π and squeezing degree for r, and $\omega = 10$, $\lambda = 0.1$ and $\bar{n} = 1$.

In conclusion, we have investigated the CV entanglement of the two mesoscopic Josephson junctions driven by a two-mode cavity field, where each CV mode coupled to one mesoscopic Josephson junction via the field-junction interaction. The time-dependent characteristic function in the Wigner representation is obtained by solving the master equation of the coupling system. The entanglement transfer from a bipartite state of a CV system to a pair of localized junctions is studied for both the cases of zero temperature and finite temperature. We show that when the field is initially prepared in a two-mode squeezed vacuum state and the two Josephson junctions initially in a two-mode separable thermal state, the state of the junction subsystem can evolve into the two-mode Gaussian entangled state due to entanglement transfer from the cavity field. It is shown that no matter how mixed the initial state of the junctions at a specific time the entanglement of the field can be completely transferred to junctions and the junctions can evolve into highly entangled state.

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