Realization of Greenberg–Horne–Zeilinger (GHZ) and W Entangled States with Multiple Superconducting Quantum-Interference Device Qubits in Cavity QED *

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An alternative scheme is proposed for generating the Greenberg–Horne–Zeilinger (GHZ) and W types of the entangled states with multiple superconducting quantum-interference device (SQUID) qubits in a single-mode microwave cavity field. In this scheme, there is no transfer of quantum information between the SQUIDs and the cavity, the cavity is always in the vacuum and thus the requirement on the quality of cavity is greatly loosened. In addition, during the process of the generation of the W entangled state, the present method does not involve a real excitation of intermediate levels. Thus, decoherence due to energy relaxation of intermediate levels is minimized.

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Entanglement plays an important role in quantum information processing (QIP) and quantum secret communications, such as quantum cryptography,[1] quantum teleportation,[2] error correction protocols,[3] and quantum dense coding.[4] In recent years, much attention has been paid to generation of entangled states with a large number of qubits, especially to realizations of the Greenberg–Horne–Zeilinger (GHZ)[5] and W[6] types of entangled states. As a promising solid-state candidate, superconducting quantum-interference devices (SQUIDs) are demonstrated to have long decoherence times.[7−9] In this Letter, methods are presented to prepare multiple SQUID qubits in the GHZ and W types of the entangled states.

First, let us consider a system composed of a three-level SQUID (as shown in Fig. 1) coupled to a single-mode cavity field (Assume that all other cavity modes are well decoupled to the three energy levels of the SQUID. The SQUIDs considered here are the identical rf SQUIDs, which consist of a Josephson tunnel junction enclosed by a superconducting loop). We may denote the three levels as $|0\rangle$, $|1\rangle$ and $|2\rangle$. Assume that the cavity mode is coupled to the $|0\rangle \leftrightarrow |2\rangle$ transition but far-off resonant with the transitions $|0\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |2\rangle$. The Hamiltonian of the coupled system is given by

$$H = H_s + H_c + V, \tag{1}$$

where H_s, H_c and V are the Hamiltonians of the SQUID and the single-mode cavity, and the interaction term, respectively.

The Hamiltonian of the SQUID (with junction capacitance C and loop inductance L) can be written as[10,11]

$$H_s = \frac{Q^2}{2C} + \frac{(\Phi - \Phi_e)^2}{2L} - E_J \cos\frac{2\pi}{\Phi_0}, \tag{2}$$

where the total charge Q on the capacitor and the magnetic flux Φ threading the ring are the conjugate variables of the system with the commutation relation $[\Phi, Q] = i\hbar$; $E_J \equiv I_c\Phi_0/2\pi$ is the Josephson coupling energy (I_c is the critical current of the junction and $\Phi_0 = h/2e$ is the flux quantum), and Φ_e is the static (or quasistatic) external flux applied to the ring.

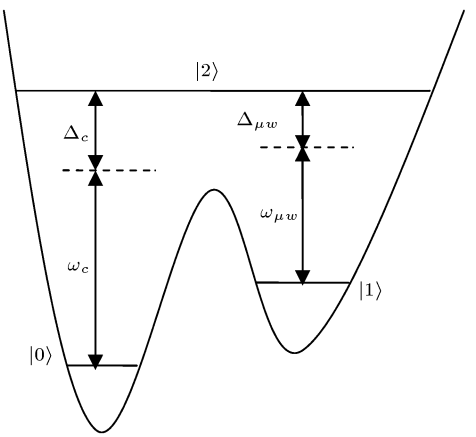

Fig. 1. Simplified level diagram of an rf SQUID with the -type three levels $|0\rangle$, $|1\rangle$ and $|2\rangle$. The transitions $|0\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ are driven by a cavity field and a microwave, respectively.

The Hamiltonian of the single-mode cavity field is (assuming $\hbar = 1$)

$$H_c = \omega_c\Big(a^+a + \frac{1}{2}\Big), \tag{3}$$

where ω_c is the frequency of the cavity field, a^+ and a are the creation and annihilation operators of the mode.

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The interaction term is given by[12]

$$V = \lambda_c \Phi_c \sum_{n,m} |n\rangle\langle n|\Phi - \Phi_e|m\rangle\langle m|, \quad (n, m = 0, 2) \tag{4}$$

where Φ_c is the magnetic flux threading the ring, which is generated by the magnetic component of the cavity field, and $\lambda_c = -1/L$ is the coupling parameter between the quantized cavity field and the SQUID. Under the rotation-wave approximation, the Hamiltonian of the system in the interaction picture can be written as[12]

$$H_I = g_{20}(e^{-i\Delta t}a^+|0\rangle\langle 2| + e^{i\Delta t}a|2\rangle\langle 0|), \tag{5}$$

$$g_{20} = \frac{1}{L}\sqrt{\frac{\omega_c}{2\mu_0}}\langle 0|\Phi|2\rangle \int_S B_c(r)\cdot dS, \tag{6}$$

where S is any surface that is bounded by the SQUID ring, r is the position vector of a point on S; $B_c(r)$ is the magnetic component of the normal mode of the cavity; g_{20} is the coupling constant between the SQUID and the cavity field; $\Delta = \omega_c - \omega_{20}$ is the detuning from the transition between the levels $|0\rangle$ and $|2\rangle$ with ω_{20} being the Raman frequency between $|0\rangle$ and $|2\rangle$.

Now we consider an SQUID driven by a classical microwave pulse that is resonant (the classical microwave frequency $\omega_{\mu w} = \omega_{20}$) with the transition between the levels $|0\rangle$ and $|2\rangle$. Under the rotation-wave approximation, we can show the Hamiltonian in the interaction picture as follows:[12]

$$H_I = \Omega_{02}(|0\rangle\langle 2| + |2\rangle\langle 0|), \tag{7}$$

$$\Omega_{02} = \frac{1}{2L}\langle 1|\Phi|2\rangle \int_s B_{\mu w}(r)dS, \tag{8}$$

where Ω_{02} is the frequency of the Rabi oscillation between $|0\rangle$ and $|2\rangle$, and $B_{\mu w}(r)$ is the component of the cavity mode. The state evolution can be written easily from Eq. (6) as

$$|0\rangle \to \cos(\Omega_{02}t)|0\rangle - i\sin(\Omega_{02}t)|2\rangle, \tag{9}$$

$$|2\rangle \to -i\sin(\Omega_{02}t)|1\rangle + \cos(\Omega_{02}t)|2\rangle. \tag{10}$$

If a three-level SQUID is coupled to a single-mode quantized cavity field and a classical microwave pulse (as shown in Fig. 1), the transition $|0\rangle \leftrightarrow |2\rangle$ is driven by the cavity field with coupling constant g_{20} and the transition $|1\rangle \leftrightarrow |2\rangle$ is driven by a classical microwave field with Rabi frequency Ω_{12}. In the interaction picture, the Hamiltonian of the coupled systems can be shown as

$$H_I = g_{20}(ae^{i\Delta_c t}|2\rangle\langle 0| + \text{H.c}) + \Omega_{12}(e^{i\Delta_{\mu w}t}|2\rangle\langle 1| + \text{H.c.}), \tag{11}$$

where g_{20} and Ω_{12} are the same as those described before, $\Delta_c = \omega_{20} - \omega_c$ is the detuning of the cavity field from the transition $|0\rangle \leftrightarrow |2\rangle$, $\Delta_{\mu w} = \omega_{21} - \omega_{\mu w}$ is the detuning of the classical microwave field from the transition $|1\rangle \leftrightarrow |2\rangle$ and ω_{21} is the transition frequency between the levels $|2\rangle$ and $|1\rangle$. In the case $\Delta_c \gg g_{20}$ and $\Delta_{\mu w} \gg \Omega_{21}$, the upper level $|2\rangle$ can be adiabatically eliminated, we can obtain the Roman coupling of the two lower flux states,[13,14]

$$H_I = \gamma(a|1\rangle\langle 0|e^{i(\Delta_c - \Delta_{\mu w})t} + a^+|0\rangle\langle 1|e^{-i(\Delta_c - \Delta_{\mu w})t}), \tag{12}$$

where $\gamma = \frac{1}{2}\Omega_{21}g_{20}\left(\frac{1}{\Delta_c} + \frac{1}{\Delta_{\mu w}}\right)$. Considering that the system of two identical SQUIDs and couples to a single-mode cavity and each SQUID is driven by a classical microwave pulse with the frequency $\omega_{\mu w}$, we assume that distance between the two SQUIDs is much larger than the linear dimension of each SQUID ring in such a way, so that the interaction between the two SQUIDs is negligible. Under the condition $\Delta_c - \Delta_{\mu w} \gg \gamma$, there is no exchange of energy between the SQUIDs and the cavity mode. The effective Hamiltonian of the system is given by[15,16]

$$H_{\text{eff}} = \lambda\Big[\sum_{i=1,2} -a^+a|0\rangle_i\langle 0| + aa^+|1\rangle_i\langle 1| + |1\rangle_1\langle 0| \otimes |0\rangle_2\langle 1| + |0\rangle_1\langle 1| \otimes |1\rangle_2\langle 0|\Big], \tag{13}$$

where the first and second terms describe the photon-number dependent Stark shifts induced by the off-resonant Roman coupling, while the third and fourth terms describe the dipole coupling between the two SQUIDs mediated by the cavity mode and the classical field. The parameter λ of Eq. (13) is expressed as $\lambda = \gamma^2/(\Delta_c - \Delta_{\mu w})$. If the cavity is initially in the vacuum state, then Hamiltonian (13) reduces to

$$H_{\text{eff}} = \lambda\Big[\sum_{i=1,2} |1\rangle_i\langle 1| + |1\rangle_1\langle 0| \otimes |0\rangle_2\langle 1| + |0\rangle_1\langle 1| \otimes |1\rangle_2\langle 0|\Big]. \tag{14}$$

From Hamiltonian (14), we can see that only the state of the SQUID system undergoes an evolution under Hamiltonian (14), and no quantum information transfer occurs between the SQUIDs and the cavity mode. Therefore, the following state evolution can be easily obtained from Eq. (14),

$$\begin{aligned}
&|0\rangle_1|0\rangle_2 \to |0\rangle_1|0\rangle_2, \quad |0\rangle_1|2\rangle_2 \to |0\rangle_1|2\rangle_2,\\
&|2\rangle_1|0\rangle_2 \to |2\rangle_1|0\rangle_2, \quad |2\rangle_1|1\rangle_2 \to e^{-i\lambda t}|2\rangle|1|1\rangle_2,\\
&|1\rangle_1|2\rangle_2 \to e^{-i\lambda t}|1\rangle_1|2\rangle_2, \quad |1\rangle_1|1\rangle_2 \to e^{-i2\lambda t}|1\rangle_1|1\rangle_2,\\
&|0\rangle_1|1\rangle_2 \to e^{-i\lambda t}[\cos(\lambda t)|0\rangle_1|1\rangle_2 - i\sin(\lambda t)|1\rangle_1|0\rangle_2\\
&|1\rangle_1|0\rangle_2 \to e^{-i\lambda t}[\cos(\lambda t)|1\rangle_1|0\rangle_2 - i\sin(\lambda t)|0\rangle_1|1\rangle_2].
\end{aligned} \tag{15}$$

Let us consider a model of N $(j = 1, 2, 3, \ldots, N)$ identical SQUIDs coupled to a single-mode cavity field (as shown in Fig. 2). We assume that the interaction between any SQUIDs is negligible, and the N SQUIDs are initially in the states $|0\rangle_1|1\rangle_{2,3,\ldots,N}$.

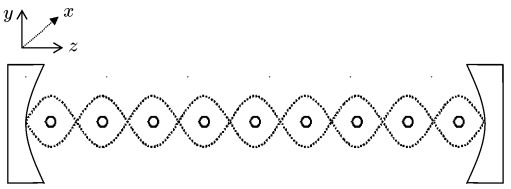

Fig. 2. Schematic setup of N SQUIDs ($j = 1, 2, 3, \ldots, N$) in a standing-wave cavity field. The magnetic of the cavity mode and the pulse are in the Y direction. Each SQUID is placed in the $X - Z$ plane. All the SQUIDs experience identical coupling to the cavity mode and the classical microwave pulse.

The GHZ entangled state with N SQUIDs can be realized in the following steps.

Step I: Adjust the level spacing of the two SQUIDs ($j = 1, 2$), in order to let the state of the two SQUIDs undergoes an evolution for an interaction time $\tau_1 = \dfrac{\pi}{4\lambda}$ under Hamiltonian (14), and then adjust the level spacing of the SQUIDs back to the original configuration. We can discard the common phase $e^{-i\lambda t}$ in the result. Step Ⅱ: A resonant π microwave pulse ($2\Omega_{20}\tau_2 = \pi$ where τ_2 is the pulse duration) with $\omega_{\mu w} = \omega_{20}$ is applied to the SQUIDs ($j = 1$), resulting in the transition $|0\rangle_1 \to -i|2\rangle_1$. Then apply a resonant microwave pulse to the SQUID ($j = 1$) again, resulting in the evolution of the SQUID state $|2\rangle_1 \to -i|1\rangle_1$ and $|1\rangle_1 \to -i|2\rangle_2$. Finally, apply a resonant microwave pulse with $\omega_{\mu w} = \omega_{20}$ for the interaction time $\dfrac{\pi}{2\,\Omega_{20}}$, resulting in the transition $|2\rangle_1 \to -i|0\rangle_1$. Step Ⅲ: Apply a resonant π microwave pulse to SQUID ($j = 1$), resulting in the transition $|1\rangle_1 \to -i|2\rangle_1$. Step Ⅳ: Adjust the level spacing of the SQUIDs ($j = 1, 3$), let the state of the two SQUIDs undergo an evolution for an interaction time $\pi/2\lambda$ under Hamiltonian (14), and then adjust the level spacing of the two SQUIDs back to the original configuration. The common phase $e^{-i\lambda t}$ can also be discarded. Step V: Apply a resonant π microwave pulse with $\omega_{\mu w} = \omega_{21}$ to the SQUID ($j = 1$), then apply a resonant π microwave pulse with $\omega_{\mu w} = \omega_{20}$ to the SQUID ($j = 1$) again, resulting in the transition $|2\rangle_1 \to -i|1\rangle_1$, $|1\rangle_1 \to -i|2\rangle_I$ and $|2\rangle_1 \to -i|0\rangle_1$.

Lastly, we repeat steps Ⅲ–V to the SQUIDs ($j = 1$, 4), then to the SQUIDs ($j = 1, 5$), until the SQUIDs ($j = 1, N$).

The evolution of the system after the above several operations can be shown as follows:

$$
\begin{aligned}
&|0\rangle_1|1\rangle_{2,3,\ldots,N} \to \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 - i|1\rangle_1|0\rangle_2)|1\rangle_{3,4,\ldots,N}\\
&\to \frac{1}{\sqrt{2}}(-|1\rangle_1|1\rangle_2 + i|0\rangle_1|0\rangle_2)|1\rangle_{3,4,\ldots,N}\\
&\to \frac{1}{\sqrt{2}}(i|2\rangle_1|1\rangle_2|1\rangle_3 + i|0\rangle_1|0\rangle_2|1\rangle_3)|1\rangle_{4,5,\ldots,N}\\
&\to \frac{1}{\sqrt{2}}(i|2\rangle_1|1\rangle_2|1\rangle_3 + |1\rangle_1|0\rangle_2|0\rangle_3)|1\rangle_{4,5,\ldots,N}\\
&\to \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_2|1\rangle_3 - |0\rangle_1|0\rangle_2|0\rangle_3)|1\rangle_{4,5,\ldots,N}\\
&\to \frac{1}{\sqrt{2}}(-|1\rangle_1|1\rangle_2|1\rangle_3|1\rangle_4 - i|0\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4)|1\rangle_{5,6,\ldots,N}\\
&\to \cdots \to \frac{1}{\sqrt{2}}[(-1)^{N-1}|1\rangle_{1,2,\ldots,N} + (i)^{N-1}|0\rangle_{1,2,\ldots,N}].
\end{aligned}
\tag{16}
$$

It is clear that the operations accomplish an N-SQUID-qubit GHZ entangled state.

Assume the three-SQUID qubit are initially in the state $|0\rangle_1|1\rangle_{2,3}$. Then the W type of entangled state with three SQUIDs can be realized by the following steps.

Step i: By adjusting the levels of the SQUIDs ($j = 1, 2$), let the state of the two SQUIDs undergo an evolution for interaction time $\dfrac{1}{\lambda}\arccos\left(\dfrac{\sqrt{N-1}}{\sqrt{N}}\right)$ under Hamiltonian (14) and then adjust back to the original configuration. The common phase $e^{-i\lambda t}$ in the result can be discarded.

Step ii: Repeat step i until that the two SQUIDs ($1, j = 3$) undergo an evolution for interaction time $\dfrac{1}{\lambda}\arccos\left(\dfrac{\sqrt{N+1-j}}{\sqrt{N+2-j}}\right)$ under Hamiltonian (14). We can obtain the three-SQUID W entangled state as follows:

$$
\begin{aligned}
|0\rangle_1|1\rangle_{2,3,\ldots,N} \to \Big(&\frac{\sqrt{N-1}}{\sqrt{N}}|0\rangle_1|1\rangle_2\\
&- i\frac{1}{\sqrt{N}}|1\rangle_1|0\rangle_2\Big)|1\rangle_3 \to \cdots \to\\
&- \frac{i}{\sqrt{N}}(i|0\rangle_1|1\rangle_2|1\rangle_3 + |1\rangle_1|1\rangle_2|0\rangle_3\\
&+ e^{-i\arccos\left(\frac{\sqrt{N-2}}{\sqrt{N-1}}\right)}|1\rangle_1|0\rangle_2|1\rangle_3\Big).
\end{aligned}
\tag{17}
$$

In the general case of N SQUIDs which are initially in the state $|0\rangle_1|1\rangle_{2,3,\ldots,N}$, repeat step i to the two SQUIDs ($1, j = 3$), then to the SQUIDs ($1, j = 4$), until that the SQUIDs ($1, j = N$) undergo an evolution for interaction time $\dfrac{1}{\lambda}\arccos\left(\dfrac{\sqrt{N+1-j}}{\sqrt{N+2-j}}\right)$ under Hamiltonian (14). For simplicity, we do not consider the phase $e^{-i\lambda t}$. The states of the system after the above steps are summarized in the following:

$$
\begin{aligned}
&|0\rangle_1|1\rangle_{2,3,\ldots,N} \to \Big(\frac{\sqrt{N-1}}{\sqrt{N}}|0\rangle_1|1\rangle_2\\
&- i\frac{1}{\sqrt{N}}|1\rangle_1|0\rangle_2\Big)|1\rangle_{3,4,\ldots,N} \to \cdots\\
&\to \Big(\frac{\sqrt{N-j}}{\sqrt{N}}|0\rangle_1|1\rangle_{2,3,\ldots,N}\\
&- i\frac{1}{\sqrt{N}}|1\rangle_1|0\rangle_2|1\rangle_{3,4,\ldots,N} - \cdots\\
&- i\frac{1}{\sqrt{N}}|1\rangle_{1,2,\ldots,j-1}|0\rangle_j|1\rangle_{j+1,j+2,\ldots,N}\Big) \to \cdots
\end{aligned}
$$

$$\rightarrow -\frac{i}{\sqrt{N}}\Big(i|0\rangle_1|1\rangle_{2,3,\ldots,N} + \sum_{j=2}^{N}|1\rangle_{1,2,\ldots,j-1}|0\rangle_j|1\rangle_{j+1,j+2,\ldots,N}\Big), \quad (18)$$

which shows that a universal N-SQUID-qubit W type of entangled state is realized.

Some experimental matters may need to be discussed here. Without loss of generality, let us consider preparing an N-qubit GHZ entangled state with N SQUIDs. For the working method, the total operation time $\tau_{\rm GHz} = (N-2)(3\tau_2+2\tau_1+2\tau_a)+3\tau_2+\tau_1+2\tau_a$ (for simplicity, we choose $\Omega_{20} \sim \Omega_{21} \sim \Omega$) should be much shorter than the energy relaxation time γ_2^{-1} of the level $|2\rangle$, and the lifetime of the cavity mode $\kappa^{-1} = Q/\omega_c$, where Q is the quality factor of the cavity. Here τ_a is the typical time required for adjusting the level spacing of single SQUIDs. In addition, direct coupling between SQUIDs needs to be negligible.

In principle, these requirements can be satisfied due to the following reasons: (1) The SQUIDs can be designed so that the level $|2\rangle$ has a sufficiently long energy relaxation time γ_2^{-1} and thus the spontaneous decay of the SQUIDs is negligible during the operation. (2) The lifetime κ^{-1} of the cavity mode can be increased by using a high-Q cavity. (3) The typical time τ_a required for adjusting the level spacings of the SQUIDs can be shortened by rapid adjustment of the level spacings of the SQUIDs.

For the sake of definitiveness, let us consider the experimental feasibility of realizing a 10-qubit GHZ entangled state using SQUIDs with $C = 36\,{\rm fF}$, $L = 36\,{\rm pH}$, $\Phi_e = 0.4993\Phi_0$, $\beta_L = 1.10$, and $R \sim 10^8\Omega$, which are demonstrated in a recent experiment.[17] Here R is the junction's damping resistance and β_L is the SQUID potential shape parameter. Note that SQUIDs with these parameters are readily available at the present time.[7–9] For the parameter presented here, the energy relaxation time γ_2^{-1} is $\sim 1.6\times10^{-5}\,{\rm s}$, the coupling matrix element between $|0\rangle$ and $|2\rangle$ is $\sim 2.9\times10^{-2}$, and the transition frequency is about 49.2 GHz. Thus we can choose $\omega_c/2\pi = 49\,{\rm GHz}$ as the cavity frequency. For a superconducting standing-wave cavity with a volume $45\times1\times1\,{\rm mm}^3$ and an SQUID with a $45\times45\,\mu{\rm m}^2$ loop located at one of the antinodes of the cavity-mode magnetic field. The coupling constant can be calculated using Eq. (6), which gives $g \sim 1.2\times10^9\,{\rm s}^{-1}$, i.e. about $0.1\Delta_c$. By choosing the frequency and amplitude of the microwave pulse appropriately such that $\Delta_{\mu w} = 10\Omega$ and $g = 1.1\Omega$ for each SQUID, we have $\Delta_c - \Delta_{\mu w} \sim 10\gamma \sim 1.16\times10^9\,{\rm s}^{-1}$, i.e. $\lambda = \gamma^2/(\Delta_c - \Delta_{\mu w}) \sim 1.16\times10^7\,{\rm s}^{-1}$. Then the pulse-SQUID interaction time τ_2 is $\sim 1.43\times10^{-9}\,{\rm s}$ and $\tau_1 \sim 6.77\times10^{-8}\,{\rm s}$. With the choice of $\tau_2 \sim \tau_a$, one has $\tau_{\rm GHz} \sim 1.2\times10^{-6}\,{\rm s}$, which is much shorter than $\gamma_2^{-1} \sim 1.6\times10^{-5}\,{\rm s}$ and $\kappa^{-1} \sim 2.6\times10^{-5}\,{\rm s}$ for a cavity with $Q = 8\times10^6$, which is readily available in some laboratories. For instance, a superconducting cavity with a quality factor $Q = 10^8$ has been demonstrated in Ref. [18].

Now let us turn to consider the experimental feasibility of a -SQUID-qubit W entangled state. As a rough estimate, we can also obtain the total operation time:

$$\tau_w = \sum_{j=2}^{10}\frac{1}{\lambda}\arccos\left(\frac{\sqrt{N+1-j}}{\sqrt{N+2-j}}\right) \leq \sum_{j=2}^{10}\frac{1}{\lambda}\arccos\left(\frac{1}{\sqrt{2}}\right) = 9\tau_1 = 6.1\times10^{-7}\,{\rm s}. \quad (19)$$

Obviously, the total operation time τ_w is much shorter than the lifetime of the cavity mode κ^{-1}.

In conclusion, we have demonstrates that the realization of 10-SQUID-qubit GHZ and W types of entangled states is possible by using the cavity described above.

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