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Pattern Structure and Properties of Invasion Percolation in Porous Media *

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The percolation clusters with varying occupy probability are constructed. Invasion percolation (IP) in percolation cluster is investigated by means of IP algorithm without trapping. The pattern constructions of IP in percolation clusters are obviously different from that of viscous fingering (VF) in percolation clusters. The fractal dimension D_f of IP increases with increasing the percolation probability P. The geometry and the topology of the porous media have strong effects on the pattern structure of IP. For large M (the node numbers occupied by the injected fluid), the rescaled value is $R_g/M^{1/D_f}$, which asymptotically approaches a constant value, and R_g is the gyration radius of IP cluster. Moreover, the chemical dimension D_l and the shortest path exponent $d_{\min}$ are obtained.

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The flow phenomena of fluids through random porous media are related to many problems of great scientific and industrial importance, including extraction of oil, soil sciences, powder technology and materials science, etc.[1] Invasion percolation (IP), a model introduced by Wilkinson and Willemsen[2] for describing the evolution of the interface between an invading and a defending fluid in a porous medium, has provided deep insight into such phenomena. Commonly, there are two types of IP. (i) The defending liquid is considered as incompressible and is trapped in the medium surrounded by the invader. The invader cannot penetrate into a trapped regime. This system is called the IP with trapping. (ii) The defending liquid is fully compressible, and therefore the invader can also enter a trapped regime. We will refer to this as the IP without trapping.

Recent years, several papers have dealt with IP with memory,[3] viscous forces,[4] correlated buoyancy,[5] and long-range correlations.[6] However, the porous media flow occurs in a discrete, highly chaotic network of pores and pore throats. Real porous media have a wide range of porosity or of the concentration of open pores (occupying probability). As this concentration P decreases, i.e., approaching the percolation threshold P_c, there appears a correlation length $\xi \sim |P - P_c|^{-\nu}$ such that the percolation lattice of open pores has a fractal geometry on length scales $L < \xi$. In many cases, physical phenomena exhibit a sharp crossover from a fractal behaviour ($L < \xi$) to that of a homogeneous random system ($L > \xi$). In the present study, the varying percolation clusters with varying occupation probabilities are constructed firstly. Then, the pattern structure, fractal dimension and chemical distance of an IP cluster are obtained by the IP algorithm.

In a square lattice (512×512) with a given percolation probability P, when $P \geq 0.593$, a spanning cluster occurs; when $P = 1$, the occupying ratio of the site reaches 100%. The connectivity of the porous media varies according to the values of P.[7–9] Those sites and clusters beyond the spanning cluster correspond to the non-connection areas of the sedimentary rocks. Our simulation is only concerned with the spanning cluster. To generate IP clusters in two-dimensional percolation clusters, we perform the following steps. (i) For a percolation probability P ($0.593 \leq P \leq 1$), a spanning cluster is constructed by the Hoshen–Kopelman algorithm.[10] (ii) We assign random numbers in the range $(0, 1)$ to each site of a spanning cluster. Then (iii) we select site of injection (the central site of the lattice and it belongs to a spanning cluster) for the invading fluid and sites of extraction in the spanning cluster for the defending fluid. (iv) We search along the perimeter of the invasion cluster for the site with the lowest random number. This perimeter site is added to the invasion cluster. Then we repeat (iii) and (iv) until the invasion fluid reaches an exit site.

In Fig. 1 we show two typical IP patterns in percolation clusters with $P = 0.8$ and 0.6. It is obvious that the larger the occupation probability P, the larger the sweep efficiency of IP in the percolation cluster. The reason for this is that the larger the P value is, the better the connectivity of the porous media could be. The pattern constructions of IP in percolation clusters are obviously different from that of viscous fingering (VF)[7,8] in percolation clusters. The reason for this is

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that the IP patterns in percolation clusters have more and larger trapping regions.

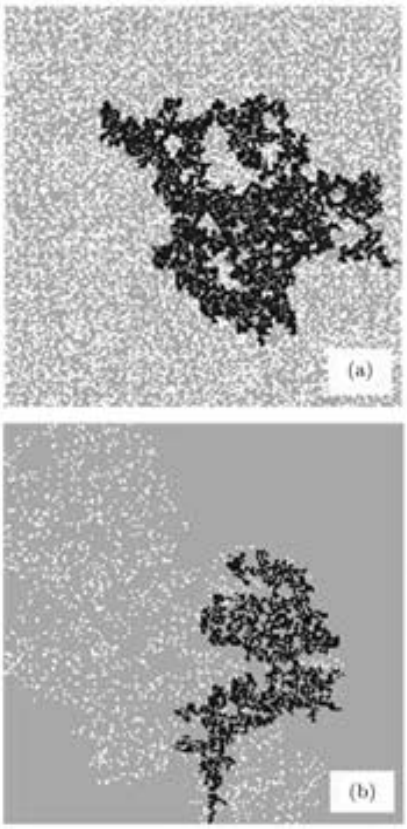


Fig. 1. Style IP patterns with (a) $P = 0.8$ and (b) $P = 0.6$ in a percolation cluster (a 512×512 lattice).

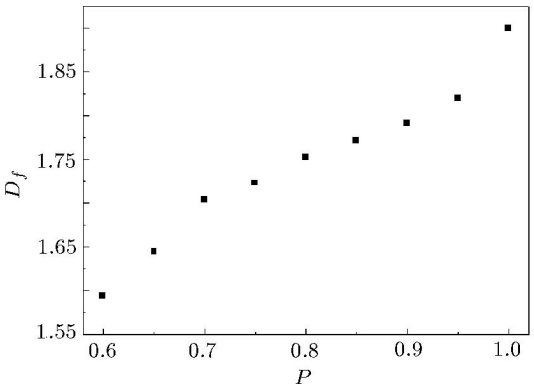


Fig. 2. Fractal dimension D as a function of occupation probability P.

To calculate the fractal dimension D_f of the IP clusters, the scale relations are to be taken into account. That is, $M \sim R_g^{D_f}$, where M represents the node numbers (or mass) occupied by the injected fluid in the area swept by IP and R_g is the gyration radius of the clusters. By means of the least squares method, D_f can be calculated. The numerical relations between the varying percolation probabilities P and the fractal dimension D_f are calculated as shown in Fig. 2, in which the data come from the results averaged 1000 realization. It can be clearly seen from Fig. 2 that the fractal dimension D_f increases with increasing the percolation probability P. $D_f = 1.89$ when $P = 1$. This result is the same as that of Refs. [11–13]. However, our work is different from Refs. [11–13] that studied only IP when $P = 1$. Our network model is also different from Ref. [6] that studied IP with long-range correlation. The fractal dimensions of the IP in percolation clusters are different from that of the VF[7,8] in percolation clusters, which vary with the change of viscosity ratio of two-phase fluid. The above-mentioned results mean that the geometry and the topology of the porous media have strong effects on the pattern structure of IP. Figure 3 shows the gyration R_g divided by M^{1/D_f} versus the number of sites M in the IP cluster. For large M, the rescaled value $R_g/M^{1/D_f}$ asymptotically approaches a constant value.[13] The horizontal lines are plotted for three different percolation probabilities P.

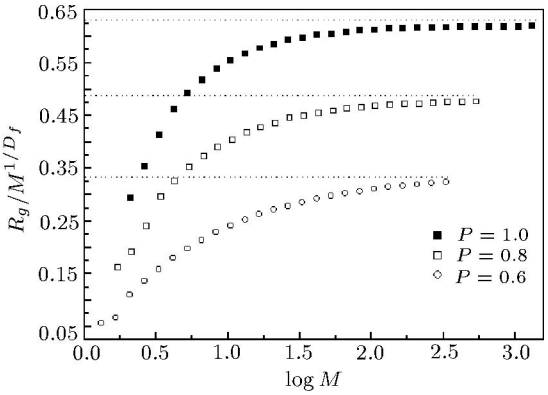


Fig. 3. Radius of gyration R_g divided by M^{1/D_f} versus the number of sites M in the IP cluster.

The fractal dimension, however, is not sufficient to characterize a fractal fully.[14] An important fractal substructure is the shortest path on the fractal between two distant fractal points. The length l of this path is called the chemical distance.[13,15] The asymptotic scaling behaviour of $M \sim l^{D_l}$ defines the chemical dimension D_l of the IP cluster. Hence, by combining of $M \sim l^{D_l}$ and $M \sim R_g^{D_f}$ one obtains that $l \sim R_g^{D_f/D_d} = R_g^{d_{\min}}$, where $d_{\min} = D_f/D_d$ is called the shortest path exponent. Figure 4 shows a relation that is the chemical distance l divided by $R_g^{d_{\min}}$ versus the gyration radius R_g in the IP cluster. When we have large R_g, we can obtain the rescaled value $l/R_g^{d_{\min}}$, in which $l/R_g^{d_{\min}}$ asymptotically approaches a constant value. Our simulation shows that $D_l = 1.671$, 1.52 and 1.31, $d_{\min} = 1.133$, 1.15 and 1.21, when $P = 1.0$, 0.8 and 0.6, respectively.

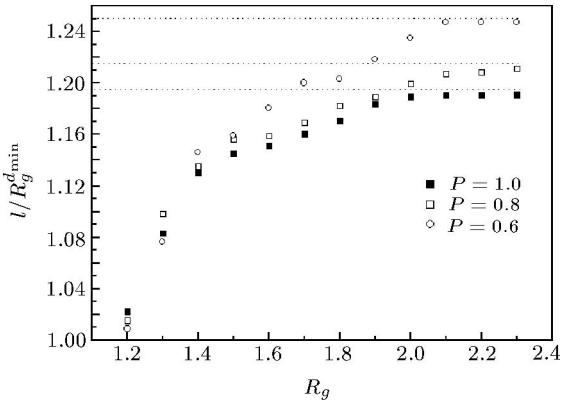


Fig. 4. Chemical distance l divided by $R_g^{d_{\min}}$ versus the gyration radius R_g in the IP cluster.

In summary, the porous media constructed here is the percolation cluster. The IP pattern construction without trapping in percolation cluster is studied. The geometry and the topology of the porous media have strong effects on the pattern structure of IP. The fractal dimension D_f increases with increasing the percolation probability P. For large M, the rescaled value is $R_g/M^{1/D_f}$, which asymptotically approaches a constant value. The chemical dimension D_l and the shortest path exponent $d_{\min}$ are obtained for the occupation probability $P = 1.0$, 0.8 and 0.6, respectively.

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