Elastic Wave Propagation in Two-Dimensional Ordered and Weakly Disordered Phononic Crystals *

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Elastic wave propagation in two-dimensional solid-solid ordered and weakly disordered phononic crystals is studied by using finite-difference time-domain method. Theoretical results show that obvious band gaps in the ordered crystal could be found, while in the weakly disordered ones the band gaps could partially vanish. Furthermore, with increase of disorder, band gaps are destructed badly and prominently in the high frequency regime while slightly in the low regime. Comparing the energy transmission dependent on time, we find that the coda wave phenomenon is prominent in the ordered crystal while weakened in the weakly disordered ones, and the physical properties are discussed.

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In the past several decades, elastic wave propagation in ordered periodic media and in disordered media has attracted great attention.[1−3] Band gaps in the frequency regime are discovered in ordered periodic media[4,5] and this may bring much wider applications, such as the creation of a vibrationless environment, the improvement in transducer and acoustic frequency filter design. Nowadays, disorder has become one of the most attractive focuses in phononic crystals research[6,7] for its rich physical properties, such as localization, multiple scattering,[6] and coda wave phenomenon,[5] etc.

In this Letter, both the ordered and the weakly disordered crystals are studied and compared with each other. The approach is based on the finite-difference time-domain (FDTD) method,[8,9] which has been applied widely in phononic crystal research and has proven to be effective. Comparing the transmission spectra of the ordered and weakly disordered crystals, we find that certain frequency band gaps in the ordered crystal will partially vanish in the weakly disordered ones. In addition, band gaps in the high frequency regime are more sensitive to perturbation than in the low frequency regime. Moreover, with increasing degree of disorder, band gaps are badly affected. The time-dependent energy transmission also shows that the coda wave phenomenon, an obvious evidence of multiple scattering,[5] is prominent in ordered crystals but faint in weakly disordered ones.

The physical model studied at present is shown in Fig. 1, in which several arrays of steel cylinders are periodically stuffed in the central rectangular region of a slab of epoxy resin matrix, and a Gaussian pulse launched by a transmitter on the left side of the plate is received by a transducer on the right side of the slab. The crystal is thus regarded to be the two-component composite with the lattice constant $a = 6\,\mathrm{mm}$, and the cylinder radius $R = 2\,\mathrm{mm}$.

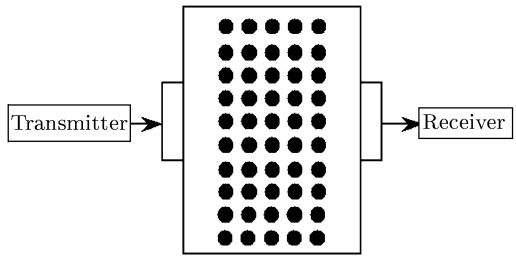


Fig. 1. The sandwiched structure. The black parts are cylinders and the blank is epoxy.

To produce the "weak disorder", named by Sanchis,[10] several different degrees of perturbation to cylinder locations are introduced to the ordered crystal. In such a weakly disordered model, the periodicity of the square lattice is maintained while the cylinder locations are chosen at random in a certain cell, plotted in dotted square in Fig. 2(a), which defines the degree of disorder. The cylinder coordinates in a square lattice are

$$x = x_0 + \varepsilon(a/2 - R), \quad y = y_0 + \varepsilon'(a/2 - R),$$

where x_0 and y_0 are the cylinder coordinates positioned in the ordered crystal, $a/2 - R$ is the half distance between two adjacent cylinders in ordered crystals, i.e. the largest range for cylinders to move in; ε and ε' are the random numbers within $[-\phi, \phi]$ with ϕ being the perturbation coefficient in $[0, 1]$ and defining the degree of the weak disorder. When $\phi = 0$, no perturbation is introduced, corresponding to the ordered

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media. When $\phi = 1.0$, In contrast, the largest perturbation is introduced into crystals and the highest degree of the weak disorder occurs.

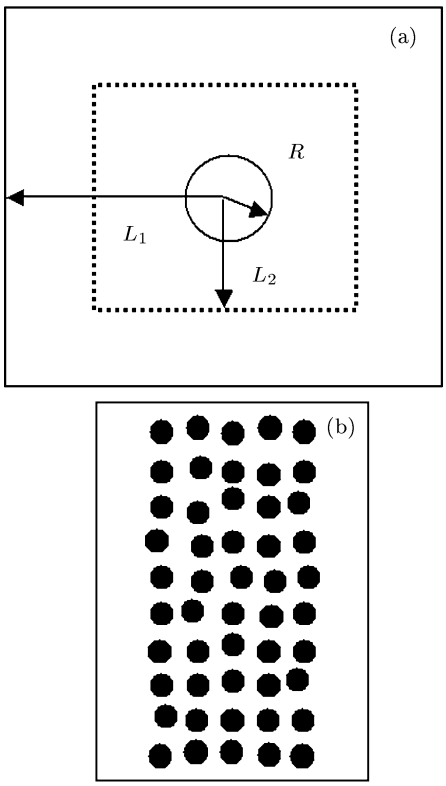


Fig. 2. Weakly disordered crystal. (a) An example lattice where $L_1 = a/2$ and $L_2 = \phi(a/2 - R)$. The solid-line square represents the crystal lattice and the circle part is a cylinder. The dashed-line square describes the range in which the axle of the cylinder could be perturbed. (b) Weakly disordered crystal sample ($\phi = 1.0$).

The computational results of the spectra in the crystals are compared in Fig. 3. In the ordered crystal ($\phi = 0$), the frequency transmission shows prominent band gaps (from 120 to 240 kHz, and from 400 to 500 kHz), which are proven in our published experiment.[11] In weakly disordered crystals, the band gap in the high frequency regime (from 400 to 500 kHz) disappears while the band gap in the low frequency regime (from 120 to 240 kHz) is maintained and affected slightly. With increasing degree of disorder, band gaps are badly affected. When $\phi = 0.5$, there is still a concave part between 400 kHz and 500 kHz, which almost cannot be treated as a band gap. When $\phi = 1.0$, the results show no obvious difference from that in the high frequency regime.

It is well known that Bragg multiple scattering plays a determinative role for elastic wave in phononic crystals to produce band gaps. When periodicity of crystals is destroyed, the conditions for the wave interaction to Bragg multiple scatters are weakened, and the band gaps are destroyed. For waves whose wavelength is relatively short in high frequency regime, Bragg multiple scattering is affected sensitively by weak disorder, which leads to disappearance of the band gaps in the high frequency regime. In contrast, since weakly disordered crystals still maintain the periodicity of the square lattice in the media, Bragg multiple scattering of the waves whose wavelength is relatively long in the low frequency regime are affected slightly, and the band gaps are maintained.

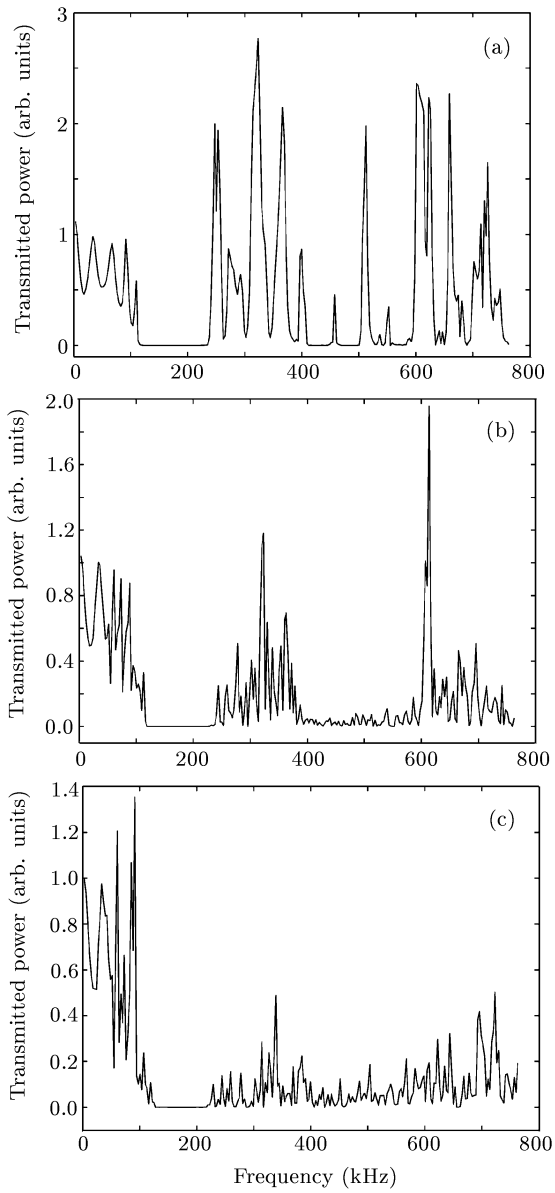


Fig. 3. Transmission power spectra: (a) ordered crystal ($\phi = 0$), (b) weakly disordered crystal ($\phi = 0.5$), and (c) weakly disordered crystal ($\phi = 1$).

To make more quantitative analysis from the numerical results, we calculate the energy ratio (energy in the band gap divided by the total energy) versus perturbation coefficient, as shown in Fig. 4.

When the perturbation coefficient $\phi = 0$, the en-

ergies in both the high band gaps and the low band gaps are almost zero, which means the energy cannot transmit through the band gaps. With the increase of the perturbation coefficient, the energy in the band gaps increases and some energy can get through the band gaps, which represents that the band gaps are destroyed. As shown in Fig. 4, the energy in the high band gaps increases more strongly than that in the low band gaps, which ensures and quantitatively represents the perturbation effect on the band gaps.

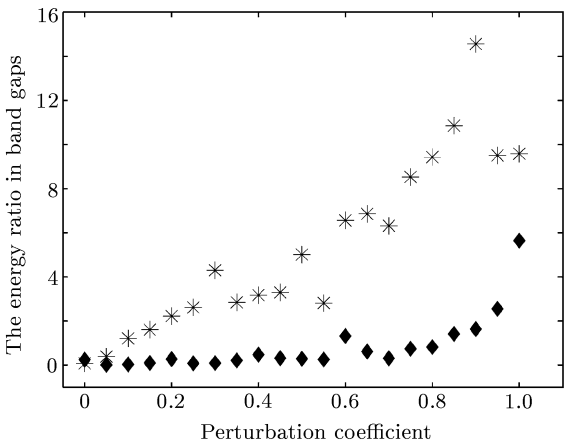


Fig. 4. Energy ratio in band gaps dependent on perturbation coefficient. The asterisks represent the energy in the high band gap and the diamonds represent the energy in the low band gap.

To understand the physical properties more comprehensively, we compare the energy transmission of the ordered and the weakly disordered media versus time in Fig. 5. The solid line presents the energy transmission in the ordered crystal, which has the highest peak and strong oscillation. After 70 μs, the plot shows strong multiple scattering in the coda wave. The dashed line is for the weakly disordered crystal ($\phi = 0.5$), which has a lower peak and weaker oscillation. The bold line is for 130 μs, the highest degree of disorder in the weak disorder which has the lowest peak and weakest oscillation. After 130 μs, the transmitted energy in the ordered crystal is lower than that in the weakly disordered crystal ($\phi = 1.0$), while oscillates stronger than that in the weakly disordered one.

In the early period ($t < 50\,\mu$s), the waves are often scattered. In the period when coda wave phenomenon appears, the waves bounce more frequently between cylinders, and strong Bragg multiple scattering occurs. With increasing degree of disorder, the conditions for wave to generate Bragg scattering are weakened, which leads to non-obvious coda wave phenomenon. Even for the filling fraction, a decisive parameter for crystals to produce band gaps[12] is not changed in the weakly disordered crystals, Bragg multiple scattering is still affected seriously, and then band gaps are destroyed. Therefore, it can be concluded that with the same filling fraction, the perturbation to the locations of scatters still destroys the Bragg multiple scattering in crystals, which is attributed to the destruction of the band gaps.

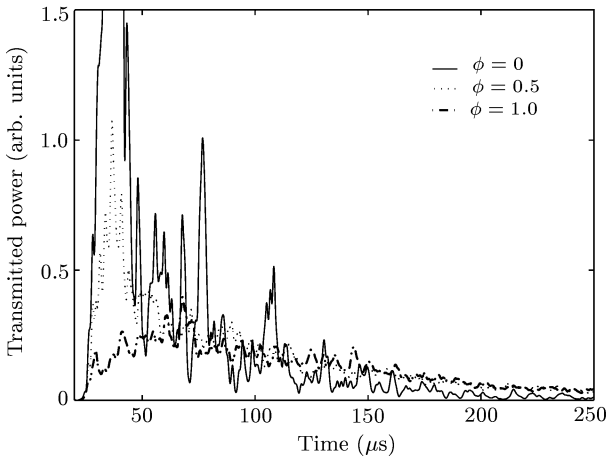


Fig. 5. Energy transmission versus time. The solid line is for the ordered crystal, the dashed line is for the weakly disordered crystal ($\phi = 0.5$), and the bold line is for the weakly disordered crystal ($\phi = 1.0$).

In conclusion, we present a comparing study of wave propagation in the ordered and the weakly disordered media. The spectra in the frequency regime show that perturbation to the positions of cylinders has serious effects on band gaps. When perturbation is introduced, the conditions of Bragg scattering are weakened, and then band gaps vanish. Moreover, high frequency is more sensitive to the perturbation than low frequency. Comparing the energy transmission dependent on time, we find that the coda wave phenomenon in weakly disordered crystals is weakened, which ensures that perturbation to the locations of cylinders in weakly disordered crystals weakens the Bragg multiple scattering.

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