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Separation of Isotopes via Dynamical Delocalization *

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Based on the dynamical de-localization phenomenon we show that different isotopes of a material can be separated. We explain that the suggested scheme can be realized in the laboratory using presently available experimental facilities.

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Dynamical localization, in the language of its discoverers, manifests localization of a quantum particle and saturation of its width when its classical counter part displays diffusion and chaos. The connection of the phenomena with the Anderson localization and its experimental realization in atom optics opens the field for important applications. Atoms in phase and amplitude modulated standing wave field and atoms bouncing on modulated atomic mirror made up of evanescent wave field display the phenomena. In this Letter, we show that the phenomena of dynamical delocalization of atoms leads to develop new technique for isotope separation. Various separation techniques are developed based on the properties of atoms. Centrifuges and diffusion are based on difference of atomic masses.[1] Laser isotope separation relies on level shift of different isotopes in atomic or molecular spectral lines.[2] In some schemes, it combines high resolution selection with Zeeman and hyperfine quantum beats.[3] Wave packet isotope separation works on difference of period during quantum evolution.[4] Optical centrifuge for molecules relies on molecular dissolution through rotations. This scheme distinguishes the molecules from each other due to their moment of inertia, and acts as optical centrifuge.[5] Laser-induced isotope separation is realized experimentally.[7] It is shown that isotopes are easily separated by controlling their evolution in Fermi accelerator.[8]

In order to study evolution of atoms on the atomic mirror, the trap is switched off and the super cooled atoms move towards the mirror under influence of the gravitational field. The atomic mirror is an outcome of the total internal reflection of the laser light field from the surface of a dielectric prism. The light field hits the flat surface of the dielectric prism making an angle θ_1 with the normal to the mirror surface. The normal to the surface is in the direction of the gravitational field. As a result, we express the wave number along the surface of the prism as

$$k_{||} = k_0 n_1 \sin\theta_1, \tag{1}$$

and along the normal to the surface, in the z direction, as

$$k_{\perp} = ik_0\sqrt{n_1^2 \sin^2\theta_1 - 1}, \tag{2}$$

which is a complex number. Here k_0 is the wave number of the electromagnetic field in the free space, and n_1 is the index of refraction of the dielectric prism.

In the presence of the complex wavenumber, the electric field experiences an exponential decay in the z direction, outside the mirror surface. On considering V_0 as the electric potential at the prism-vacuum interface, we express the electric potential in the z direction above the prism surface as

$$V_{\rm opt} = V_0 e^{ik_{\perp}z} = V_0 e^{-\kappa z}, \tag{3}$$

where $k_{\perp} = i\kappa$. Hence, the atom, initially in the ground state and moving towards the prism, experiences an exponentially increasing force due to the optical field

$$F_{\rm opt} \equiv -\nabla V_{\rm opt} = V_0 \kappa e^{-\kappa z}, \tag{4}$$

as it moves towards the surface of the prism. Therefore, it turns back when its kinetic energy exhausts against the height of the potential. Hence, the evanescent wave serves as a mirror for the atom. The frequency of the external field, ω, is chosen to be large enough compared to the frequency ν of the dipole transition of the atom. The atom experiences this as a large detuning. The probability of spontaneous emission when the atom is in the excited state is inversely proportional to the square of the detuning $\delta = \omega - \nu$, that is,

$$P_{sp} = \gamma \frac{\Omega_{\rm max}^2}{4\delta^2} \tau_{\rm ref}. \tag{5}$$

Therefore, with the rise of the detuning the probability of finding the electron in the excited state becomes smaller, which reduces the spontaneous emission rate. Here γ is the decay constant of the excited state and $\tau_{\rm ref} = 2/\kappa v_z$ is the characteristic time of atom field interaction during the process of reflection off the mirror. The quantity v_z describes the velocity of the atom

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in z direction. Since the evanescent wave field $E(z)$ changes with the position as $e^{-\kappa z}$, as a consequence the Rabi frequency of the atom $\Omega = dE(z)/\hbar$ changes with the position of the atom. Hence, $\Omega_{\max}$ corresponds to the maximum Rabi frequency seen by the atom at its turning point on the surface of the mirror. Here the light shift due to the external field compensates for the kinetic energy of the atom of mass m, that is

$$\frac{\hbar\Omega_{max}^2}{4\delta} = \frac{1}{2}mv_z^2. \tag{6}$$

This dependence of spontaneous emission on detuning makes it possible to neglect the spontaneous emission in the experiments in the presence of a larger detuning. In our work we consider the limit of large detuning leading to a very little probability of finding the atom in the excited state. Therefore, we neglect the possibility of spontaneous emission and take the atom in the ground state. Far above the surface of the prism the evanescent light field is negligible and the atom undergoes an attractive constant gravitational force $F_{gr} = -\nabla(mgz) = -mg$. However, in the region of the evanescent wave field, the atom experiences an exponentially changing force as expressed in Eq. (4). The force due to the optical field is repulsive on the atom which is in the ground state due to blue detuning of the laser light field from the transition frequency of the atom, that is, $\delta > 0$. Hence, the gravitational field together with the exponential field makes a cavity for the atoms. The atom exhibits bounded motion inside this cavity.

At the onset of the experiment the atoms move along the $\tilde{z}$-direction under the influence of gravity and bounce off the atomic mirror. Since the atomic mirror results from a laser field incident on a glass prism under an angle of total internal reflection and creates an evanescent wave whose intensity $I(\tilde{z}) = I_0 \exp(-2k\tilde{z})$, decays over a distance k^{-1} outside from the prism. A sinusoidal modulation of the intensity with amplitude ϵ and frequency ω changes the intensity $I(\tilde{z})$ to $I(\tilde{z}, \tilde{t})$, i.e.

$$I(\tilde{z}, \tilde{t}) = I_0 \exp(-2k\tilde{z} + \epsilon \sin\omega\tilde{t}). \tag{7}$$

A large detuning between the laser light field and the atomic transition frequency ensures that the atom rarely leaves the ground state. The dynamics of the centre-of-mass motion of the atom in the ground state follows then from the Hamiltonian

$$H = \frac{\tilde{p}^2}{2m} + mg\tilde{z} + \frac{\hbar\Omega_{\text{eff}}}{4}e^{-2k\tilde{z}+\epsilon\sin\omega\tilde{t}}. \tag{8}$$

Here $\tilde{p}$ is the momentum of the atom of mass m along the $\tilde{z}$ axis, and g denotes the gravitational acceleration.

We introduce the dimensionless position and momentum coordinates $z \equiv \tilde{z}\omega^2/g$ and $p \equiv \tilde{p}\omega/(mg)$ and time $t \equiv \omega\tilde{t}$ together with the dimensionless intensity $V_0 \equiv \hbar\omega^2\Omega_{\text{eff}}/(4mg^2)$, steepness $\kappa \equiv 2kg/\omega^2$ and the modulation depth $\lambda \equiv \omega^2\epsilon/(2kg)$ of the evanescent wave. In these variables, the classical dynamics follows from the Hamiltonian equations of motion,

$$\dot{z} = p,$$

$$\dot{p} = -1 + \kappa V_0 \exp[-\kappa(z - \lambda\sin t)]. \tag{9}$$

The corresponding Quantum dynamics of the atoms provide a Lamb-Dicky parameter $\eta \equiv \hbar\omega^3/(mg^2)$.[8]

From the analytical study of the atomic dynamics, we know that the dynamical localization occurs within a window of modulation strength, i.e. $0.24 < \lambda < \lambda_c$. Above the critical modulation $\lambda_c(= \sqrt{\eta}/2)$, we can find quantum delocalization.[7,8] For two different isotopes of mass m_1 and $m_2 = m_1 + \Delta m$, two different critical modulation strengths occur, i.e. $\lambda = \lambda_{c_1}$ for atoms of mass m_1 and $\lambda = \lambda_{c_2}$ for atoms of mass m_2. We may express λ_{c_2} as

$$\lambda_{c_2} \approx \lambda_{c_1} - \Delta\lambda, \tag{10}$$

where $\Delta\lambda \equiv \dfrac{\Delta m}{2m}\lambda_{c_1}$. Hence, if we chose set of experimental parameters such that modulation strength λ is smaller than λ_{c_1} but greater than λ_{c_2}, atoms of the isotope of mass m_1 will stay localized, whereas the atoms of the other isotope of mass m_2 will show classical diffusion and spread above the modulated atomic mirror, as shown in Fig. 1. We know the location of the localized isotope, therefore, by using an intense laser we can extract them and store in a chamber.

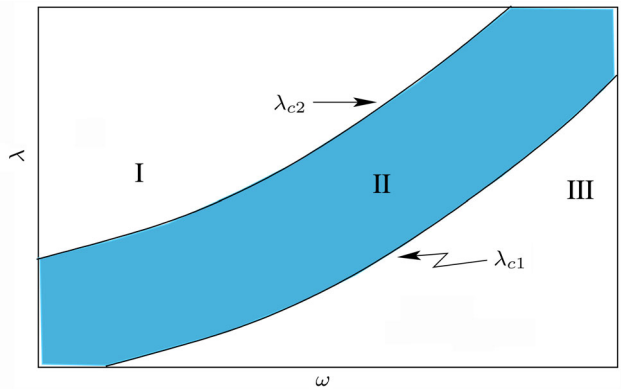

Fig. 1. Three dynamical regimes, in λ-ω plane, marked by two isotopes (in arbitrary units). In region I, two isotopes are dynamically localized as the values of frequency ω, corresponding to the values of λ smaller than λ_{c_2}. In region II, the lighter isotope is localized, whereas the heavier one is delocalized, making the isotope separation window. In region III, the two isotopes display dynamical delocalization as the values of ω correspond to larger values of modulation strength λ, here isotopes are taken beyond the separation window.

For the two isotopes, Br^{79} and Br^{81}, we can determine the two corresponding critical modulation

strengths as a function of frequency of external modulation, ω, measured in hertz. In experiment we may tune any modulation frequency from 0 to 2 MHz. Hence we find three different regimes: first, the localization regime, where the two isotopes are dynamically localized; second, delocalization and localization (DL) regime, where the lighter isotope is localized and the heavier delocalized; and third, delocalization regime, where the two isotopes display dynamical delocalization. We may set experimental parameters such that we find a modulation strength, λ, in DL-regime. As a result we observe that the lighter atoms of Br^{79} isotope is localized, and the heavier atoms of Br^{81} undergo diffusion, and spread over the atomic mirror.

Numerical investigation of the system estimates the quantum mechanical momentum space distribution for the lighter isotope as exponentially localized

$$\psi(p) \sim e^{-p/\ell}, \tag{11}$$

where ℓ describes localization length. However, the estimated behaviour in position space is

$$\psi(z) \sim e^{-\sqrt{z}/\ell}. \tag{12}$$

For the heavier isotope the quantum delocalization takes place and momentum distribution obeys the Maxwellian distribution

$$P_{cl}(p) = \left(1/\sqrt{2\pi E_T}\right) \exp[-p^2/(2E_T)], \tag{13}$$

as in Brownian motion. Here $E_T \equiv \omega^2 k_B T/(mg^2)$ is the scaled effective thermal energy, k_B and T are Boltzmann's constant and the effective temperature, respectively. The corresponding position distribution follows the exponential barometric distribution

$$P_{cl}(z) = (1/E_T) \exp[-z/E_T]. \tag{14}$$

The characteristics of quantum delocalization are associated with the Boltzmann statistics of the phase space distribution which reads

$$P(z,p) = N \exp\{-[p^2/2 + V_{app}(z)]/E_T\}. \tag{15}$$

Here V_{app} is the potential comprising gravitational potential and the optical potential, and $N \equiv (2\pi E_T^3)^{-1/2}$ is the normalization constant as in classical domain.

We have explained a new method to separate different isotopes of a material based on dynamical effects in modulated gravitational cavity. Isotopes of a material display different dynamical localization limits, hence the difference of their masses leads to quantum diffusion regimes differently. Numerical calculations show that the square of the width in momentum space of the localized lighter isotope and its width in position space display a saturation beyond quantum break time. In contrast, the heavier isotope which observes quantum delocalization exhibits a gradual rise of the square of its width in time, following a linear relation, however it is the width in position space which displays the same behaviour.

The technique proposed here can be applied to different materials, and further, can be extended to separate more than two isotopes of a material. The $\Delta\lambda$ regime appears so small that it is directly proportional to the mass difference Δm, however its direct proportionality to λ_{c_1} may lead to tune it to larger values, for example, by choosing larger value of frequency ω.

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