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Numerical Simulation of Self-Pumped Phase Conjugate Plane–Curve Loop Mirror Based on Photorefractive Nonlinearity *

Mehran Vahdani Moghaddam[1**], Zeynab Chenari[1], Hamid Latifi[1], Vladimir Vladimirovich Shuvalov[2], Konestantin Valentinovich Rudenko[2]

[1]*Laser and Plasma research institute, Shahid Beheshti University, Evin, Tehran 19835-63113, Iran*
[2]*M.V. Lomonosov Moscow State University, Vorobévy gory, 119992, Moscow, Russia*

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We deal with computer simulation of a transient process in a self-pumped phase conjugate plane–curve loop mirror based on $BaTiO_3$. In optimal circumstances the nonlinear reflectivity and fidelity of such a mirror respectively achieve 0.80–0.90 and 0.95–0.98. The generation of conjugate wave-front occurs due to scattering from the dynamic hologram which is produced in the region of self-intersection of forward and backward beams. In such a model the scenario of passing to unstable generation regimes is similar to the self-pumped phase conjugate plane–plane loop mirror and substantially differs from a single-crystal double phase conjugate mirror.

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Phase distortions caused by optical inhomogeneities can be compensated for by using the so-called nonlinear phase-conjugate-mirror (PCM) based on photorefractive-crystals (PRCs).[1] Such mirrors can operate at cw radiation intensities up to a few mW/cm^2.[2] Moreover, some of them do not require any pump waves (self-pumped double and loop PCM, PCMs with a linear resonator, etc.),[3] being in fact threshold-free (with respect to the radiation intensity but not to the nonlinear interaction constants) and sometimes mirror-free parametric oscillators. For example, in single- and two-crystal double PCMs,[4] two non-interfering light waves (incoherent or having different frequency) simultaneously are conjugated. In this case the development of generation is a classical example of self-organization which proceeds in nonlinear medium-light systems. By varying either the parameters or geometry of the mirror, the response time might be sufficiently changed,[5] or quite complicated self-oscillation regimes can be obtained with characteristic times up to a few hours and days.[2]

It was shown in Refs. [6,7] that the dynamic of formation nonlinear response in double PCM is substantially more complicated than that of a self-pumped phase conjugate (SPPC) plane–plane loop mirror. In single-crystal double PCM[1] based on a PRC, apart from a dynamic hologram formed in the region of self-intersection of input beams, additional refractive-index grating also appear which is typically for 'two-crystal' PCM.[3] The competition between two phase conjugated channels leads to complex spatiotemporal dynamics of generated nonlinear waves. Generation of two conjugate waves with the efficiency up to 70–80% and the appearance of a resemblance of dynamic chaos in a system of thin soliton-like filaments become possible. However, in the SPPC plane–plane loop mirrors based on the same PRC, only a dynamic hologram, which is formed in the region of self-intersection of forward and backward beams, operates. Therefore, the nonlinear reflectivity R and fidelity H of such mirror can be achieved respectively to be $R_{\max} = 0.80$–0.90 and $H_{\max} = 0.90$–0.95. In this case the transition to unstable regimes occurs only due to self-action processes.

The fidelity, as a further key point in phase conjugate, has been addressed theoretically and experimentally.[8,9] Indeed, in the SPPC plane–plane loop mirrors, the conjugate fidelity is poor.[9] The insertion of optical elements in the loop (SPPC loop mirrors with a three-prism system in the loop) improves the conjugate fidelity. However, because of the size problems, the optical length of the loop can not be made to be smaller than a specific value. This prevents us from correcting highly aberrated beams. Indeed after one lap in the loop, the propagation of the diverging aberrated beam affects the overlap of the beams inside the crystal.[9]

Great effort has been taken to compacting the loop PCMs. Earlier instead of SPPC plane–plane loop mirrors is purposed SPPC plane–curve loop geometry.[9] In the following, by using the numerical simulation we show that in the SPPC plane–curve loop mirror as well as SPPC plane–plane loop mirror, there is no competition between several phase conjugate channels and the scenario of passing to unstable generation regimes substantially differs from that described in Ref. [6].

The geometry of the problem is shown in Fig. 1. We assume that the forward light wave with the amplitude A_f and the wave vector $k = k_x, k_y$ propagates from the input face of a PRC ($z = 0$ plane) at a small angle $\alpha/2$ in the positive direction of the z axis. It is assumed that an optical system consisting of spherical and plane mirrors, respectively M1 and M2, which forms the backward wave (propagating in the nega-

*Supported by the Laser and Plasma Research Institute (LAPRI), Shahid Beheshti University.
**Email: M_vahdani@sbu.ac.ir
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tive direction of the z axis) with the amplitude A_b, is mounted directly behind the output face of the PRC ($z = L$ plane). The mirrors change the direction of propagation of incident wave and then focus it near the crystal after one lap in the loop. In the paraxial ray approximation the backward beam is supposed to read

$$A_b(x, z = L, t) = A_f(-x, z = L, t) \exp[-ikx \sin\alpha] \cdot \exp[-ik(x - x_c)^2/R], \quad (1)$$

where x_c is the central position of incident beam along the x direction and R is the curvature of spherical mirror.

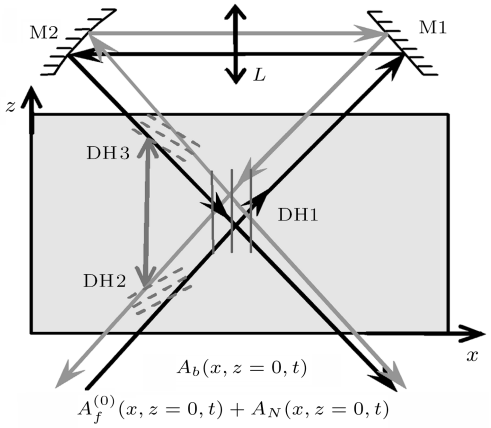

Fig. 1. Interaction geometry of the waves $A_{f,b}(x, z, t)$ in a SPPC plane–curve loop mirror PCM: DH1, DH2, and DH3: dynamic holograms for the first (DH1, the self-intersection region) and second (DH2 and DH3, see Ref. [6]) possible generation channels.

In Ref. [10], the nonlinear response of the PRC was calculated by using the system of microscopic equations obtained for the two-dimensional case (the so-called slit beams[11]) taking into account only transmission dynamic holograms (the grating vector is directed along the x axis) by neglecting the photovoltaic effect. The problem to be solved was transferred to the class of self-consistent problems taking into account the relation of the intensity distribution $I_{f,b}(x, z, t) = |A_{f,b}(x, z, t)|^2$ with the crystal field $E_{sc}(x, z, t)$, which is specified by standard reduced wave equations for the amplitude $A_{f,b}(x, z, t)$ of the interacting waves. The reduced equations were written in the paraxial approximation by neglecting the spatially homogeneous addition to the refractive index η of the PRC caused by the external E_0. As in Ref. [7], we consider the situation in which the total intensity is determined by the sum of intensities $I(x, z, t) = I_f(x, z, t) + I_b(x, z, t)$, i.e., the case of incoherent or orthogonally polarized counter propagating waves.

We solve the self-consistent problem by calculating numerically the evolution of distribution $A_{f,b}(x, z, t)$ and $\delta\eta(x, z, t)$ in time. All the variables are described discretely (the number of the grid nodes of the PRC aperture $H = 4\,\text{mm}$ is 8192 and on its length $L = 4\,\text{mm}$ is 512). The problem was solved within the framework of the adiabatic approximation by using the method of separation over physical factors and the fast Fourier transform.[13,14] As in Ref. [7], the step in time ($\Delta \cong 0.2\,\text{s}$) was much smaller than the evolution time of the PRC state, and most of the parameters were not varied and their values were determined by the PRC type ($BaTiO_3$, see Table 1 in Ref. [7]).

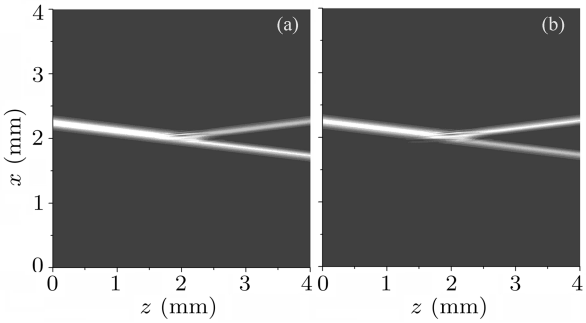

Fig. 2. Maps of the distribution $I_f(x, z)$ (a) and $I_b(x, z)$ (b) at the instant $t = 800\,\text{s}$ upon stationary phase conjugate of a Gaussian beam of the input intensity $I_f(x, z = 0)$; $\alpha = 14°$, $I_{\max} = 35\,\text{mW/cm}^2$, $\langle I_N\rangle/I_{\max} = 10^{-4}$, $E = 1\,\text{Vcm}^{-1}$, $R = 16\,\text{mm}$.

The intensity distributions $|A_f^{(0)}(x, z = 0, t)|^2$ of the signal radiation at a wavelength of $0.514\,\mu\text{m}$ on the input face of the PRC is assumed to be Gaussian (the beam width is $2\rho_0 = 200\,\mu\text{m}$ for $I_{\max} = 35$–$90\,\text{mW/cm}^2$. It is assumed that the curvature of the spherical mirror is chosen $R = 16\,\text{mm}$ and the external electrostatic field E_0 in the PRC was directed along the x axis and is comparatively weak ($E_0 = 1\,\text{Vcm}^{-1}$). The period of dynamic holograms written in the PRC is specified by the convergence angle $\alpha = 14°$ of the beams, which is also fixed. In the SPPC plane–plane loop mirror, the generation of the field $A_b(x, z, t)$ with the spatial structure $A_{f,b}(x, z = 0, t)$ close to that of the conjugated input signal $A_f(x, z)$ was easily achieved. Figure 2 shows the 'maps' of intensity distributions $I_f(x, z)$ (Fig. 2(a)) and $I_b(x, z)$ (Fig. 2(b)) in the PRC after the transition process ($t = 800\,\text{s}$), where the darker regions correspond to lower intensities. A stationary refractive-index grating DH1 was formed in the self-intersection region of forward (A_f) and backward (A_b) beams in the (Fig. 1). In Fig. 3, the spatial distributions of dynamic holograms in the plane $z = L/2$ are presented for both SPPC plane–plane (a) and plane–curve loop mirror (b). Under optimal condition the fidelity $H(t)$ and nonlinear reflectivity $R(t)$ [7] of conjugated wave on the input face of the PRC achieve the maximum values 0.95–0.98 and 0.80–0.90, respectively (curve 1 in Fig. 6). Note that $H(t)$ and $R(t)$ are calculated by transmitting preliminary the output wave $A_b(x, z = 0, t)$ through a spatial filter in the $z = 0$ plane. According to the results of our numerical simulation, the values $H(t)$ and $R(t)$

in the SPPC plane–curve loop mirror shows fairly enhancement with respect to the SPPC plane–plane loop mirror, and this is why the loop mirror system consists in two plane mirrors has poor conjugate fidelity (Fig. 4).

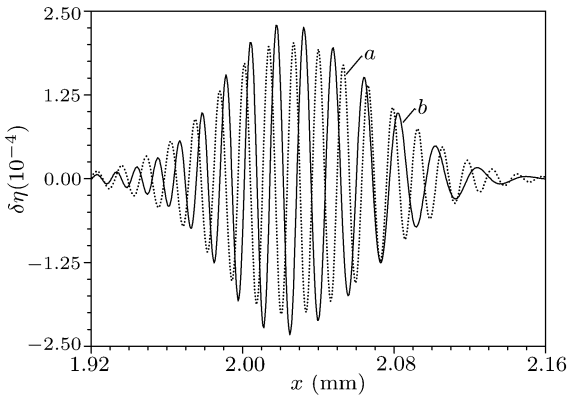

Fig. 3. Spatial distribution of dynamic holograms upon stationary conjugation of a Gaussian beam in the SPPC plane–plane (a) and SPPC plane–curve loop mirror (b) in the section $z = L/2$; $2\rho_0 = 200\,\mu$m, $\alpha = 14°$, $I_{\max} = 35\,\mathrm{mW/cm^2}$, $\langle I_N\rangle/I_{\max} = 10^{-4}$, $E = 1\,\mathrm{Vcm^{-1}}$, $R = 16\,$mm.

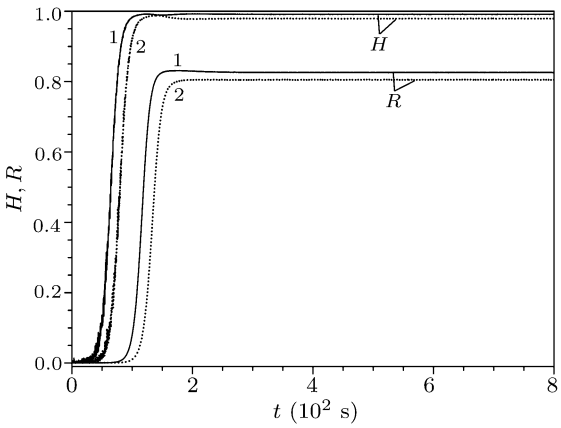

Fig. 4. Transient processes for H and R upon stationary conjugation of a Gaussian beam in the loop (curve-1) and SPPC plane–curve loop mirror (curve-2); $\alpha = 14°$, $I_{\max} = 35\,\mathrm{mW/cm^2}$, $\langle I_N\rangle/I_{\max} = 10^{-4}$, $E = 1\,\mathrm{Vcm^{-1}}$, $R = 16\,$mm.

The time during which the phase transition occurs in the PCM depends on the relative noise level $\langle I_N\rangle/I_{\max}$ of the input radiation A_f (Fig. 5). As $\langle I_N\rangle/I_{\max}$ increases in the range under study, the conjugated component forms faster. The changes of growth kinetics of $R(t)$ and $H(t)$ in the time upon variations of $\langle I_N\rangle/I_{\max}$ for SPPC plane–plane and SPPC plane–curve loop mirror are considerably different. As was shown in Ref. [7] in the SPPC plane–plane loop mirror, increment of $\langle I_N\rangle/I_{\max}$ almost does not affect the dependence $H(t)$, while $R(t)$ rapidly reaches the maximum value $R_{\max}$ only due to the increase of amplitude of the initial noise seed. However, in the SPPC plane–curve loop geometry, $H(t)$ and $R(t)$ both are affected by the change of $\langle I_N\rangle/I_{\max}$ (Fig. 5(a)).

As the input-beam intensity $I_{\max}$ increases, the efficiency, which is more important, and stability of conjugate drastically decrease (Fig. 6). In this case in the forward and backward beams, a complicated irregular system of thin soliton-like filaments begin to appear gradually due to self-action (Fig. 7), in which then a resemblance of dynamic chaos appears. Many additional dynamic holograms are produced in the PRC due to numerous self- and mutual-intersections of the filaments, and phase conjugation is no longer efficient.

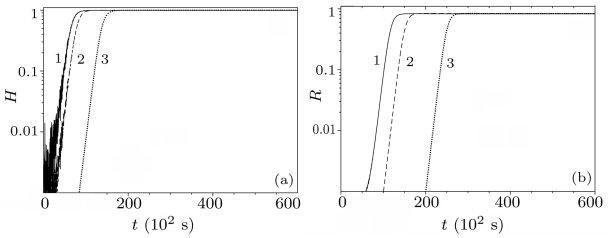

Fig. 5. Delay of the SPPC plane–curve loop mirror start relative to the noise level $\langle I_N\rangle/I_{\max}$. Dependences $H(t)$ (a) and $R(t)$ (b) for $\alpha = 14°$, (1) $\langle I_N\rangle/I_{\max} = 10^{-1}$, (2) $\langle I_N\rangle/I_{\max} = 10^{-4}$, (3) $\langle I_N\rangle/I_{\max} = 10^{-8}$.

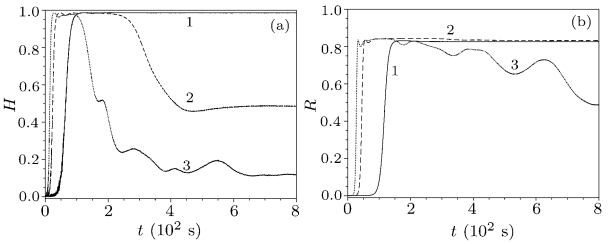

Fig. 6. Transformation of H and R dependences upon variations of the input-beam intensity $I_{\max}$ for $E = 1\,\mathrm{Vcm^{-1}}$, (1) $I_{\max} = 35\,\mathrm{mW/cm^2}$, (2) $I_{\max} = 60\,\mathrm{mW/cm^2}$, and (3) $I_{\max} = 90\,\mathrm{mW/cm^2}$, $\alpha = 14°$, $\langle I_N\rangle/I_{\max} = 10^{-4}$.

We present the results of numerical simulation on kinetics of transient processes in the SPPC plane–curve loop mirror based on a $BaTiO_3$. It is shown that the reflectivity and fidelity of such a mirror respectively can achieve $R_{\max} = 0.80$–0.90 and $H_{\max} = 0.95$–0.98, which shows fairly enhancement rather than SPPC plane–plane loop mirror. The scenario of passing to unstable generation regimes in the SPPC plane–curve loop mirror substantially differs from that described earlier[6] for single crystal double PCM but is similar to SPPC plane–plane loop mirror.[7] This is explained by the fact that there is no evident competition between two generation channels of the conjugate wave in SPPC plane–curve loop mirror. If the conjugate wave is generated in the mirror, this is certainly related to scattering from the dynamic hologram DH1 (Fig. 1) produced in the self-intersection region of the forward and backward beams. The additional gratings such as DH2 and DH3 are produced in the SPPC

plane–curve loop mirror only in the unstable generation regime, which is probably caused by the excess (compared to a single-crystal double PCMs) nonlinear phase shift in the backward wave. Our results experimentally are confirmed by the SPPC loop and plane–curve loop mirrors where it is shown that the generation and development of PC wave is only due to scattering of feed-backed signal from dynamic holograms which is formed in the self-intersection region.[15,16] However it is in contrast with the PC wave formation in double SPPC mirrors where the generation of PC waves is due to self-bending of one beam into the other, i.e. spontaneously generated loop of light formed within the crystal, resulting in phase conjugation emission from both beams.[17,18]

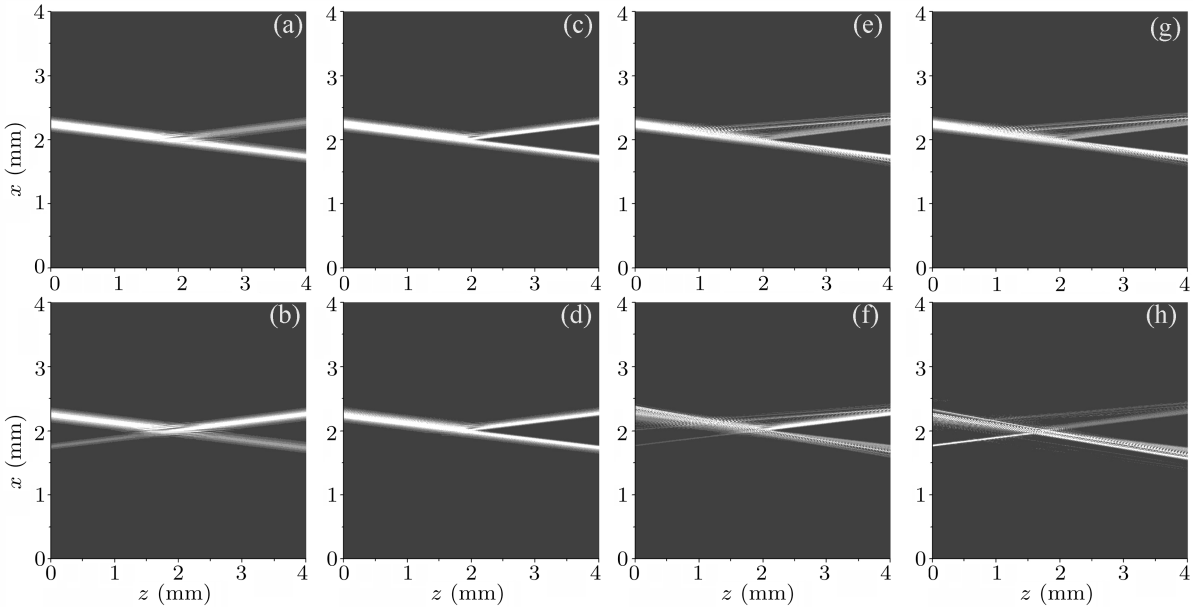

Fig. 7. Development of instabilities in the distribution $I_f(x,z)$ (a,c,e,g) and $I_b(x,z)$ (b,d,f,h) at instants $t = 30\,\mathrm{s}$ (a,b), 96 s (c,d), 169 s (e,f) and 546 s (g,h) for $\alpha = 14°$; $I_{\max} = 90\,\mathrm{mW/cm^2}$, $\langle I_N\rangle/I_{\max} = 10^{-4}$; $E_0 = 1\,\mathrm{Vcm^{-1}}$.

Our calculations confirms that phase conjugation in self-pumped PCMs is always accompanied by the drastic broadening of the spatial spectrum of the interacting light fields. Taking into account that the energy of the latter is only redistributed among different components of the spatial spectrum, the accuracy of description of high-frequency spatial harmonics proves to be very important.

In this work we do not aim to experimentally compare the order of magnitudes of resulted parameters. The reason is because all PRCs inevitably contain many defects: the characteristics of crystals, even of the same type, can sufficiently differ in magnitude. In the majority of cases all the necessary parameters, which are required for the simulation of PRC have not been measured and the result interpretation of any experiment in terms of multiparametric model seems to be a complex task.

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