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Localization Exponent for the Second Landau Level in the Quantum Hall Effect *

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At temperature above 1 K, we measured the temperature dependence of the longitudinal and Hall resistivity ρ_{xx}, ρ_{xy} in the regime of the quantum Hall plateau-to-plateau transitions. The localization exponent v is extracted with an approach based on the variable range hopping theory. We find the quantity $v \approx 2.3$ at the second Landau level, which is proven to be accurately universal.

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The localization problem of Landau levels and the relationship between the phenomenon of integer quantum Hall effect have attracted considerable and growing interest.[1,2] It is now well accepted that the existence of localized states at Landau level tails is essential to the basic quantization phenomenon. Quantum Hall plateau transitions, where Hall resistivity ρ_{xy} goes from the one quantized value to another by tuning the magnetic field B, is now understood to be a localization–delocalization transition through a very narrow band of extended states at the middle of a Landau level.[3] In analogy with other localization transitions, a power-law divergence of localization length $\xi \propto |B - B_c|^{-v}$ is expected as B approaches to the critical point B_c, where v is the localization exponent. In the critical region, resistivity tensor scales as $\rho_{uv} = \rho_c f(L/\xi)$, where f is a scaling function. Since the experimental data are measured at finite temperatures, the quantum phase coherence length induced by the electron–phonon or electron–electron scattering sets an effective finite system size L which scales with temperature as $L \propto T^{-\frac{p}{2}}$ or $T^{-\frac{1}{z}}$, where p or z is called the dynamical exponent. Then we derive $\rho_{uv} = \rho_c f(|B - B_c|T^{-\kappa})$, the scaling function of both the longitudinal resistivity ρ_{xx} and the Hall resistivity ρ_{xy}, where the critical exponent is expressed as $\kappa = \dfrac{p}{2v}$ or $\kappa = \dfrac{1}{zv}$.

There are many theoretical works to understand this quantum critical phenomenon and the values of v, p, and the corresponding κ. The most extensively studied quantity is the noninteracting localization exponent v, which has been studied by different models of random disorder potential (white noise disorder Anderson model, quantum percolation network model, etc.) and different calculation techniques (recursive Green function, transfer matrix, Chern number etc.).[4−8] All of these theoretical works give an exponent $v \approx 2.3$ for the lowest Landau level ($N = 0$, here N denotes the Landau level index). For higher Landau levels, there is also argument that the exponent $v \approx 2.3$ seems to be universal.[9] However, a much larger exponent $v \approx 5.5$ for the second Landau level ($N = 1$) is claimed by several groups,[4,10−13] which calls into the problem the Landau level independence of v. The issue is somewhat controversial, and the actual value of v for higher Landau levels remains an open question.

On the other hand, the above scenario describing the quantum Hall plateau transitions as a quantum critical phenomenon is remarkably well verified experimentally in the original work on InGaAs–InP samples.[14] A remarkable quantum phase transition has been demonstrated by showing a scaling behaviour in temperature of the maximum slope in the Hall resistivity ρ_{xy}, and of the half width for the longitudinal resistivity ρ_{xx} between neighbouring Hall plateaus: $\left.\dfrac{d\rho_{xy}}{dB}\right|_{B_c} \propto T^{-\kappa}$ and $\Delta B \propto T^{\kappa}$ with the critical exponent $\kappa = 0.42$. Notice that almost all experimental works measure the critical exponent κ only.[14−17] For lack of sufficient knowledge about the dynamical exponent p, the information about the value of quantity v cannot be obtained *separately* by the relation $\kappa = p/2v$. Thus to determine experimentally whether v is universal for higher Landau levels, another extrapolation method is required.

To deepen the understanding of universality of localization exponent v and to avoid the problems mentioned above, we extend the study to higher temperature and utilize a direct experimental approach. In contrast to the previous papers in which we have reported the plateau-to-plateau transitions and the corresponding exponents κ using the above-mentioned or-

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dinary experimental method,[18,19] in this Letter we measure the localization exponent v directly and determine the universality of v for the second Landau level ($N = 1$). The sample we measured was grown by molecular-beam epitaxy and consists of a 25-nm-wide GaAs well bounded on each side by undoped and Si δ-doped layers of AlGaAs. At low temperature about 4 K, it has a mobility $\mu = 2.1\times10^5\,\mathrm{cm^2V^{-1}s^{-1}}$ and the electron density n_s is fixed at $2.8\times10^{11}\,\mathrm{cm^{-2}}$. Hall-bar patterns with an aspect ratio of 5 : 2 were fabricated by standard photolithography, and the contacts were made by deposition of Au alloy. The sample is of high quality as it shows very strong integer quantum Hall states. In order to measure ρ_{xx} and ρ_{xy}, we use a standard ac lock-in technique with electric current $I = 10$ nA and frequency of 13 Hz. In the present experiment we use a ^{4}He cryogenic system from 1.7 K to 4 K, and a further experiment at lower temperature (from several mK to 1 K) using Oxford Dilution Refrigerator will appear soon.[20] We emphasize that although most of the quantum Hall plateau transition experiments are performed at lower temperatures (below 1 K), we are interested in the transition behaviour at somewhat higher temperature (above 1 K). Nevertheless, the sample can also clearly show the scaling phenomenon in the plateau transitions at temperature range above 1 K, which means that this localization related to quantum phase transition is robust.

To extract the correlation exponent v, we use a method based on the variable range hoping scaling theory. It can be concluded from the wide plateau that most of the electron states are localized at low temperatures. Actually Paalanen *et al.* estimated that in a GaAs–AlGaAs heterostructures at 50 mK, 95% of the states in each Landau level are localized.[21] Thus the mechanism for the conductivity is variable range hopping (VRH) in the presence of Coulomb interaction. When the localization length ξ becomes much smaller than the effective temperature length L,[19] VRH dominates the conductivity in the transition regime. By assuming VRH transport responsible for broadening of the σ_{xx} peaks, Polyakov and Shklovskii[22,23] arrive at an explicit expression of scaling function

$$\sigma_{xx} = \sigma_0 \exp\Big(-\sqrt{\frac{T_0}{T}}\Big), \quad T_0(B) \propto \frac{e^2}{\varepsilon\xi(B)}. \qquad (1)$$

The characteristic temperature T_0 is determined by the Coulomb interaction at length scale given by the localization length ξ. Here $\sigma_0 \propto 1/T$ is a temperature dependent factor. As mentioned above, the localization length $\xi(B)$ diverges as $\xi(B) \propto |B - B_c|^{-v}$ when B approaches to the critical point B_c. Combining these relations immediately yields

$$\sigma_{xx} = \sigma^* s \exp(-\sqrt{T^* s}), \quad s = \frac{|B - B_c|^v}{T}, \qquad (2)$$

where s is the scaling variable, σ^* and T^* are constants. Using this formula between conductivity σ and ξ, we can easily and directly obtain the localization length ξ and corresponding localization exponent v.

We focus on the transition from $i = 2$ to $i = 3$ and from $i = 4$ to $i = 3$ integer quantum Hall states, which correspond to Landau levels $1\uparrow$ and $1\downarrow$. Here we denote the filling factor $i = nh/eB$, and the spin-split Landau level index as $N\uparrow$ or $N\downarrow$. Firstly we convert the measured resistivity data ρ's to conductivity σ's using the standard matrix conversion,

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}, \quad \sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} \qquad (3)$$

and plot the σ traces at different temperatures from 1.79 K to 2.69 K in Fig. 1. A σ_{xx} peak that widens with T and accompanying step in σ_{xy} clearly shows the transition between the neighbouring plateaus around filling factors $i = 2$, 3, 4.

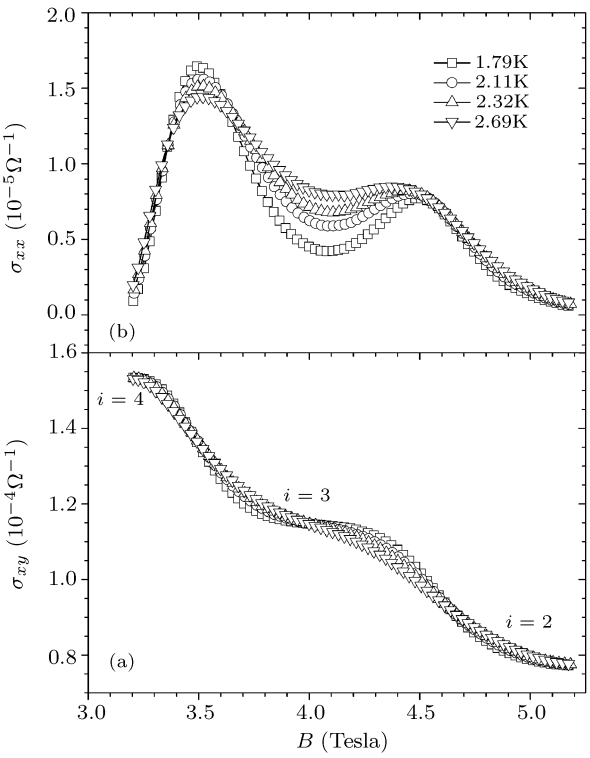


Fig. 1. Longitudinal conductivity and Hall conductivity vs magnetic field B at different temperatures T for the transitions between plateaus $i = 2$, $i = 3$ and $i = 4$.

Let us now turn to a closer analysis of the σ data. As shown in Fig. 2, the data at $1\uparrow$ as well as $1\downarrow$ Landau levels fit well the predicted temperature behaviour of Eq. (2) for the same exponent $v \approx 2.3$. Notice that the fitting is perfect for the $1\downarrow$ Landau level and good for the $1\uparrow$ Landau level. We think that this is strong evidence that the localization behaviour and the localization exponent $v \approx 2.3$ are universal for the second Landau level($N = 1$). Further, from these

fits we are able to extract the localization length ξ in the plateau transition regime and plot it against the distance to the critical point B_c in Fig. 3 for $1\downarrow$ Landau level. We find that it follows the power-law behaviour $T_0(B) \sim 1/\xi(B) \propto |B - B_c|^v$ with the exponent $v = 2.3 \pm 0.3$.

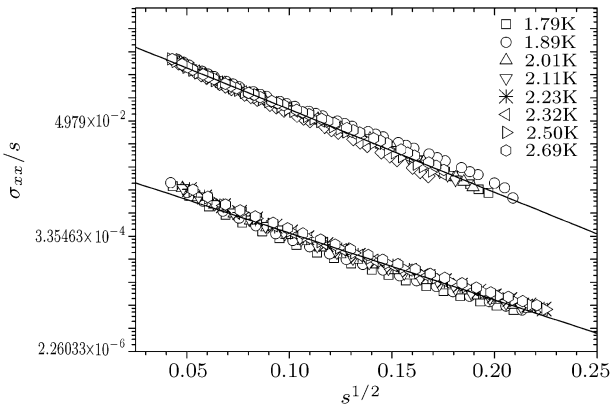


Fig. 2. Longitudinal conductivity σ_{xx} as a function of the scaling variable s in the plateau transition regime. The dashed line is indicated for fitting to Eq. (2). Upper: at $1\downarrow$ Landau level. Lower: at $1\uparrow$ Landau level. The data for the $1\downarrow$ Landau level are shifted for clarity.

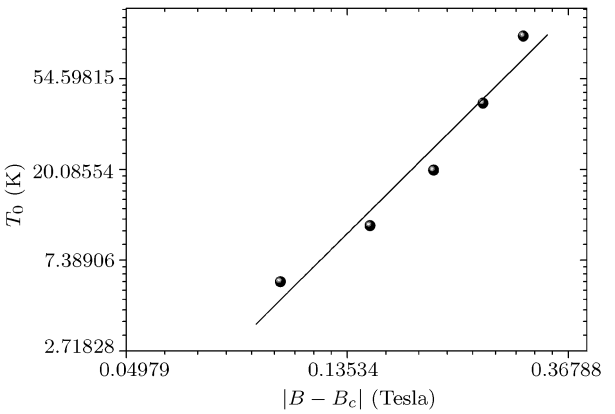


Fig. 3. Characteristic temperature T_0 (corresponding to the localization length ξ) as a function of the distance of magnetic field B to the critical point B_c for plateau transition at $1\downarrow$ Landau level.

Although from our previous experiment we found that the critical exponent $\kappa = p/2v$ varies from 0.66 to 0.71 for the transitions at $1\uparrow$ and $1\downarrow$ Landau levels,[18] the localization exponent $v \approx 2.3$ is clearly independent of the Landau level index. To the best of our knowledge, there is only one experiment measuring the localization exponent v directly by studying the plateau transitions as a function of the sample size at very low temperature[24] (from 25 mK to 1 K). We also find that $v \approx 2.3$ is independent of the Landau levels and disorder strength. In contrast to their result which is obviously very limited because there are only four data of sample size, our experiment gives strong support to the universality of localization exponent $v \approx 2.3$ for the second Landau level ($N = 1$).

We propose an explanation for the result that is observed in our experiment. For the lowest Landau level ($N = 0$), the situation is well understood, and the theoretical calculated exponent $v \approx 2.3$ is independent of the range and strength of the disorder potential. However calculations for v at higher Landau levels (for example, at the second Landau level $N = 1$) which give a value of $v \approx 5.5$ often begin with the assumption that the two-dimensional electrons move in the short range disorder potential (uncorrelated δ function like potential). Careful theoretical analysis gives that if we include a finite range in the disorder potential, v will be found to decrease quickly from 5.5 to 2.3. When the range of disorder potential is about 4 times as the magnetic length $l_B = \sqrt{\dfrac{\hbar}{eB}}$, a value of $v \approx 2.3$ is obtained.[12,13] Actually, in the GaAs–AlGaAs samples, the disorder of the system is dominated by the potential of the ionized impurities. Being screened by the two-dimensional electrons, the Coulomb potential fluctuation becomes slowly varying with a large correlation length of the order of 100 nm. Since the magnetic length l_B at higher Landau levels is always about several hundreds of Å, it is safe to assume that the disorder potential is usually of finite range, i.e., the range is comparable to or larger than the magnetic length. Thus it is quite possible that due to the presence of finite range disorder potential, $v \approx 2.3$ is indeed universal for higher Landau levels, which is consistent with our experimental result. However, more experimental work will be helpful in resolving this issue.

In conclusion, we have measured the temperature dependence of the resistivity ρ of a two-dimensional electron system in the regime of the quantum Hall plateau transition. We extract quantitative information of the localization exponent v by an approach based on the variable range hopping theory and the main result is that we find an accurate universality of the localization exponent $v \approx 2.3$ for the second Landau level. However we focus on the behaviour at high temperatures; similar experiment at lower temperatures (from several mK to 1 K) will be discussed in the near future.[20]

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