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Projective Synchronization in Drive–Response Networks via Impulsive Control *

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Impulsive projective synchronization in $1+N$ coupled chaotic systems are investigated with the drive–response dynamical network (DRDN) model. Based on impulsive stability theory, some simple but less conservative criteria are achieved for projective synchronization in DRDNs. Furthermore, impulsive pinning scheme is also adopted to direct the scaling factor onto the desired value. Numerical simulations on generalized chaotic unified system are illustrated to verify the theoretical results.

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Synchronization in coupled two chaotic systems has attracted much attention for both theoretical interest and practical applications since the seminal work of Pecora and Carroll.[1] So far, different types of chaos synchronization[1−5] have been studied. Among them, projective synchronization (PS) is one of most interesting problems. On the other hand, impulsive control synchronization has been well developed.[6,7]

In this Letter, impulsive projective synchronization (IPS) in DRDNs is investigated. A class of chaotic systems are called the partially linear systems in which the state vector can be broken into two parts $(\boldsymbol{u}, z)$, the equation for z is nonlinearly related to the other variables, while the equation for the rate of change of the vector $\boldsymbol{u}$ is linearly related to $\boldsymbol{u}$ via a matrix M that depends on the variable z. PS may occur when two systems are linked through the variable z.

The drive–response dynamical network model in Ref. [4] is described by the equation

$$\begin{aligned}\dot{\boldsymbol{u}}^d &= M(z)\cdot\boldsymbol{u}^d, \quad \dot{z} = f(\boldsymbol{u}^d, z),\\ \dot{\boldsymbol{u}}_i^r &= M(z)\cdot\boldsymbol{u}_i^r + c\sum_{j=1}^{N} a_{ij}\boldsymbol{u}_j^r,\ i=1,2,\ldots,N,\end{aligned} \tag{1}$$

where the superscripts d and r represent the drive and response systems, respectively, $\boldsymbol{u}_i^r (i=1,2,\ldots,N)$ denote the multi-dimensional state variables of the response networks systems. N is the number of coupled nodes and the constant $c>0$ is the coupling strength. $A=(a_{ij})_{N\times N}$ is real matrix reflecting the topology of network. We give the impulses at instant time t_k in response networks:

$$\begin{aligned}\dot{\boldsymbol{u}}^d &= M(z)\cdot\boldsymbol{u}^d, \quad \dot{z} = f(\boldsymbol{u}^d, z),\\ \dot{\boldsymbol{u}}_i^r &= M(z)\cdot\boldsymbol{u}_i^r, \quad i=1,2,\ldots,N, t\neq t_k,\\ \delta\boldsymbol{u}_i &= c\sum_{j=1}^{N} a_{ij}\boldsymbol{u}_j^r, \quad t=t_k, k=1,2,3,\ldots.\end{aligned} \tag{2}$$

where the matrix A is irreducible satisfying $a_{ij}\geq 0, i\neq j, a_{ii}=-\sum_{j=1,j\neq i}^{N} a_{ij}, i=1,2,\ldots,N$. It is obvious that matrix A is semi-negative definitive and eigenvalues of A are all negative except an eigenvalue 0 with multiplicity 1. The system (2) is said to be realized impulsive projective synchronization (IPS), if there exists a common scaling factor constant $\alpha(\neq 0)$ such that $\lim_{t\to\infty}\|u_i^r-\alpha u^d\|=0$ for any $i(1\leq i\leq N)$.

For sake of simplicity and clarity, we only consider a three-dimensional chaotic system in the DRDNs, which is described by

$$\begin{aligned}\dot{x} &= m_{11}x + m_{12}y, \quad \dot{y} = m_{21}x + m_{22}y,\\ \dot{z} &= f(x,y,z), \quad \dot{x}_i = m_{11}x_i + m_{12}y_i\\ \dot{y}_i &= m_{21}x_i + m_{22}y_i, \quad i=1,2,\ldots,N, t\neq t_k,\\ \delta x_i &= c\sum_{j=1}^{N} a_{ij}x_j, \quad \delta y_i = c\sum_{j=1}^{N} a_{ij}y_j,\\ t &= t_k,\ k=1,2,3,\ldots,\end{aligned} \tag{3}$$

where x, y, z, x_i and y_i are the state variables; m_{11}, m_{12}, m_{21} and m_{22} are the functions of z. Notation c, N and $A=(a_{ij})_{N\times N}$ are the same as the above-mentioned ones.

Let $e_i = xy_i - x_i y, i=1,2,\ldots,N$, then $\delta e_i = x\delta y_i - \delta x_i y = c\sum_{j=1}^{N} a_{ij}e_j, i=1,2,\ldots,N$. If e_i tends to 0 in evolution, PS between every system in response networks and the drive system would be realized. Further, if the whole response networks realize complete synchronization, we can obtain the results that the drive system and the response networks system achieve PS with a common scaling factor.

Denote $\boldsymbol{e} = (e_1, e_2, \ldots, e_N)^T$, then $\delta\boldsymbol{e} = (\delta e_1, \delta e_2, \ldots, \delta e_N)^T = cA\boldsymbol{e}$, the system (3) can be rewritten in vector form:

$$\begin{aligned}\dot{\boldsymbol{e}} &= (m_{11}+m_{22})\boldsymbol{e}, \quad t\neq t_k, \quad \dot{z} = f(x,y,z),\\ \delta\boldsymbol{e} &= cA\boldsymbol{e} = I_k(\boldsymbol{e}), \quad t=t_k,\ k=1,2,3,\ldots.\end{aligned} \tag{4}$$

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We have translated the problem of realizing PS of (3) into the stability problem of origin of the error system (4).

Theorem 1. Let the impulses be equidistant and separated by interval Δ, then $Q \in R^{n\times n}$ is a symmetric and positive definite matrix. Here $\varepsilon_0 > 0$ is an upper bound of $m_{11}(z) + m_{22}(z)$ for any t. Suppose that there exist constant scalars $d \geq 0$, $\gamma > 1$ such that (i) $\Omega_1 = (I + cA)^T Q(I + cA) - dQ \leq 0$, (ii) $2\varepsilon_0\Delta + ln(\gamma d^2) \leq 0$, where I denotes the identity matrix. Then the origin of the error system (4) is asymptotically stable, i.e., system (3) realizes IPS.

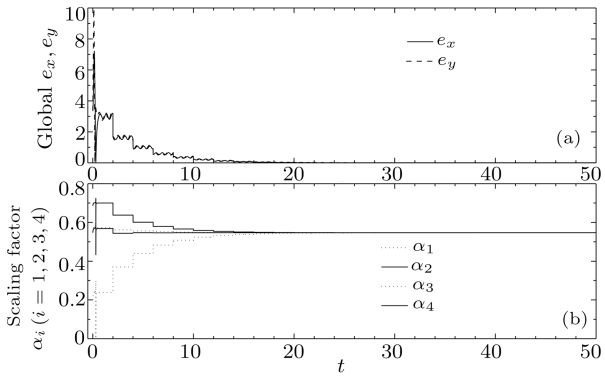


Fig. 1. Simulation results of IPS with impulsive interval $\Delta = 2$ and feedback gain $c = 0.4$: (a) temporal evolution of the synchronization error $e_x = \sqrt{\sum_{i=1}^{3}(x_{i+1} - x_i)^2}$, $e_y = \sqrt{\sum_{i=1}^{3}(y_{i+1} - y_i)^2}$, (b) the evolution of the scaling factor α_i.

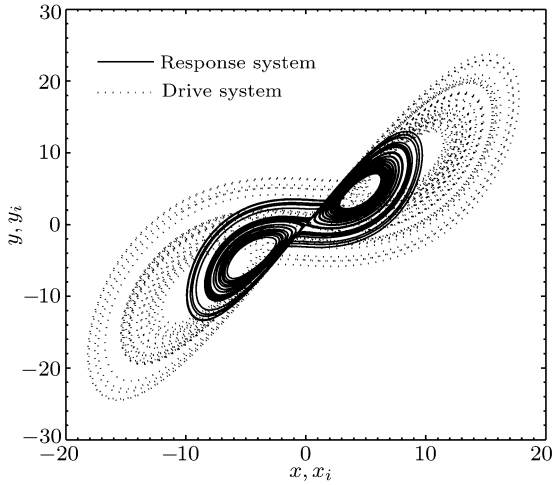


Fig. 2. Trajectories of IPS in the $x - y$ plane.

Proof. Construct a Lyapunov function $V(t, \boldsymbol{e}) = \boldsymbol{e}^T Q\boldsymbol{e}$, where Q is a symmetric and positive definite matrix; $\lambda_{\min}(Q)\|\boldsymbol{e}\|^2$ and $\lambda_{\max}(Q)\|\boldsymbol{e}\|^2$ are the functions, $\lambda_{\min}$ and $\lambda_{\max}$ denote the smallest and largest eigenvalues of a square matrix, respectively. $V(t, \boldsymbol{e})$ implies that $\lambda_{\min}(Q)\|\boldsymbol{e}\|^2 \leq V(t, \boldsymbol{e}) \leq \lambda_{\max}(Q)\|\boldsymbol{e}\|^2$. For $t \neq t_k$, the derivative of $V(t, e)$ along the solution of (4) is $D^+V(t, \boldsymbol{e}) = 2(m_{11} + m_{22})\boldsymbol{e}^T Q\boldsymbol{e} \leq \varepsilon_0 V(t, \boldsymbol{e})$. For $t = t_k$, we have $D^+V(t_k, \boldsymbol{e} + \delta\boldsymbol{e}) = \boldsymbol{e}^T\Omega_1\boldsymbol{e} + dV(t_k, \boldsymbol{e}) \leq dV(t_k, \boldsymbol{e})$.

Let $\dot{\lambda}(t) = \varepsilon_0$, $d_k = d$,$k = 1, 2, \ldots$, it follows the conditions of impulsive stability theorems,[6,7] then the origin of the error system (4) is asymptotically stable. This completes the proof.

Many chaotic partially linear systems such as Lorenz system and Chen system satisfy that $m_{11} + m_{22} < 0$, then $\varepsilon_0 > 0$ can be any small, let $Q = I$ in theorem 1, then the following corollary holds.

Corollary 1. Suppose that $m_{11} + m_{22} < 0$ and there exist constants $d(0 \leq d < 1)$ such that $\Omega_1 = (I + cA)^T(I + cA) - dI \leq 0$, then for any interval $\Delta \geq 0$, the origin of the error system (4) is asymptotically stable.

It has been known that the scaling factor of PS in two coupled partially linear systems is unpredictable[2]. The case is the same here. From an application point of view, we need to control the ultimate state of synchronized dynamics of model (3). The goal is to realize $(x_i, y_i)^T \to \alpha(x, y)^T$ as $t \to \infty$, $i = 1, 2, \ldots, N$, where α is a desired scaling factor. We apply the pinning control[8] on just any one of the nodes in response networks of system (3). Without loss of generality, node 1 is selected to be under pinning control. The impulses in Eq. (3) are replaced by $\delta x_1 = c\sum_{j=1}^{N} a_{1j}x_j - ck(x_1 - \alpha x), \delta y_1 = c\sum_{j=1}^{N} a_{1j}y_j - ck(y_1 - \alpha y), \delta x_i = c\sum_{j=1}^{N} a_{ij}x_j$, $\delta y_i = c\sum_{j=1}^{N} a_{ij}y_j$, $i = 2, \ldots, N$, $t = t_k$, $k = 1, 2, 3, \ldots$. Here α can be any desired value, $k > 0$ is a constant. The corresponding pinning matrix can be written as

$$A = \begin{pmatrix} a_{11} - k & a_{12} & \cdots & a_{1N}, \\ a_{21} & a_{22} & \cdots & a_{2N}, \\ \vdots & \vdots & \ddots & \vdots, \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix}.$$

Similarly, we obtain the following theorem and corollary.

Theorem 2. Let the impulses be equidistant and separated by interval Δ, then $Q \in R^{n\times n}$ is a symmetric and positive definite matrix. Here $\varepsilon_0 > 0$ is an upper bound of $m_{11}(z) + m_{22}(z)$ for any t. Suppose that there exist constant scalars $d \geq 0$,$\gamma > 1$ such that (i) $\Omega_2 = (I + c\tilde{A})^T Q(I + c\tilde{A}) - dQ \leq 0$, (ii) $2\varepsilon_0\Delta + ln(\gamma d^2) \leq 0$. Then IPS with desired scaling factor α can be achieved in pinning drive–response network.

Corollary 2. Suppose that $m_{11} + m_{22} < 0$ and there exist constants $d(1 > d \geq 0)$ such that $\Omega_2 = (I+c\tilde{A})^T(I+c\tilde{A}) - dI \leq 0$, then for any interval $\Delta \geq 0$, IPS with desired scaling factor α can be achieved in the pinning drive–response model.

Remark 1. It is worth noting that any width of impulsive interval $\Delta \geq 0$ can be arbitrarily selected to implement PS with desired scaling factor under certain conditions in corollary 2. This has theoretically solved the open problem remained in Ref. [5] when $N = 1$ in pinning response networks.

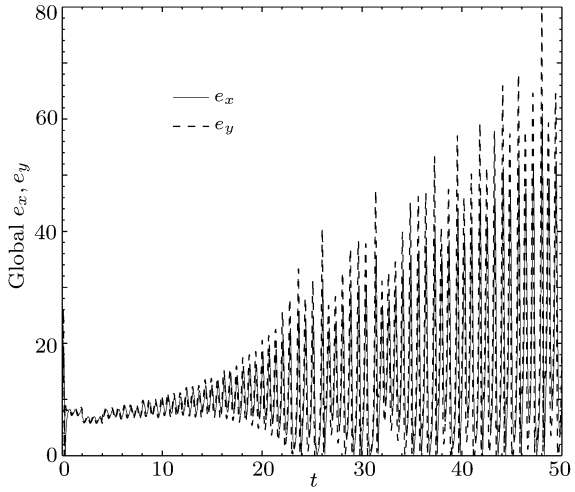


Fig. 3. Temporal evolution of error e_x, e_y with gain $c = 1.35$.

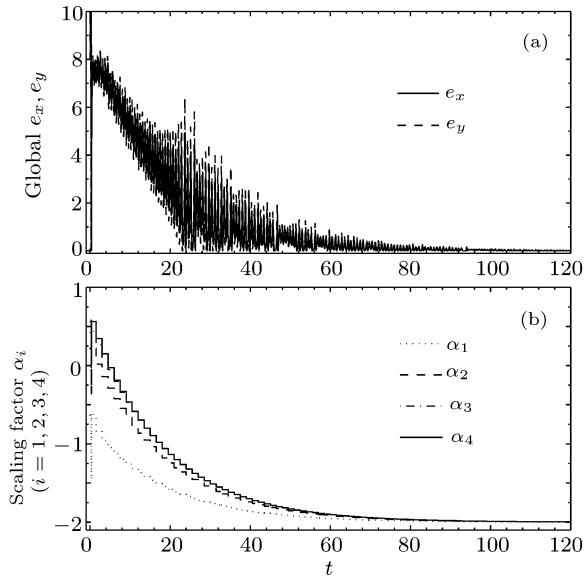


Fig. 4. Simulation results of IPS with designed scaling factor $\alpha = -2$, $\Delta = 2$, $c = 0.4$: (a) temporal evolution of the synchronization error e_x,e_y, (b) the evolution of the scaling factor α_i.

We place the general unified chaotic system[9] on nodes of networks

$$\begin{aligned} \dot{x} &= (25\theta + 10)(y - x), \\ \dot{y} &= (28 - 35\theta)x - xz + (29\theta - 1)y, \\ \dot{z} &= xy - \frac{1}{3}(\theta + 8)z, \end{aligned} \tag{5}$$

where $\theta \in [-0.016, 1.15]$. Set $\theta = 0$ here and $m_{11} + m_{22} = -11 < 0$, the impulsive coupling configuration matrix

$$A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -3 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}.$$

In this numerical simulation, we can take any interval $\Delta > 0$. Obviously, if $0 < c < 0.5$, there exists the constant $d(0 \leq d < 1)$ to ensure that the condition of corollary 1 is satisfied. Here, we let the impulsive coupling $c = 0.4$, $\Delta = 2$ and the initial values are $(1, 2, 3, 4, 5, 6, 7, 8, 1, 9, 10)$. The numerical results are depicted in Figs. 1 and 2. Figure 1(a) shows temporal evolution of the response networks synchronization errors $e_x = \sqrt{\sum_{i=1}^{3}(x_{i+1} - x_i)^2} \to 0$ and $e_y = \sqrt{\sum_{i=1}^{3}(y_{i+1} - y_i)^2} \to 0$. Figure 1(b) depicts that all the scaling factors $\alpha_i = \max\{x_i/x, y_i/y\}$ tend to the unpredictable $\alpha = 0.5465$. Figure 2 shows the trajectories of IPS in the $x - y$ plane. However, if we set the gain $c = 1.35$, which is obviously not satisfied the condition of corollary 1, the synchronization will not be achieved, as shown in Fig. 3.

The example of pining control is illustrated with the desired scaling factor $\alpha = -2$. Let $\Delta = 1, 5$, $k = 1$; θ,c,A and the initial values are the same as mentioned above. Figure 4(a) shows the final synchronization of the response networks. Figure 4(b) illustrates that the pinning model with unified system has achieved IPS with the desired scaling factor $\alpha = -2$. Figure 5 indicates the trajectories of IPS with $\alpha = -2$ in the $x - y$ plane.

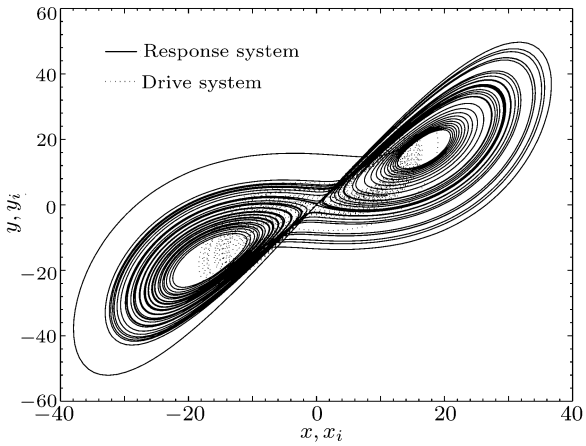


Fig. 5. Trajectories of IPS in the $x-y$ plane with designed scaling factor $\alpha = -2$.

Remark 2. Numerical results are rather general here, for any interval $\Delta > 0$, $\theta \in [-0.016, 1.15]$ and the scaling factor $\alpha \neq 0$, IPS with desired scaling factor always occurs as is expected and observed in the stable region of $0 < c < 0.5$.

In summary, the scheme of impulsive projective synchronization (IPS) has been investigated with a drive–response network (DRDN) model. Furthermore, impulsive pinning control method is adopted to control the desired scaling factor. Some simple but effective and less conservative sufficient conditions are theoretically derived. Numerical simulations on unified chaotic system support the theory results. The main advantage is any width of impulsive interval Δ, which will be easily applied to real situations.

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