New Three-Mode Einstein–Podolsky–Rosen Entangled State Representation and Its Application in Squeezing Theory *

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(Received 22 April 2002)

By extending the Einstein–Podolsky–Rosen bipartite entanglement to the tripartite case, we construct the common eigenvector of the tripartite centre-of-mass coordinate and two mass-weighted relative momenta, which is a new entangled state of continuum variables. The classical dilation transform of variables in such a state induces a new three-mode squeezing operator related to a three-mode bosonic operator realization of SU(1,1) Lie algebra.

PACS: 03. 65. Ud, 42. 50. Dv

In recent years, quantum entanglement has attracted much attention from physicists due to its weird, queer but fascinating property. The conception of quantum entanglement originated from Einstein, Podolsky and Rosen (EPR)[1] who criticized in 1935 that quantum mechanics was not complete. They took the relative coordinate of two particles and the total momentum as an example, and found the bipartite total wavefunction. They revealed that a measurement performed on one particle would provide information of the other particle without disturbing it. This is entanglement. In 1994, in Ref. [2] the common eigenvector of the two-particle relative position $X_1 - X_2$ and the total momentum $P_1 + P_2$ in the two-mode Fock space was found. This is

$$|\eta\rangle = \exp\Big[-\frac{1}{2}|\eta|^2 + \eta a_1^\dagger - \eta^* a_2^\dagger + a_1^\dagger a_2^\dagger\Big]|00\rangle_{12}, \quad (1)$$

where $\eta = \eta_1 + \mathrm{i}\eta_2$ is a complex number; $|00\rangle_{12}$ is the two-mode vacuum state; $(a_i, a_i^\dagger)$, $i = 1, 2$, are two-mode Bose annihilation and creation operators, related to (X_i, P_i) by $X_i = \frac{1}{\sqrt{2}}(a_i + a_i^\dagger)$, $P_i = \frac{1}{\sqrt{2}i}(a_i - a_i^\dagger)$. The $|\eta\rangle$ state obeys the eigenvector equations

$$\begin{aligned}(X_1 - X_2)|\eta\rangle &= \sqrt{2}\eta_1|\eta\rangle,\\ (P_1 + P_2)|\eta\rangle &= \sqrt{2}\eta_2|\eta\rangle.\end{aligned} \quad (2)$$

A remarkable property endowed with $|\eta\rangle$ is that a classical c-number scaling transformation $\eta \to \eta/\mu$ maps on to a two-mode squeezing operator in two-mode Fock space, i.e.

$$\mathrm{e}^{\theta(a_1^\dagger a_2^\dagger - a_2 a_1)} = \frac{1}{\mu}\int \frac{\mathrm{d}^2\eta}{\pi}|\eta/\mu\rangle\langle\eta|, \quad \mu = \mathrm{e}^\theta. \quad (3)$$

Or we can say that the two-mode squeezing operator has a natural representation in the entangled state $|\eta\rangle$ representation.[3] Physically, this is related to the fact that an idler-mode and a signal-mode, composing a two-mode squeezed state, generated from a parametric down-conversion process, are entangled with each other in the frequency domain.[4] For the theory of two-mode squeezed states we refer to Refs. [5] and [6].

The purpose of this work is to extend the EPR bipartite entanglement to the tripartite case. We construct the common eigenvector of the tripartite centre-of-mass coordinate and two mass-weighted relative momenta, since they are permutable, and then we prove that this is a new entangled state of continuum variables. We also show that the dilation transformation of the variables of such a state induces a new three-mode squeezing operator. Accordingly, we can see the intrinsic relation between the entangled states of continuum variables and squeezed states.[5,6] This is important in extending the conceptions of both squeezed and entangled states.

Let X_{cm} be the tripartite centre-of-mass coordinate

$$X_{\mathrm{cm}} = \mu_1 X_1 + \mu_2 X_2 + \mu_3 X_3, \quad (4)$$

where $\mu_i = m_i/M$ $(i = 1, 2, 3)$ is the reduced mass of each particle, $M = \sum_i m_i$. We see that it is permutable with two mass-weighted relative momenta

$$\begin{aligned}[P_1/\mu_1 - P_2/\mu_2, X_{\mathrm{cm}}] &= 0,\\ [P_1/\mu_1 - P_3/\mu_3, X_{\mathrm{cm}}] &= 0,\end{aligned} \quad (5)$$

where P_i is the momentum of particle i. We search for the common eigenvector $|\chi, p_2, p_3\rangle$ of the three compatible operators: X_{cm}, $P_1/\mu_1 - P_2/\mu_2$, and $P_1/\mu_1 - P_3/\mu_3$. The latter two operators are the mass-weighted relative momenta of particles 1 and 2, particles 1 and 3, respectively. After much effort, we find that the eigenvector with eigenvalues χ, p_2, p_3 (real numbers) is

$$|\chi, p_2, p_3\rangle = \pi^{-\frac{3}{4}}\sqrt{\frac{\mu_1\mu_2\mu_3}{\lambda}}\exp\Big\{-\frac{1}{2\lambda}[(\mu_2^2 + \mu_3^2)\mu_2^2 p_2^2 + (\mu_1^2 + \mu_2^2)\mu_3^2 p_3^2 - 2\mu_2^2\mu_3^2 p_2 p_3] - \frac{\chi^2}{2\lambda}$$

* Supported by the President Foundation of Chinese Academy of Sciences.

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$$+\frac{\sqrt{2}\chi}{\lambda}\sum_{i=1}^{3}\mu_i a_i^\dagger+\frac{\mathrm{i}\sqrt{2}p_2\mu_2}{\lambda}[\mu_1\mu_2 a_1^\dagger$$
$$-(\mu_1^2+\mu_3^2)a_2^\dagger+\mu_3\mu_2 a_3^\dagger]+\sum_{i=1}^{3}\Big(\frac{1}{2}-\frac{\mu_i^2}{\lambda}\Big)a_i^{\dagger 2}$$
$$+\frac{\mathrm{i}\sqrt{2}p_3\mu_3}{\lambda}[\mu_1\mu_3 a_1^\dagger+\mu_2\mu_3 a_2^\dagger-(\mu_1^2+\mu_2^2)a_3^\dagger]$$
$$-\frac{2}{\lambda}\sum_{i<j}^{3}\mu_i\mu_j a_i^\dagger a_j^\dagger\Big\}|000\rangle, \tag{6}$$

where $\lambda=\sum_{i=1}^{3}\mu_i^2$, $|000\rangle$ is the three-mode vacuum state. In fact, operating a_i on $|\chi,p_2,p_3\rangle$ respectively yields

$$a_1|\chi,p_2,p_3\rangle=\Big\{\frac{\sqrt{2}\mu_1}{\lambda}[\chi+\mathrm{i}(\mu_2^2p_2+\mu_3^2p_3)]$$
$$+a_1^\dagger-\frac{2\mu_1}{\lambda}\sum_i\mu_i a_i^\dagger\Big\}|\chi,p_2,p_3\rangle, \tag{7}$$

$$a_2|\chi,p_2,p_3\rangle=\Big\{\frac{\sqrt{2}\mu_2}{\lambda}[\chi-\mathrm{i}(\mu_1^2+\mu_3^2)p_2+\mathrm{i}\mu_3^2p_3]$$
$$+a_2^\dagger-\frac{2\mu_2}{\lambda}\sum_i\mu_i a_i^\dagger\Big\}|\chi,p_2,p_3\rangle, \tag{8}$$

$$a_3|\chi,p_2,p_3\rangle=\Big\{\frac{\sqrt{2}\mu_3}{\lambda}[\chi+\mathrm{i}\mu_2^2p_2-\mathrm{i}(\mu_1^2+\mu_2^2)p_3]$$
$$+a_3^\dagger-\frac{2\mu_3}{\lambda}\sum_i\mu_i a_i^\dagger\Big\}|\chi,p_2,p_3\rangle. \tag{9}$$

As expected, it then follows that

$$(P_1/\mu_1-P_2/\mu_2)|\chi,p_2,p_3\rangle=p_2|\chi,p_2,p_3\rangle, \tag{10}$$
$$(P_1/\mu_1-P_3/\mu_3)|\chi,p_2,p_3\rangle=p_3|\chi,p_2,p_3\rangle, \tag{11}$$
$$(\mu_1X_1+\mu_2X_2+\mu_3X_3)|\chi,p_2,p_3\rangle=\chi|\chi,p_2,p_3\rangle. \tag{12}$$

Using the technique of integration within an ordered product (IWOP) of operators[7−9] and the normal product form of the three-mode vacuum state

$$|000\rangle\langle 000|=:\mathrm{e}^{-a_1^\dagger a_1-a_2^\dagger a_2-a_3^\dagger a_3}:, \tag{13}$$

we can prove that the states $|\chi,p_2,p_3\rangle$ span a complete set, i.e.

$$\int\int\int_{-\infty}^{\infty}\mathrm{d}\chi\mathrm{d}p_2\mathrm{d}p_3|\chi,p_2,p_3\rangle\langle\chi,p_2,p_3|$$
$$=\pi^{-\frac{3}{2}}\frac{\mu_1\mu_2\mu_3}{\lambda}\int\int\int_{-\infty}^{\infty}\mathrm{d}\chi\mathrm{d}p_2\mathrm{d}p_3:\exp\Big\{-\frac{\chi^2}{\lambda}$$
$$-\frac{1}{\lambda}[(\mu_1^2+\mu_3^2)\mu_2^2p_2^2+(\mu_1^2+\mu_2^2)\mu_3^2p_3^2-2\mu_2^2\mu_3^2p_2p_3]$$
$$+\frac{\sqrt{2}\chi}{\lambda}\sum_{i=1}^{3}\mu_i(a_i^\dagger+a_i)$$
$$+\frac{\mathrm{i}\sqrt{2}p_2}{\lambda}[\mu_1\mu_2^2a_1^\dagger-\mu_2(\mu_1^2+\mu_3^2)a_2^\dagger$$
$$+\mu_3\mu_2^2a_3^\dagger-\mu_1\mu_2^2a_1+\mu_2(\mu_1^2+\mu_3^2)a_2-\mu_3\mu_2^2a_3]$$
$$+\frac{\mathrm{i}\sqrt{2}p_3}{\lambda}[\mu_1\mu_3^2a_1^\dagger+\mu_2\mu_3^2a_2^\dagger-\mu_3(\mu_1^2+\mu_2^2)a_3^\dagger$$
$$-\mu_1\mu_3^2a_1^\dagger-\mu_2\mu_3^2a_2^\dagger+\mu_3(\mu_1^2+\mu_2^2)a_3^\dagger]$$
$$-\frac{2}{\lambda}\sum_{i<j}^{3}\mu_i\mu_j(a_i^\dagger a_j^\dagger+a_ia_j)+\sum_{i=1}^{3}\Big(\frac{1}{2}-\frac{\mu_i^2}{\lambda}\Big)$$
$$\cdot(a_i^{\dagger 2}+a_i^2)-\sum_{i=1}^{3}a_i^\dagger a_i\Big\}:$$
$$=:\exp\{0\}:=1. \tag{14}$$

It is noted that all the creation operators commute with all the annihilation operators within the normal ordering symbol, so they can be considered c-number parameters while the integration is going on; this is the essential point of the IWOP technique. Furthermore, it is not difficult to prove the orthogonal property

$$\langle\chi,p_2,p_3|\chi',p_2',p_3'\rangle=\delta(\chi-\chi')\delta(p_2-p_2')\delta(p_3-p_3'). \tag{15}$$

Equations (14) and (15) altogether indicate that $|\chi,p_2,p_3\rangle$ is capable of making a new quantum mechanical representation.

Let us explain why $|\chi,p_2,p_3\rangle$ is an entangled state. Making a onefold Fourier integration of $|\chi,p_2,p_3\rangle$ over $d\chi$, we have

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{d}\chi\mathrm{e}^{\mathrm{i}xp}|\chi,p_2,p_3\rangle=\sqrt{\frac{\mu_1\mu_2\mu_3}{2\pi^{3/2}}}\exp\Big\{-\frac{\lambda}{2}p^2$$
$$+\mathrm{i}\sqrt{2}\lambda p\sum_{i=1}^{3}\mu_i a_i^\dagger+\frac{1}{2}\sum_{i=1}^{3}a_i^{\dagger 2}$$
$$-\frac{1}{2\lambda}[(\mu_1^2+\mu_3^2)\mu_2^2p_2^2+(\mu_1^2+\mu_2^2)\mu_3^2p_3^2-2\mu_2^2\mu_3^2p_2p_3]$$
$$+\frac{\mathrm{i}\sqrt{2}p_2}{\lambda}[\mu_1\mu_2^2a_1^\dagger-\mu_2(\mu_1^2+\mu_3^2)a_2^\dagger-\mu_3\mu_2^2a_3^\dagger]$$
$$+\frac{\mathrm{i}\sqrt{2}p_3}{\lambda}[\mu_1\mu_3^2a_1^\dagger+\mu_2\mu_3^2a_2^\dagger-\mu_3(\mu_1^2+\mu_2^2)a_3^\dagger]\Big\}|000\rangle. \tag{16}$$

In comparison with the expression of the momentum eigenstate

$$|p\rangle_i=\pi^{-1/4}\exp\Big[-\frac{1}{2}p^2+\mathrm{i}\sqrt{2}pa_i^\dagger+\frac{1}{2}a_i^{\dagger 2}]|0\rangle_i, \tag{17}$$

we see

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{d}\chi\mathrm{e}^{\mathrm{i}\chi v}|\chi,p_2,p_3\rangle$$
$$=\sqrt{\frac{\mu_1\mu_2\mu_3}{2\pi}}|\mu_1\Big(\frac{\mu_2^2p_2+\mu_3^2p_3}{\lambda}+v\Big)\rangle_1$$
$$\otimes\Big|\mu_2\Big(\frac{\mu_2^2p_2+\mu_3^2p_3}{\lambda}+v-p_2\Big)\rangle_2$$
$$\otimes\Big|\mu_3\Big(\frac{\mu_2^2p_2+\mu_3^2p_3}{\lambda}+v-p_3\Big)\rangle_3. \tag{18}$$

Its inverse transformation is

$$|\chi,p_2,p_3\rangle = \sqrt{\frac{\mu_1\mu_2\mu_3}{2\pi}}\int_{-\infty}^{+\infty}\Big|\mu_1\Big(\frac{\mu_2^2p_2+\mu_3^2p_3}{\lambda}+v\Big)\rangle_1 \otimes\Big|\mu_2\Big(\frac{\mu_2^2p_2+\mu_3^2p_3}{\lambda}+v-p_2\Big)\Big\rangle_2 \otimes\Big|\mu_3\Big(\frac{\mu_2^2p_2+\mu_3^2p_3}{\lambda}+v-p_3\Big)\Big\rangle_3 \mathrm{e}^{-\mathrm{i}\chi v}\mathrm{d}v. \tag{19}$$

Making the variable change $\frac{\mu_2^2p_2+\mu_3^2p_3}{\lambda}+v\to p$ we have

$$|\chi,p_2,p_3\rangle = \sqrt{\frac{\mu_1\mu_2\mu_3}{2\pi}}\mathrm{e}^{\frac{\mathrm{i}(\mu_2^2p_2+\mu_3^2p_3)\chi}{\lambda}}\int_{-\infty}^{+\infty}|\mu_1p\rangle_1 \otimes|\mu_2(p-p_2)\rangle_2\otimes|\mu_3(p-p_3)\rangle_3\mathrm{e}^{-\mathrm{i}\chi p}\mathrm{d}p. \tag{20}$$

This is the Schmidt decomposition of $|\chi,p_2,p_3\rangle$ which shows that once particle 1 is measured in the state $|\mu_1p\rangle_1$, particles 2 and 3 immediately collapse to the coordinate eigenstate $|\mu_2(p-p_2)\rangle_2\otimes|\mu_3(p-p_3)\rangle_3$. According to the standard theory of Schmidt decomposition of an entangled state,[10] we know that $|\chi,p_2,p_3\rangle$ is an entangled state.

We now make a classical transformation $\chi\to\chi/\mu$, $p_2\to p_2/\mu$, $p_3\to p_3/\mu$ in the state $|\chi,p_2,p_3\rangle$ to build a ket-bra integral

$$\int\int\int_{-\infty}^{+\infty}\frac{\mathrm{d}\chi\mathrm{d}p_2\mathrm{d}p_3}{\mu^{3/2}}|\chi/\mu,p_2/\mu,p_3/\mu\rangle\langle p,\chi_2,\chi_3|\equiv S_3. \tag{21}$$

Using Eq. (6) and the IWOP technique we perform the integration in Eq. (21)

$$\begin{aligned}S_3 =& \int\int\int_{-\infty}^{\infty}\frac{\mathrm{d}\chi\mathrm{d}p_2\mathrm{d}p_3}{\mu^{3/2}}|\chi/\mu,p_2/\mu,p_3/\mu\rangle\langle p,\chi_2,\chi_3|\\ =&\int\int\int_{-\infty}^{\infty}\frac{\mathrm{d}\chi\mathrm{d}p_2\mathrm{d}p_3}{3(\pi\mu)^{3/2}}:\exp\Big\{-\frac{1}{2\lambda}\Big(1+\frac{1}{\mu^2}\Big)\chi^2\\ &+\frac{\sqrt{2}}{\lambda}\Big[\frac{1}{\mu}(\mu_1a_1^\dagger+\mu_2a_2^\dagger+\mu_3a_3^\dagger)+\mu_1a_1\\ &+\mu_2a_2+\mu_3a_3\Big]\chi-\frac{\mu_2^2}{2\lambda}(\mu_1^2+\mu_3^2)\Big(1+\frac{1}{\mu^2}\Big)p_2^2\\ &+\frac{\sqrt{2}}{\lambda}\Big[\frac{\sqrt{2}}{2}\mu_2^2\mu_3^2p_3\Big(1+\frac{1}{\mu^2}\Big)\\ &+\frac{\mathrm{i}}{\mu}(\mu_1\mu_2^2a_1^\dagger-\mu_2(\mu_1^2+\mu_3^2)a_2^\dagger+\mu_3\mu_2^2a_3^\dagger\Big)\\ &-\mathrm{i}(\mu_1\mu_2^2a_1-\mu_2(\mu_1^2+\mu_3^2)a_2+\mu_3\mu_2^2a_3)]p_2\\ &-\frac{\mu_3^2}{2\lambda}(\mu_1^2+\mu_2^2)\Big(1+\frac{1}{\mu^2}\Big)p_3^2\\ &+\frac{\sqrt{2}}{\lambda}\Big[\frac{\mathrm{i}}{\mu}(\mu_1\mu_3^2a_1^\dagger+\mu_2\mu_3^2a_2^\dagger-\mu_3(\mu_1^2+\mu_2^2)a_3^\dagger\Big)\\ &-\mathrm{i}(\mu_1\mu_3^2a_1+\mu_2\mu_3^2-\mu_3(\mu_1^2+\mu_2^2)a_2a_3)\Big]p_3\\ &-\frac{2}{\lambda}\sum_{i<j}^{3}\mu_i\mu_j(a_i^\dagger a_j^\dagger+a_ia_j)\\ &+\sum_{i=1}^{3}\Big(\frac{1}{2}-\frac{\mu_i^2}{\lambda}\Big)(a_i^2+a_i^{\dagger2})-\sum_{i=1}^{3}a_i^\dagger a_i\}:\\ =&\mathrm{sech}^{3/2}\theta:\exp\Big\{\tanh\theta\Big[\frac{1}{2}\sum_{i=1}^{3}\Big(1-\frac{2\mu_i^2}{\lambda}\Big)a_i^{\dagger2}\\ &-\frac{2}{\lambda}\sum_{i<j=1}^{3}\mu_i\mu_ja_i^\dagger a_j^\dagger\Big]+(\mathrm{sech}\theta-1)\sum_{i=1}^{3}a_i^\dagger a_i\\ &-\tanh\theta\Big[\frac{1}{2}\sum_{i=1}^{3}\Big(1-\frac{2\mu_i^2}{\lambda}\Big)a_i^2\\ &-\frac{2}{\lambda}\sum_{i<j=1}^{3}\mu_i\mu_ja_ia_j\Big]\Big\},\end{aligned} \tag{22}$$

where

$$e^\theta=\mu,\ \mathrm{sec\ h}\,\theta=\frac{2\mu}{1+\mu^2},\ \tanh\theta=\frac{\mu^2-1}{1+\mu^2}. \tag{23}$$

Furthermore, using the operator identity

$$\mathrm{e}^{\alpha a_i^\dagger a_i}=:\exp[(\mathrm{e}^{\alpha}-1)a_i^\dagger a_i]:, \tag{24}$$

We can simplify Eq. (22) as

$$\begin{aligned}S_3 =&\exp[\tanh\theta\cdot A^\dagger]\exp[\ln\,\mathrm{sech}\,\theta\cdot B]\\ &\cdot\exp[-\tanh\theta\cdot A],\end{aligned} \tag{25}$$

where

$$\begin{aligned}A^\dagger\equiv&\frac{1}{2}\sum_{i=1}^{3}\Big(1-\frac{2\mu_i^2}{\lambda}\Big)a_i^{\dagger2}-\frac{2}{\lambda}\sum_{i<j=1}^{3}\mu_i\mu_ja_i^\dagger a_j^\dagger,\\ B\equiv&\sum_{i=1}^{3}a_i^\dagger a_i+\frac{3}{2}.\end{aligned} \tag{26}$$

These obey an $SU(1,1)$ Lie algebra

$$[A^\dagger,B]=-2A^\dagger,\ [B,A]=-2A,\ [A^\dagger,A]=-B, \tag{27}$$

which agrees with the unitarity of $S_3^\dagger=S_3^{-1}$. Thus we can further compact the three-mode squeezing operator (25) as

$$S_3=\exp[\lambda(A^\dagger-A)], \tag{28}$$

Due to Eq. (15) we see that S_3 squeezes $|\chi,p_2,p_3\rangle$ to

$$S_3|\chi,p_2,p_3\rangle=\frac{1}{\mu^{3/2}}|\chi/\mu,p_2/\mu,p_3/\mu\rangle. \tag{29}$$

Moreover, using Eq. (25) we see

$$S_3a_1S_3^{-1}=a_1\cosh\theta+\Big[a_1^\dagger-\frac{2\mu_1}{\lambda}\sum_{i=1}^{3}\mu_ia_i^\dagger\Big]\sinh\theta,$$

$$S_3a_2S_3^{-1}=a_2\cosh\theta+\Big[a_2^\dagger-\frac{2\mu_2}{\lambda}\sum_{i=1}^{3}\mu_ia_i^\dagger\Big]\sinh\theta,$$

$$S_3 a_3 S_3^{-1} = a_3 \cosh\theta + \left[a_3^\dagger - \frac{2\mu_3}{\lambda}\sum_{i=1}^{3}\mu_i a_i^\dagger\right]\sinh\theta. \tag{30}$$

It then follows

$$S_3 \sum_{i=1}^{3}(a_i/\mu_i)S_3^{-1} = \sum_{i=1}^{3}\Big[a_i/\mu_i \cosh\theta + (a_i^\dagger/\mu_i - \frac{6}{\lambda}\mu_i a_i^\dagger)\Big]\sinh\theta, \tag{31}$$

and

$$S_3 \sum_{i=1}^{3} a_i\mu_i S_3^{-1} = \sum_{i=1}^{3}\Big[a_i\mu_i\cosh\theta + a_i^\dagger\mu_i \cdot \Big(1 - \frac{6}{\lambda}\mu_i^2\Big)\sinh\theta\Big]. \tag{32}$$

By introducing the two quadratures

$$P = \frac{1}{\sqrt{3}}\sum_{i=1}^{3} p_i/\mu_i,\ X = \frac{1}{\sqrt{3}}\sum_{i=1}^{3} x_i\mu_i, \tag{33}$$

we see

$$S_3 P S_3^{-1} = P\cosh\theta - \frac{1}{\sqrt{3}}\sum_{i=1}^{3} p_i/\mu_i\Big(1 - \frac{6\mu_i^2}{\lambda}\Big)\sinh\theta, \tag{34}$$

and

$$S_3 X S_3^{-1} = X\cosh\theta + \frac{1}{\sqrt{3}}\sum_{i=1}^{3} x_i\mu_i\Big(1 - \frac{6\mu_i^2}{\lambda}\Big)\sinh\theta. \tag{35}$$

In particular, when $\mu_1 = \mu_2 = \mu_3 = 1/3$, Eqs. (34) and (35) give us the standard squeezing transformation $S_3 P S_3^{-1} = \mu P$, $S_3 X S_3^{-1} = X/\mu$ for the three-mode quadratures. Operating S_3 in Eq. (25) on the vacuum state yields a new squeezed state

$$S_3|000\rangle = \mathrm{sech}^{3/2}\theta \exp[A^\dagger \tanh\theta]|000\rangle. \tag{36}$$

The quantum fluctuation of the operator quadratures in the state $S_3|000\rangle$ is

$$\langle(\Delta X)^2\rangle = \frac{1}{4}\mu^2,\ \langle(\Delta P)^2\rangle = \frac{1}{4}\mu^{-2}, \tag{37}$$

thus the minimum uncertainty relation still remains

$$\Delta X\Delta P \equiv \sqrt{\langle(\Delta X)^2\rangle\ \langle(\Delta P)^2\rangle} = \frac{1}{4}. \tag{38}$$

In summary, by extending the EPR bipartite entanglement to the tripartite case, we construct the common eigenvector of the tripartite centre-of-mass coordinate and two mass-weighted relative momenta, which is a new entangled state of continuum variables. The classical dilation transformation of variables in such a state induces a new three-mode squeezing operator which involves an $SU(1,1)$ Lie algebra. Thus we see again the intrinsic relation between entanglement and squeezed states.

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