Detecting Multi-Scale Coherent Eddy Structures and Intermittency in Turbulent Boundary Layer by Wavelet Analysis *

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(Received 20 January 2005)

Multi-scale decomposition by wavelet transform has been performed to velocity time sequences obtained by fine measurements of turbulent boundary layer flow. A conditional sampling technique for detecting multi-scale coherent eddy structures in turbulent field is proposed by using multi-scale instantaneous intensity factor and flatness factor of wavelet coefficients. Although the number of coherent eddy structures in the turbulent boundary layer is very small, their energy percentage with respect to the turbulence kinetic energy is high. Especially in buffer layer, the energy percentages of coherent structures are significantly higher than those in the logarithmic layer, indicating that the buffer layer is the most active region in the turbulent boundary layer. These multi-scale coherent eddy structures share some common dynamical characteristics and are responsible for the anomalous scaling law in the turbulent boundary layer.

PACS: 47.27.Lx, 47.27.Nz

Since the discovery of the low-speed streaky structures in near-wall turbulence,[1] the traditional view of turbulence as a pure random process has changed. Now it is widely accepted that coherent structure is a kind of intrinsic properties of turbulence and plays an important role in momentum, energy and mass transfer in turbulent flows. The theoretical and experimental research on coherent structures can provide us some new viewpoints for our understanding of turbulence essence and composing effective turbulence control strategy.

Coherent structures exist not only at large scales, but also at small scales.[2−4] Indeed, as indicated by Sandborn[5] in 1959, the presence of low speed streaks might be indicated by "bursts in the overall frequencies". In recent years, it has been discovered that universal and organized small-scale coherent structures also exist in turbulent flows.[6,7] The experimental measurements and direct numerical simulation show that small-scale filamentary coherent structures exist even in homogeneous and isotropic turbulence.[8,9] Researchers have discovered that multi-scale coherent structures with strong intermittency exist in the near-wall region of turbulent channel flows and turbulent boundary layer.[10−14] The multi-scale coherent structures in turbulent shear flows have significant influence on turbulent statistical properties. The development of the research on the intermittency of multi-scale coherent structures in turbulent shear flows is helpful for bringing forward faster and more effective numerical simulation method based on the multi-scale decomposition of turbulent structures. Farge and Schneider[15] recently put forward a coherent vortex simulation method based on wavelet decomposition instead of wave number spectrum decompositions generally used before. The new method is in accordance with the physical mechanism of turbulence and has been proven to be an effective method for the direct numerical simulation of turbulence.

Wavelet transform is a new mathematical technique developed recently.[16] It provides the most suitable elementary representation of turbulent flows.[17] "Eddies" are the fundamental structures in turbulent flows. Eddy and wavelet share common features in many physical aspects, and wavelet can be regarded as the mathematical mode of an eddy structure in turbulent flows. As a new tool, wavelet transform can be devoted to identify coherent structure in wall turbulence instead of the conditional sampling methods usually used before. Liu and Jiang[18] have proven that the wavelet coefficient structure function is identical with the locally averaged velocity structure function rather than the classical velocity structure function used by Kolmogorov.[19]

Figure 1 is the contour plots of the magnitude of wavelet coefficients for the longitudinal fluctuating velocity signal in the standard (a, t) plane, where a represents the wavelet scale parameter and t denotes time. From this figure, it can be seen that there exist one-to-one eddies at different positions as well as at different scales. The large-scale eddies seem to be randomly distributed and are fairly space filling. A typical process in which a large eddy creates two or more smaller eddies can be seen clearly. This subdivision repeats until eddies reach the scale at which they are readily dissipated by the fluid viscosity. There is a kinetic energy flux from larger eddies to smaller ones. The smaller eddies obtain their energy at the expense

* Supported by the National Natural Science Foundation of China under Grants Nos 10472081 and 10232020.

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of the energy loss in larger eddies.

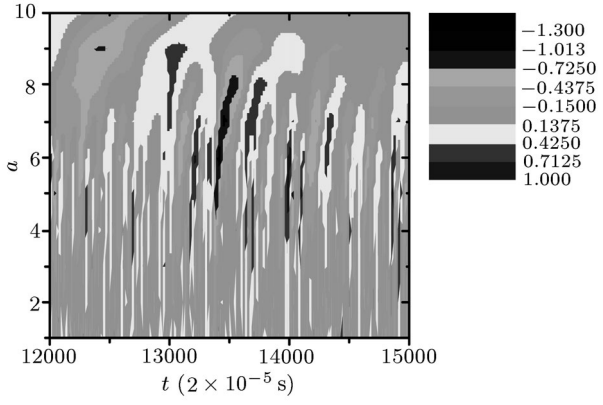


Fig. 1. Contours of the magnitude of wavelet coefficients for the longitudinal fluctuating velocity signal.

The first qualitative indicator of intermittency is the intermittency factor of wavelet coefficients:

$$I(a,t) = |W(a,t)|^2 / \langle |W(a,t)|^2 \rangle_t, \qquad (1)$$

where $W(a,t)$ represents the wavelet coefficients calculated at scale a and time point t. The intermittency factor clearly shows the intensity of the local activity of the eddy structures at each scale.

Another indicator of intermittency is the flatness factor of the wavelet coefficients:

$$FF(a) = \langle |W(a,t)|^4 \rangle_t / \langle |W(a,t)|^2 \rangle_t^2. \qquad (2)$$

A conditional sampling detection method with the intermittency factor is used to extract the phase-averaged waveform for multi-scale coherent eddy structures in wall turbulence. The method can be simply summarized as follows: the flatness factor $FF(a)$ is computed at each scale. If $FF(a)$ is less than 3, it means that there are few of coherent eddy structures at this scale and we should turn to the next scale. If $FF(a)$ is larger than 3 for a given scale, a threshold level L is imposed on $I(a,t)$ and coherent structures are distinguished for those wavelet coefficients whose $I(a,t)$ is larger than L. Then $FF(a)$ is recalculated for the wavelet coefficients whose $I(a,t)$ is less than L. If the recalculated $FF(a)$ is still larger than 3, the threshold value L is lowered and the process is iterated until $FF(a)$ is less than 3. In this way, coherent structures at each scale are detected from the disorder background turbulence.

The experiment was conducted in a recirculating wind tunnel with a 1500-mm-long working section and a 800 mm × 600 mm rectangular cross-section. The test flat plate was fixed above the lower wall of the test section at the zero incidence angle. The leading edge of the plate was symmetrically wedged. The length and width of the plate was 1500 mm and 600 mm, respectively. The measurement was taken at the location of 1000 mm downstream from the leading edge. When the mean velocity of free coming flow was 14 m/s, the boundary layer thickness was 35 mm and the Reynolds number based on the thickness of boundary layer was about 31000. The temperature-sensitive material of the hot-wire used in this experiment was tungsten filament in diameter 5 μm. The time sequence of longitudinal velocity component at different vertical locations was finely measured with resolution higher than dissipation scale. For each position, the sampling frequency was 50000 Hz and the sampling number was 4194304. The mean velocity profile of turbulent boundary layer is shown in Fig. 2, in which $u^+ = u/u_*$ and $y^+ = yu_*/\nu$ are the longitudinal velocity and the distance from the wall nondimensionalized by the skin friction velocity u_* and the kinematic viscosity ν. A typical logarithmic region can be clearly seen in this figure.

The energy contribution from eddies at scale a can be obtained by integrating the square of the modulus of wavelet coefficients over the temporal location parameter t. Figure 3 shows the energy contribution as a function of scale parameter a. It can be found that there is a scale that corresponds to the peak of energy contribution. This maximum energy is related to the large-scale coherent structures, i.e. burst, in the near wall region of the turbulent boundary layer. The flatness factor $FF(a)$ calculated by formula (2) is shown in Fig. 4. $FF(a)$ decreases from the value that is significantly larger than 3 to about 3 as the scale parameter increases. Compared to Fig. 3, $FF(a)$ at scales smaller than the scale that corresponds to the peak of energy contributions is larger than 3, which indicates that lots of coherent structures satisfying $I(a,t) \geq L$ exist at these scales. Figure 5 shows the probability distribution of multi-scale coherent eddy structures as a function of scale parameter. Probability of coherent eddy structures occurred at small scales is much larger than that occurred at larger scales. The probability of coherent eddy structure in the buffer layer is markedly larger than those in the logarithmic layer. The ratio of kinetic energy contained in coherent eddy structures at each scale to the total turbulent kinetic energy at the same scale is shown in Fig. 6. Although the occurrence probability of coherent eddy structures at each scale is very small, the ratio of kinetic energy contained in these structures is much higher. It also can be seen that the coherent structures in the buffer layer occupy more energy than those in the logarithmic layer, which implies that the buffer layer is more important than other regions in the turbulent boundary layer and is responsible for turbulence production.

The ratio of the averaged energy intensity of each coherent eddy structure to that of each non-coherent eddy structure at the same scale is shown in Fig. 7. Mean intensity of a single coherent eddy structure is

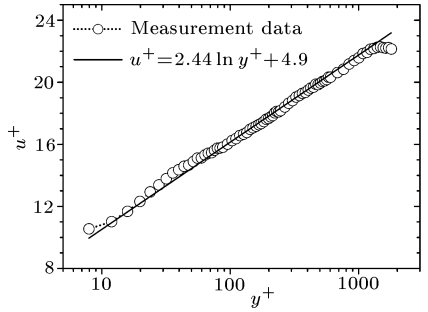


Fig. 2. Mean velocity profile in wall units.

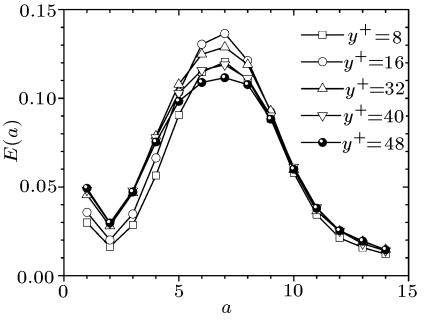


Fig. 3. Energy intensity as a function of wavelet scale at different vertical locations.

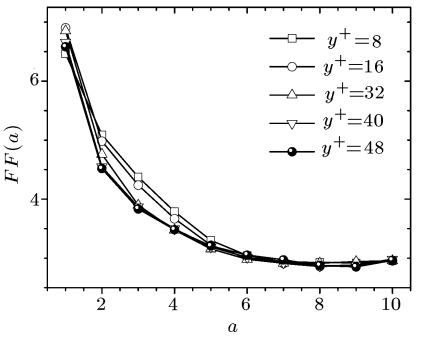


Fig. 4. Flatness factor as a function of wavelet scale at different vertical locations.

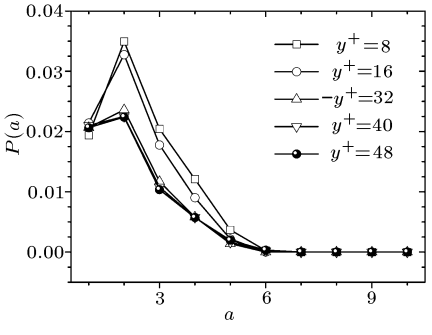


Fig. 5. Probability of multi-scale coherent eddy structures as a function of scales at different vertical locations.

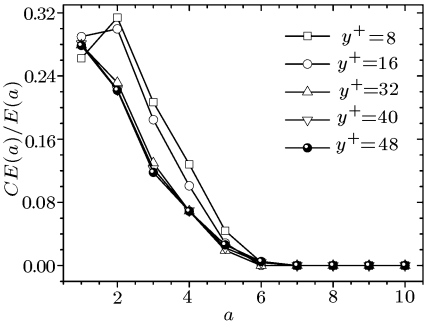


Fig. 6. Ratio of kinetic energy in coherent eddy structures at one scale to turbulent kinetic energy at the same scale.

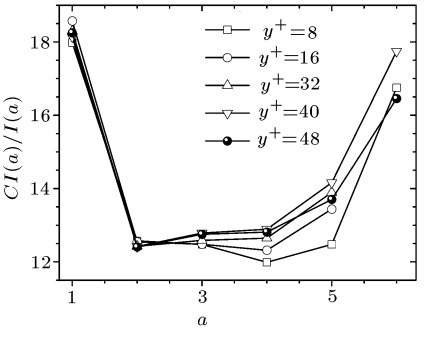


Fig. 7. Ratio of average energy intensity of a coherent eddy structure to that of a non-coherent eddy structure at the same scale.

about 12–18 times of a non-coherent eddy structure, which indicates that coherent eddy structures are much stronger than non-coherent ones at the same scale.

The phase-averaged waveforms of coherent eddy structures for different scales are presented in Fig. 8, in which the vertical axis represents phase-averaged velocity, while the abscissa axis represents the time. At $y^+ = 8$ in the buffer layer, it is obvious that the waveforms at different scales are quite similar to each other though temporal scales are quite different. This implies that the dynamic development and evolution process of coherent structures at different scales share some common characteristics, which can be summarized as follows: in temporal evolution process, the velocity first slowly decreases, then sharply increases, and at last slowly decreases.

Especially in the middle of the process, the velocity gradient is great and its effect is powerful in a short period, which means that during a burst, the high-speed inrush fluid sweeps towards the wall strongly in a short time. These coherent eddy structures are the backbone in the transport and maintenance of turbulent flows, and constitute the significant difference between turbulence and other random fields. Figure 9 presents the pth order relative scaling exponents calculated by the extended self-similarity scaling law (ESS)[20] at $y^+ = 8$ and $y^+ = 40$, including multi-scale coherent eddy structures. The anomalous scaling exponents $\zeta(p, 3)$ are discrepant with the Kolmogorov linear scaling law[19] of $\zeta(p) = p/3$. It clearly shows that when $p < 3$ the exponent is larger than the linear scaling one, while it is smaller when $p > 3$, as described by Benzi *et al.*[20] When closer to the wall, the anisotropy becomes enhanced and the coherent eddy structures are more active, so the difference between the anomalous scaling exponents and the linear scaling ones becomes distinct. Figure 10 shows the p-th order relative scaling exponents calculated after elimination of the multi-scale coherent eddy structures. It is obvious that the scaling exponents calculated at both the locations exactly fit with the linear scaling law, which means that multi-scale coherent eddy structures are responsible for the anomalous scaling law in the turbulence boundary layer.

In summary, coherent eddy structures play an important role in turbulence. They are much stronger than non-coherent eddy structures in the sense that the mean intensity of a structure is about 12–18 times as great as that of a non-coherent eddy structure at

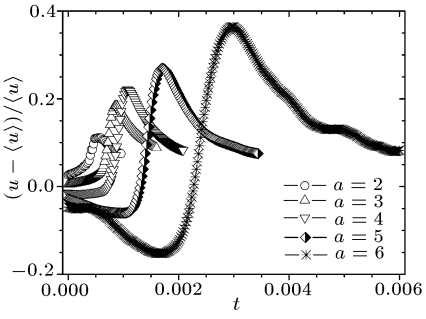


Fig. 8. Conditional phase-averaged waveforms of multi-scale coherent eddy structures at $y^+ = 8$.

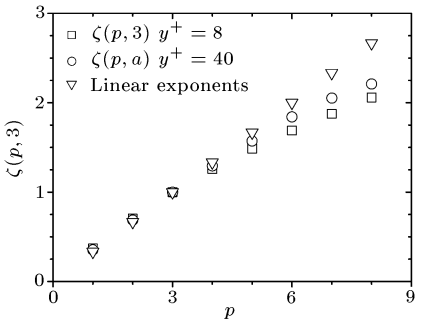


Fig. 9. Relative scaling exponents calculated by wavelet coefficients including coherent structures at y^+=8 and 40.

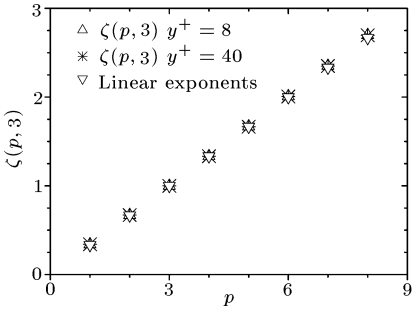


Fig. 10. Relative scaling exponents calculated without coherent structures at $y^+ = 8$ and 40.

the same scale. Although the multi-scale coherent eddy structures occurred at the probability of no more than 4% in the buffer layer, their energy percentage is very high. The energy percentage of coherent structures in the buffer layer are significantly higher than those in the logarithmic layer, which indicates that the buffer layer is the most active region in the turbulent boundary layer. In spite of the different temporal scales of multi-scale coherent eddy structures, their phase-averaged evolution waveforms are similar, which implies that the development dynamical process of coherent eddy structures at different scales have some characteristics in common. Multi-scale coherent eddy structures are also responsible for the intermittency and anomalous scaling law in the turbulent boundary layer. When those structures are eliminated, the ESS scaling exponents calculated by wavelet coefficients are in good agreement with the Kolmogorov linear scaling exponents.

We would like to thank Dr Xu Chunxiao at Tsinghua University for his help.

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