Effect of Guiding Magnetic Field on Weibel Instability *

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We derive a linear dispersion relation in the presence of a constant uniform guiding magnetic field parallel to the beam velocity direction, which shows a strong background magnetic field suppresses or even stabilizes the Weibel instability produced by two counter streams in electron-ion plasmas. The simulation results are in good agreement with the analytical ones. Also observed in the simulations are the suppression of electrostatic field, a higher level of saturation of self-generated magnetic field, and the apparent difference in phase space compared with those in the absence of guiding magnetic field.

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The Weibel instability, first predicted by Weibel in 1959, is a transverse electromagnetic instability due to an anisotropic distribution function.[1] The simple physical interpretation provided by Fried in 1959[2] treated the particle distribution function more generally as a two-stream configuration of a cold plasma. The Weibel instability is recognized as one of the important mechanisms for magnetic field generation relevant to a number of phenomena in nature.[3] In the fast ignition scheme for inertial fusion,[4,5] the Weibel instability is of a special significance and plays a crucial role in energy transportation.

The Weibel instability has been widely investigated in unmagnetized plasma. Several authors[6−9] also investigated the instability in magnetized plasma. Yang *et al.*,[10] for example, carried out linear stability analysis for the Weibel instability in relativistic magnetized plasmas with the propagation direction parallel to the background magnetic field ($k//\boldsymbol{B}_0$). However, the study on the Weibel instability in the presence of an external magnetic field perpendicular to wave vector k and parallel to the beam propagation direction has not been reported, to our best knowledge.

If a circularly polarized super-intense laser is used to generate relativistic electron beam in the fast ignition, there exists a guiding magnetic field along the beam propagation direction due to the electrons circumgyrating with laser electric field. The motivation of our work is to investigate the effect of the guiding magnetic field on the Weibel instability. We focus our interest on one-dimensional non-relativistic case for simplicity. The main conclusion is suitable for the relativistic case.

Now we derive the linear dispersion relation on the Weibel instability in the case of two counter cold electron beams with a constant guiding magnetic field, where the ions are at rest and provide global charge neutrality background. The electron beams are assumed to move along the y-axis with velocity $\boldsymbol{\nu}_{0j} = \nu_{0j}\boldsymbol{e}_y$, the perturbation wavevector $\boldsymbol{k} = k\boldsymbol{e}_x$ is along the x-axis, and the guiding magnetic field $\boldsymbol{B}_0 = B_0\boldsymbol{e}_y$. The initial current is zero, i.e. $\sum_j n_{0j}\boldsymbol{\nu}_{0j} = 0$. Fourier-analysing Maxwell equations and linearized fluid equations for each electron beam, we have

$$(\omega^2 - c^2k^2)\boldsymbol{E} + c^2\boldsymbol{k}(\boldsymbol{k}\cdot\boldsymbol{E}) = -i4\pi\omega\boldsymbol{j}, \tag{1}$$

$$n_{1j} = n_{0j}\frac{k\nu_{1jx}}{\omega}, \tag{2}$$

$$-i\omega\boldsymbol{\nu}_{1j} = -\frac{e}{m_e}\Big(\boldsymbol{E} + \frac{1}{c}\boldsymbol{\nu}_{0j}\times\boldsymbol{B} + \frac{1}{c}\boldsymbol{\nu}_{1j}\times\boldsymbol{B}_0\Big). \tag{3}$$

where the subscripts 0 and 1 are the unperturbed and perturbed cases, respectively. The perturbed velocities are given by

$$\nu_{1jx} = -\frac{ie\omega}{m_e(\omega^2-\omega_{ce}^2)}\Big(E_x + \frac{k\nu_{0j}}{c}E_y + i\frac{\omega_{ce}}{\omega}E_z\Big),$$

$$\nu_{1jy} = -\frac{ie}{m_e\omega}E_y,$$

with the gyrofrequency $\omega_{ce} = \dfrac{eB_0}{m_ec}$. The current can be expressed by

$$\boldsymbol{j} = -e\sum_j(n_{0j}\boldsymbol{\nu}_{1j} + n_{1j}\boldsymbol{\nu}_{0j}). \tag{4}$$

Substituting Eq. (4) into Eq. (1), and making use of the condition $n_{01}\nu_{01}^2 + n_{02}\nu_{02}^2 = -n_0\nu_{01}\nu_{02}$ with $n_0 = n_{01} + n_{02}$, we can obtain the dispersion relation

$$\omega^2 - c^2k^2 - \omega_{pe}^2 + \omega_{pe}^2\frac{k^2\nu_{01}\nu_{02}}{(\omega^2-\omega_{ce}^2)} = 0, \tag{5}$$

where $\omega_{pe}^2 = \dfrac{4\pi n_0e^2}{m_e}$.

Equation (5) can be normalized as

$$\omega^2 - k^2 - 1 - \frac{k^2|\nu_{01}\nu_{02}|}{(\omega^2-\omega_{ce}^2)} = 0, \tag{6}$$

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where the frequency, the wavenumber, and the light velocity in vacuum are dimensionless or in units of ω_{pe}, ω_{pe}/c, and c, respectively. Equation (6) shows a purely growing mode with growth rate

$$\gamma^2 = \frac{k^2+1+\omega_{ce}^2}{2} \cdot \left[\left(1+\frac{4(k^2|\nu_{01}\nu_{02}|-(1+k^2)\omega_{ce}^2)}{(k^2+1+\omega_{ce}^2)^2}\right)^{\frac{1}{2}}-1\right], \quad (7)$$

whenever the condition

$$\omega_{ce}^2 < \frac{k^2|\nu_{01}\nu_{02}|}{(1+k^2)}, \quad (8)$$

is satisfied, which is just the Weibel instability. If $\omega_{ce}=0$, Eq. (7) reduces to the case that there is not any guiding magnetic field.[11,12]

It is followed from the linear theory that the guiding magnetic field suppresses the Weibel instability, which is clearly seen if Eq. (7) is approximated to be

$$\gamma^2 = \frac{k^2|\nu_{01}\nu_{02}|-(1+k^2)\omega_{ce}^2}{k^2+1+\omega_{ce}^2}, \quad (9)$$

in the non-relativistic case in which $|\nu_{01}\nu_{02}| \ll 1$. When the guiding magnetic field increases to the value that satisfies the condition that the gyrofrequency of the electrons is equal to the growth rate for a mode k in the absence of guiding magnetic field,[13] the instability for the mode is completely suppressed. The reason for stabilization of the guiding magnetic field is that it produces the gyration of electrons in an x–z plane, which reduces electric current due to the increasing perturbed magnetic field, especially for a long-wavelength mode.

Inequality (8) is about the threshold condition for the Weibel instability to occur. In order to understand the role of the guiding magnetic field, we can rewrite inequality (8) as

$$k^2\left(\frac{|\nu_{01}\nu_{02}|}{\omega_{ce}^2}-\frac{c^2}{\omega_{pe}^2}\right) > 1. \quad (10)$$

The necessary condition for the instability to occur from Eq. (10) is

$$|\nu_{01}\nu_{02}|/\omega_{ce}^2 > c^2/\omega_{pe}^2 = d_e^2, \quad (11)$$

that is, the geometry average of the electron beam Larmor radius is larger than skin depth. Inequality (11) may also be rewritten as

$$\sum_j \frac{m}{2} n_{0j} v_{0j}^2 > \frac{B_0^2}{8\pi}, \quad (12)$$

which shows the threshold condition for the instability to occur, that is, the initial energy of the beam electrons must be larger than the energy of guiding magnetic field. The physics meaning is explicit. If the electron energies are too low to overcome the confinement by magnetic field, a magnetic field perturbation can not effectively deflect electron motion to form current which amplifies the perturbation.[14] When the threshold condition is satisfied, the instable mode is obtained from Eq. (10),

$$k^2 > \frac{1}{|\nu_{01}\nu_{02}|/\omega_{ce}^2-d_e^2} > \frac{1}{|\nu_{01}\nu_{02}|\omega_{ce}^2}. \quad (13)$$

The mode with short wavelength can be excited.

The dependence of growth rate on guiding magnetic field and wavenumber k is plotted in Fig. 1, where $|\nu_{01}\nu_{02}|=0.04$. Figure 1 shows that the growth rate decreases as the guiding magnetic field increases. When $\omega_{ce}=0.141$ ($B_0 \approx 0.141 m_e c\omega_{pe}/e$), mode $k=1$ is suppressed, but other $k=1$ modes still grow. When the guiding magnetic fields comes to guarantee the condition $\omega_{ce} \geq 0.2$ ($B_0 \geq 0.2 m_e c\omega_{pe}/e$), all the modes are suppressed.

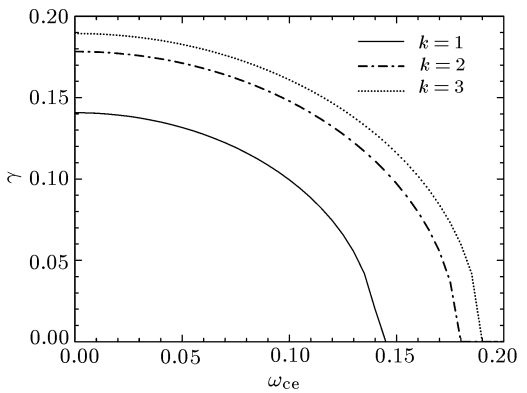

Fig. 1. The growth rate of the Weibel instability versus the electron gyrofrequency ω_{ce} for $k=1,2,3$.

The effect of the guiding magnetic field on the Weibel instability is simulated with an electromagnetic particle-in-cell (PIC) code,[15,16] in which two cold stream velocities are respectively $\boldsymbol{\nu}_{01}=0.2\boldsymbol{e}_y$ and $\boldsymbol{\nu}_{02}=-0.2\boldsymbol{e}_y$ with the same density $n_{01}=n_{02}=0.5$ to satisfy the initial zero current. The simulations were performed on $x\times z = 128\times 8$, using a total of 122880 particles. The periodic boundary conditions are imposed in both the x and z direction. Temporal and spatial scales in these simulations are normalized to the inverse electron plasma frequency ω_{pe}^{-1} and the collisionless skin depth c/ω_{pe}. In these normalized units, the box size is $12.8\times 0.8(c/\omega_{pe})^2$. The dimension in the z direction is small enough to allow the Weibel instability to grow only in the x direction. A sinusoidal perturbation with a small amplitude and a spatial period 2π ($k=1$) on electromagnetic field is initially loaded in the x direction. Here we define the mean electric and magnetic energies as

$$\xi_{Bz}(t) = \frac{1}{S}\int B_z^2 dxdz, \quad \xi_{Ex}(t) = \frac{1}{S}\int E_x^2 dxdz, \quad (14)$$

where S is the total box size.

Figure 2(a) shows the evolution of the magnetic energies of mode $k=1$ with the initial perturbation wave mode we have set, for different gyrofrequencies of guiding magnetic field $\omega_{ce}=0$, 0.08, 0.12, 0.15 re-

spectively. As is predicted, the guiding magnetic field stabilizes the Weibel instability. The growth rate decreases as the guiding magnetic field increases, which is clearly seen from the duration before saturation. In the case of $\omega_{ce} = 0.15$ ($B_0 = 0.15 m_e c\omega_{pe}/e$), the instability for mode $k = 1$ hardly grows, that is to say, the Weibel instability for single mode $k = 1$ is suppressed, which is close to the analytical one $\omega_{ce} = 0.141$ from inequality (8). The instability seems to be more complex in a nonlinear regime on the effect of guiding magnetic field. It renders the Weibel magnetic field $\boldsymbol{B}_z$ saturated with a higher level than that in the absence of the magnetic field, which is beyond our understanding presently.

Figure 2(b) shows that the evolution of electrostatic energy with time for different gyrofrequencies of guiding magnetic field $\omega_{ce} = 0$, 0.08, 0.12, 0.15 respectively. The electrostatic field is also suppressed with the increase of the guiding magnetic field. Comparing it with Fig. 2(a), we find that for the same guiding magnetic field, the electrostatic field grows at a later time and comes to a much smaller saturation level than that of the magnetic field, although it grows at a rapid rate in the order of almost twice the linear growth rate of self-generated Weibel magnetic field. For different guiding magnetic fields, the electrostatic field also begins to grow at different times and with different rates. The stronger the field, the later the time at which it begins to grow, and the less the growth rate at which it grows. When the gyrofrequency of guiding field increases to $\omega_{ce} = 0.12 (B_0 \geq 0.12 m_e c\omega_{pe}/e)$, the electrostatic energy is almost negligible compared to magnetic energy.

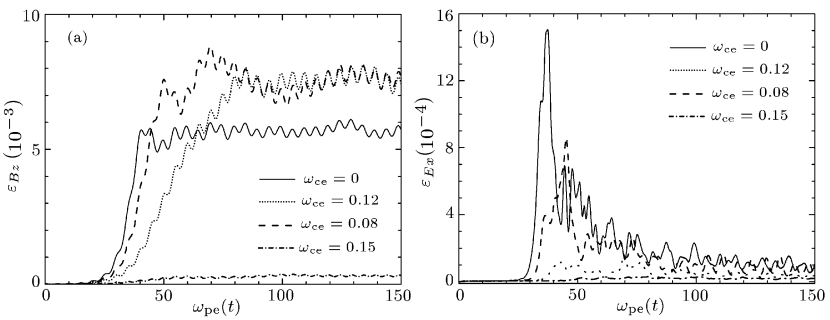

Fig. 2. (a) The magnetic energy ξ_{Bz} of Fourier component $k = 1$ and (b) the electrostatic energy ξ_{Ex}, versus time for $\omega_{ce} = 0.0$, 0.08, 0 .12, 0.15.

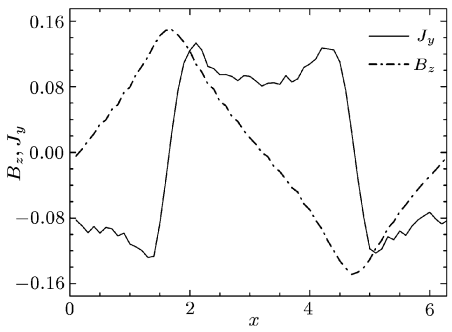

Fig. 3. The magnetic field B_z and electric current J_y for $\omega_{ce} = 0.12$ at time $t = 150\omega_{pe}$.

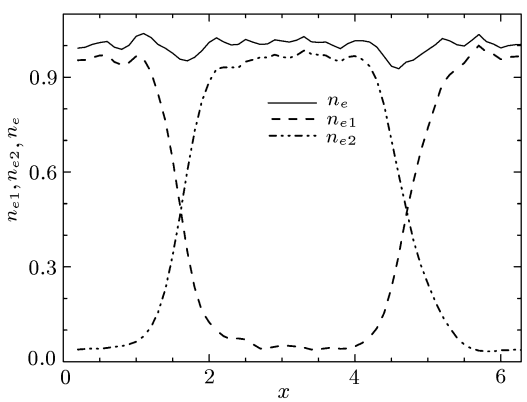

Fig. 4. The total electron density and densities of each stream for $\omega_{ce} = 0.12$ at time $t = 150\omega_{pe}$.

Figure 3 shows the space configuration of the self-generated Weibel magnetic field B_z and the corresponding electric current J_y at time $t = 150$ in case of $\omega_{ce} = 0.12$, when the Weibel instability has been saturated. Both of them are similar with those in the absence of guiding magnetic field. On the action of such a dipolar field, as shown in Fig. 4, the electron population with initial velocity $\boldsymbol{\nu}_{02} = -0.2\boldsymbol{e}_y$ is congregated and trapped between the two peaks of B_z with negative gradient located in the central region, while the population with $\boldsymbol{\nu}_{01} = 0.2\boldsymbol{e}_y$ is congregated between the two peaks of B_z with positive gradient due to the periodic structure. Although two electron populations concentrate different regions, the density of the total electron density remains uniform, which is different from the result in Ref. [13]. The reason is most possibly because of the neglect of the ion motion in our simulation. The spatial configurations of B_z and, J_y and densities described above are almost the same as that in the absence of the guiding field.

Figure 5 shows the electron distribution in the (x, v_x), (x, v_y), and (x, v_z) phase space at time $t = 150$ for $\omega_{ce} = 0$ and 0.12 respectively, in which we observe the spatial separation of the two beams due to magnetic effect ((b) and (e)). The guiding magnetic field reduces the velocity spread in v_x ((a) and (d)) and v_y ((b) and (e)), and makes each electron stream bunch in space furthermore. The most principle difference is that the guiding magnetic field deviates most of the electron kinetic energies from the x direction to the z direction shown in Figs. 5(a), 5(c), 5(d) and 5(f). As a result, the guiding magnetic field B_0 is modified by the current J_z, which are plotted in Fig. 6. The modified field B_y and the corresponding current J_z are of the typical characteristics with mode $k = 2$.

The simulation results described above is for the

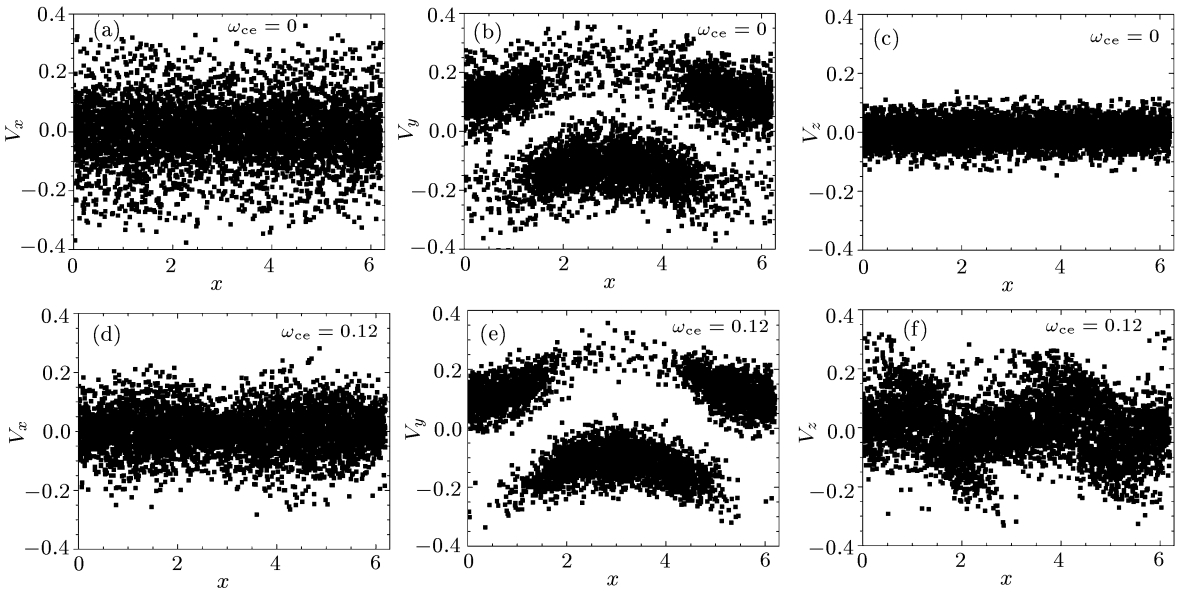

Fig. 5. The distribution in the (v_x, x), (v_y, x), (v_z, x) phase space respectively for $\omega_{ce} = 0$ frame [(a)–(c)], for $\omega_{ce} = 0.12$ [(d)–(f)] at time $t = 150\omega_{pe}$.

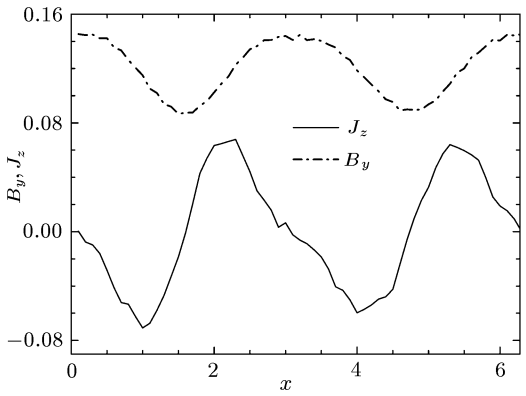

Fig. 6. The magnetic field $\boldsymbol{B}_y$ (dashed line) and electric current $\boldsymbol{J}_z$ (solid line) for $\omega_{ce} = 0.12$ at time $t = 150\omega_{pe}$.

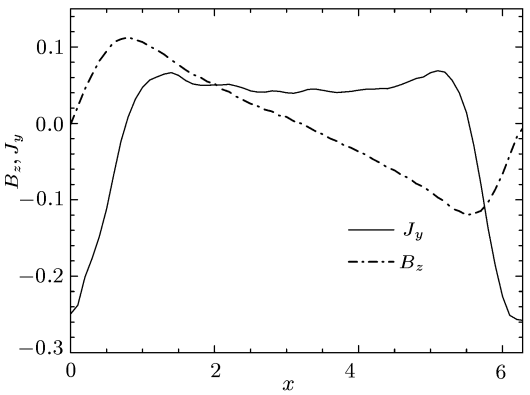

Fig. 7. The magnetic field $\boldsymbol{B}_z$ (dashed line) and electric current $\boldsymbol{J}_y$ (solid line) for $\omega_{ce} = 0.12$ at time $t = 150\omega_{pe}$ for asymmetry case.

case of two symmetry streams. In fact, for the asymmetry case, such as $\boldsymbol{\nu}_{01} = 0.4\boldsymbol{e}_y$, $\boldsymbol{\nu}_{02} = -0.1\boldsymbol{e}_y$, $n_{01} = 0.2$, $n_{02} = 0.8$, the evolution of magnetic energy and electrostatic energy with the time are the same as that discussed for the symmetrical case. The main difference is the distance between two peaks of magnetic field B_z, as shown in Fig. 7, it is obvious that the distance between the two peaks is greater than that shown in Fig. 3, which is similar to the results without the guiding magnetic field.[17] Correspondingly the configurations of B_y, electron density, electron distribution in phase space are different. However, it is still true that two electron populations are concentrated in different regions of Weibel magnetic field B_z with positive and negative gradients, respectively, and the guiding magnetic field makes them bunch more closely.

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