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# Wavelet Cross-Spectrum Analysis of Multi-Scale Disturbance Instability and Transition on Sharp Cone Hypersonic Boundary Layer *

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*Experimental measurement of hypersonic boundary layer stability and transition on a sharp cone with a half angle of 5° is carried out at free-coming stream Mach number 6 in a hypersonic wind tunnel. Mean and fluctuation surface-thermal-flux characteristics of the hypersonic boundary layer flow are measured by Pt-thin-film thermocouple temperature sensors installed at 28 stations on the cone surface along longitudinal direction. At hypersonic speeds, the dominant flow instabilities demonstrate that the growth rate of the second mode tends to exceed that of the low-frequency mode. Wavelet-based cross-spectrum technique is introduced to obtain the multi-scale cross-spectral characteristics of the fluctuating signals in the frequency range of the second mode. Nonlinear interactions both of the second mode disturbance and the first mode disturbance are demonstrated to be dominant instabilities in the initial stage of laminar-turbulence transition for hypersonic shear flow.*

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Hypersonic boundary layer stability and transition are of importance to aeronautic and astronautic engineering as it is characterized by marked increases in heat transfer and skin friction in aeronautic and astronautic vehicles.[1,2] These increases have significant impact on the performance control and structural design of the hypersonic vehicles. Unlike the case of subsonic flow, the mechanisms underlying the laminar-turbulence transition in hypersonic flow still are poorly understood.[3,4] The uncertainty in the prediction of the transition location results in overly conservative designs that are not optimally efficient. At hypersonic velocities, for Mach numbers greater than 4–4.5, a new acoustic type of instability (the so-called second mode instability)[5,6] arises in addition to the vortex instability mode (the first mode instability). According to the theoretical and numerical studies,[7,8] the second mode in hypersonic flows is the most unstable instability and is expected to govern the occurrence of the laminar-turbulence transition.

Wavelet transform is a mathematic technique developed with enormous applications in the fields of science and engineering. It convolutes time series signals with an analytic function named wavelet at a definite time and scale by means of dilations and translations of mother wavelet. It provides a two-dimensional unfolding of one-dimensional signals resolving both on spatial and temporal dimensions as independent variables. Thus wavelet transform overcomes the shortcomings of the Fourier analysis by adding the time resolution and providing simultaneous temporal characteristics of signals.

Relative to a mother wavelet $\psi(t)$, with respect to a family of wavelet, $\psi_{ab}(t)$ is the shift (by factor $b$) and dilatation (by factor $a$) of $\psi(t)$, i.e.

$$\psi_{ab}(t) = \frac{1}{\sqrt{a}}\psi\Big(\frac{t-b}{a}\Big). \tag{1}$$

The continuous wavelet transform of signal $f(t)$ with respect to a family of wavelet $\psi_{ab}(t)$ is defined as their scalar product by

$$\psi_f(a,b) = \int_{-\infty}^{+\infty} f(t)\overline{\psi_{ab}(t)}dt, \tag{2}$$

where the over bar indicates the complex conjugate operator. Wavelet analysis is an extension of the Fourier analysis by shift time windows with proper time interval (window width). This is appropriate for spectrum analysis of non-stationary signals with multi-scale local disturbances whose power spectral characteristics are not constant in time. The occurring positions and scale of temporal disturbance can be identified accurately by wavelet analysis.

The wavelet cross-spectrum is defined by wavelet coefficients $\psi_f(a_1,b)$ and $\psi_f(a_2,b)$ at two scales $a_1$ and $a_2$,

$$B_{ff}(a_1,a_2) = \int_T \psi_f(a_1,\tau)\overline{\psi_f(a_2,\tau)}d\tau. \tag{3}$$

The cross-spectrum[9,10] depends heavily on the simultaneous behaviour of the signals at two frequencies

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$f_1 = 1/a_1$ and $f_2 = 1/a_2$. In particular, when the goal is to study non-stationary signal with multi-scale local disturbance, wavelet cross-spectrum can point out the interaction level of different frequency components included in local disturbance.

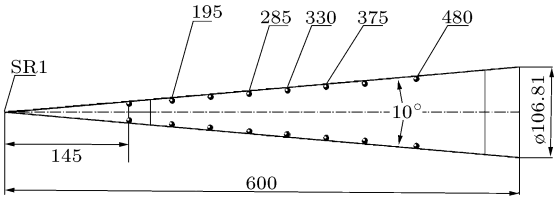



**Fig. 1.** Schematics of the sharp cone model.

Experiments were carried out in the hypersonic blow-down wind tunnel. This tunnel was designed for the Mach numbers $M_\infty = 5-15$ and the unit Reynolds number $Re_1$ at $5 \times 10^5 - 7 \times 10^7\,\mathrm{m}^{-1}$, with a set of changeable profile and conic axis-symmetric nozzles. An Ohmic heater was used for maintaining air temperature in the settling chamber up to 700–1300 K. The open-jet test section has the length of 1600 mm with the nozzle end diameter of 480 mm. The stagnation pressure $P_0$ and stagnation temperature $T_0$ were measured and kept to be constant. Here the stagnation pressure is 2.69 MPa, and the stagnation temperature $T_0$ is 702 K. The unit Reynolds number $Re_1$ is $1.3 \times 10^7\,\mathrm{m}^{-1}$.

The model is a cone sharp with half-angle of 5° and length 600 mm, as shown in Fig. 1. The model consists of a sharp cone nose of 1 mm in bluntness radius and the base part equipped with 28 Pt-thin-film thermocouple sensors to measure the wall temperature. The thermocouple sensors are located along the bottom generatrix. The first thermocouple sensor is located at 90 mm from the cone apex; and the other ones are placed in uniform separation from each other.

The output signals were digitized by 12-bit analog-to-digital converter data acquisition cards with ±5 V range and 1 MHz sampling rate. At each location of measurement, 524288 samples are recorded.

Figure 2 shows the wavelet coefficients magnitude contour of the temperature signals at different stations along one generatrix. The corresponding temperature signals are also presented to compare the disturbance occurrence locations. For the purpose of clarity, only a section of 7 ms of the signals is presented in the figure. From the plane representation of the wavelet coefficients, it can be seen that some local high magnitudes of the wavelet coefficients exist at different time in the scale range of 200 kHz to 300 kHz corresponding to the second-mode disturbance. The second mode disturbance is dominant in a hypersonic laminar boundary layer. More and more large amplitude disturbances are triggered downstream with the development of the disturbance. At the last station $X = 480$ mm, the disturbance is fully developed and the transition reaches its peak.

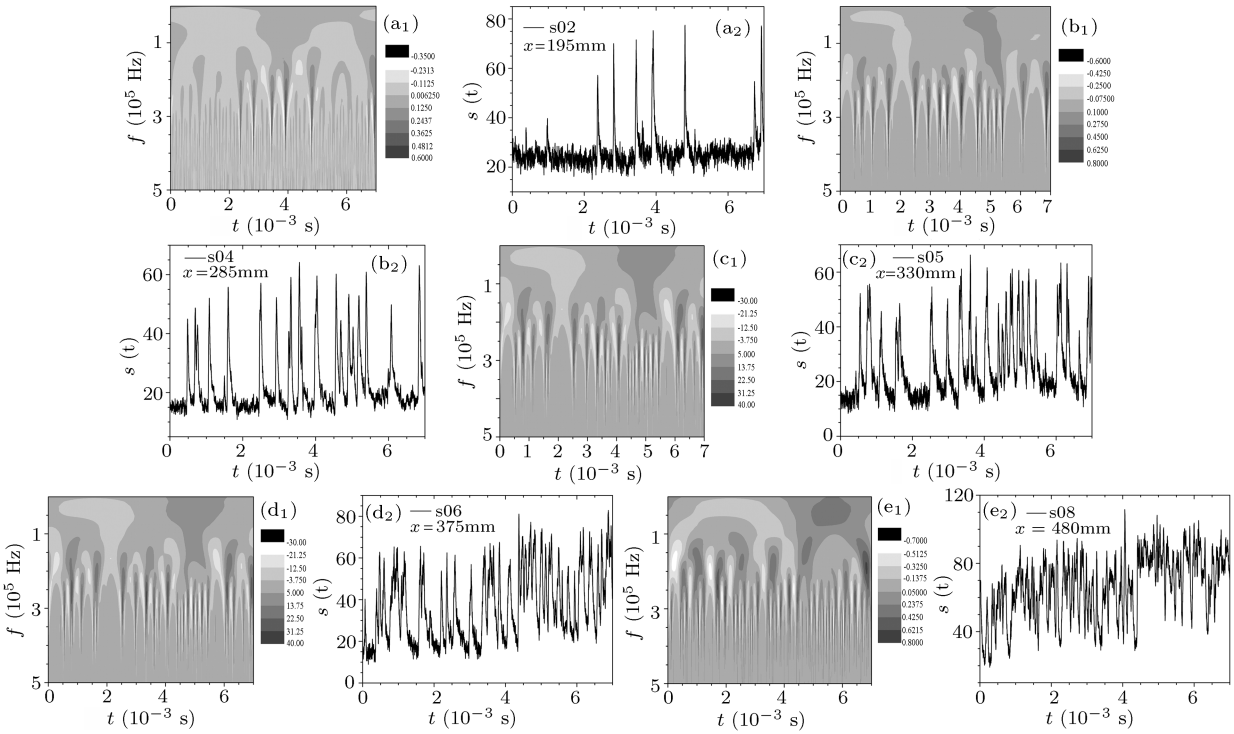



**Fig. 2.** Wavelet coefficients magnitude contour of the signal at different stations along one generatrix on the cone: (a) $X = 195$ mm, (b) $X = 285$ mm, (c) $X = 330$ mm, (d) $X = 375$ mm, (e) $X = 480$ mm

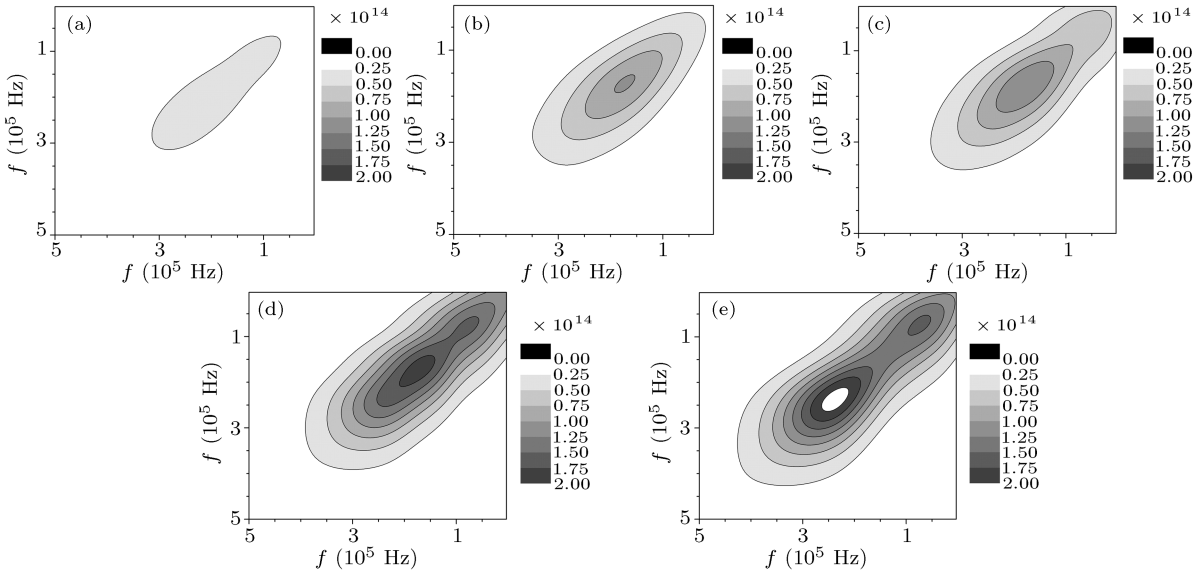



**Fig. 3.** Two-scale wavelet cross spectrum of the temperature signals at different stations along one generatrix on the cone: (a) $X = 195\,\mathrm{mm}$, (b) $X = 285\,\mathrm{mm}$, (c) $X = 330\,\mathrm{mm}$, (d) $X = 375\,\mathrm{mm}$, (e) $X = 480\,\mathrm{mm}$.

Figure 3 shows the multi-scale wavelet cross-spectrum calculated by Eq. (3) providing the development details of the frequency interaction of disturbance in hypersonic boundary layer. At the initial stage of transition, the frequency band of the disturbance is narrow, the border between the first mode and the second mode is confused. No significant frequency peak is observed in the plot of wavelet cross spectrum. However, the disturbance at different frequencies within this narrow band is important. They couple each other with nonlinearity and give rise to their enhancement. The nonlinear interaction leads to new disturbance modulation arising and the band range of disturbance frequencies becomes broader with the boundary layer developing downstream. These are qualitative indicators of nonlinear interaction growth among different frequency components in the disturbance frequency band. The disturbance develops across a broader frequency range and two significant frequency peaks appear gradually representing the first mode and the second mode respectively. There is a very clear dominant frequency band including the first mode and the second mode in the hypersonic laminar boundary layer. At $X = 480\,\mathrm{mm}$ the first mode and the second mode can be identified clearly. This also indicates that the transition reaches its peak while the second mode is centred at $f_2 = 226\,\mathrm{kHz}$ and the first mode is centred at $f_1 = 68\,\mathrm{kHz}$.

In summary, a new analysis method based on wavelet transform, termed as the wavelet cross spectra, is presented to investigate the evolution mechanism of unstable disturbance during hypersonic boundary layer transition. The measurement results show that the initial unstable disturbances cause nonlinear interactions between different frequencies. The nonlinear interactions among different frequencies broaden the band frequency range of unstable disturbances in return. Both the low-frequency and the second mode disturbance are important for interactions among different unstable frequencies in the initial stage of transition. This interaction precedes the final breakdown of the hypersonic laminar boundary layer and transition to turbulence.

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