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Electronic States of Elliptical Quantum Rings Subjected to a Magnetic Field *

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We calculate the energy states and Aharonov–Bohm oscillations of an electron in elliptical quantum rings in the presence of a uniform magnetic field by using an exact numerical diagonalization. The calculated results show that the elliptical quantum rings are flatter, larger amplitudes and periods of the Aharonov–Bohm oscillations are observed. In addition, in the limits of a circular quantum ring, the results of our approach are in good agreement with those of earlier theories.

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With the development of molecular-beam epitaxy and metal organic chemical-vapour deposition, we can now precisely control the distance between two material layers of particular compositions and doping profiles to a few angstroms. In recently, semiconductor quantum ring (QR) samples in the quantum confined regime have been fabricated on AlGaAs–GaAs heterostructures containing a two-dimensional electron gas. QR nanostructures have been intensively investigated for their potential applications in quantum information processing and low-dimensional semiconductor optoelectronic devices. QRs are promising quantum structures because of their particular topology structures, which leads to the Aharonov–Bohm (AB) effect. It is well known that the interference phenomenon, which depends on persistent currents and magnetization, can be modulated by tailoring the shape and the size of the QRs. Since the capabilities to control the electron number in the QRs and to modify its geometry are essential for observing and exploiting new phenomena, systems with these capabilities have attracted much theoretical interest.[1−23] The simplest model is a circular quantum ring (CQR) with a uniform width. In recently, many papers mainly focus on systems with perfect ring geometry, where the confinement potential does not depend on the azimuthal coordinate. However, distorted QRs must be considered because imperfections occur in growth and fabrication processes. QRs of arbitrary shape in the presence of a threading magnetic field have also been considered. Although a detailed study of the electron–electron interaction in the two-, three-, and few-electron cases will be the most interesting theoretical features of these QRs, the analysis of the one-body confinement is a necessary first step.

In this Letter, we report the calculation of electronic states in elliptical quantum rings (EQRs) in the presence of a magnetic field by using an exact diagonalization. The main aim of this study is to achieve the energy spectrum of the EQRs. Noncircular rings may be intentionally or nonintentionally produced for experimental studies and electronic transmissions through curved waveguides. Thus an accurate knowledge of EQRs is needed for further investigation.

We consider the states of an electron located on an EQR. An external magnetic field is applied perpendicularly to the plane of the ring. The vector potential of the magnetic field is taken as $\boldsymbol{A} = \frac{1}{2}\boldsymbol{B} \times \boldsymbol{r}$ and the Hamiltonian of an electron is given by

$$H = \frac{1}{2m^*}\left(\boldsymbol{P} + \frac{e}{c}\boldsymbol{A}\right)^2, \tag{1}$$

where m^* is the effective electron mass. Here we have chosen, without loss of generality, the origin of the coordinate system to be the centre of the EQR. The electron position is denoted by $\boldsymbol{r}$. In what follows, we assume that the EQR has infinitesimal thickness. The Hamiltonian of the system can be rewritten as

$$H = -\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} - i\gamma\frac{\partial}{\partial\phi} + \frac{1}{4}\gamma^2\rho^2, \tag{2}$$

where $\rho^2 = (ach\xi\cos\phi)^2 + (ash\xi\sin\phi)^2$, a and ξ are the parameters of the EQR. In our calculations, the units of distance and energy are the electronic Rydberg and Bohr radius in GaAs, $R_B = \frac{m^*e^4}{2\hbar^2\varepsilon_0^2}$, $a_0^* = \frac{\varepsilon_0\hbar^2}{m^*e^2}$, respectively; $\gamma = \frac{e\hbar B}{2m^*cR_B}$ is a dimensionless measure of the magnetic field.

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In order to explore the effects of the magnetic field on the energy state, the wave function of the electron in an EQR can be written in the form

$$\psi = \frac{1}{\sqrt{2\pi}} \sum_m C_m e^{im\phi}. \tag{3}$$

The coefficients C_m can be obtained by using an exact numerical diagonalization. In the calculation, the index m has to be truncated. In our calculations, it is found that the electronic energy states are almost constant when the index $|m|$ is larger than 10, if other parameters of an EQR are kept constant at the same time. In order to reduce the dimension of the matrix in the computation and improve the computational speed, we can take $m = -10, -9, \cdots, 9, 10$. In addition, other basic parameters of GaAs are $m^* = 0.067m_0$, $\varepsilon_0 = 12.5$, $a_0^* = 98\,\text{Å}$, $R_B = 5.83\,\text{meV}$.

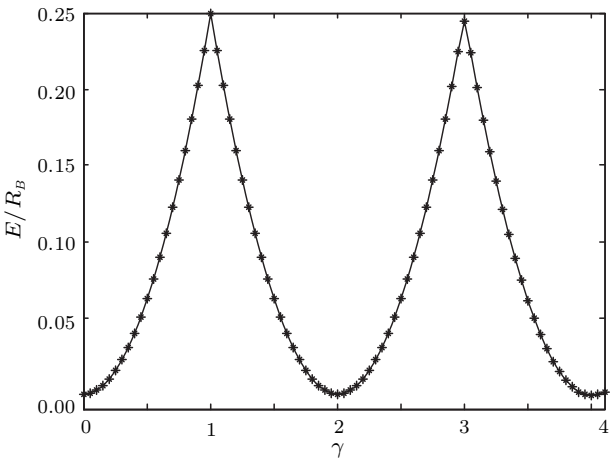


Fig. 1. Variation of the energy an electron located on the QR as a function of the magnetic field. The stars are the limit results of the EQR; the solid curve is the result of the CQR taken from Ref. [24].

The CQR is a limit case of the EQR. we compare the ground state energy of an electron located on an EQR as a function of the magnetic field with the results for an electron located on a CQR for $R = 1.0a_0^*$. It is exciting that the limit results of the EQR are in good agreement with those of Korkusinski *et al.*[24] In Fig. 1, the solid curve is the results taken from Ref. [24], and the stars are the limit results of the EQR for $a = 1.4142 \times 10^{-4}$ and $\xi = 9.5569$. The typical AB oscillations of the ground state energy with a period 2γ is clearly observed. The maximum values of the ground state energy and the oscillation periods for two curves depend on the values of the applied magnetic field. This phenomenon stems from the complicated interplay of geometrical confinement and the magnetic-field confinement. The AB effect demonstrates that the electromagnetic potentials, rather than the electric and magnetic fields, are the fundamental quantities in quantum mechanics. It is a quantum mechanical phenomenon by which a charged particle is affected by magnetic fields in regions from which the particle is excluded. Thus the AB oscillations of the ground state energy as a function of the magnetic field can be easily understood.

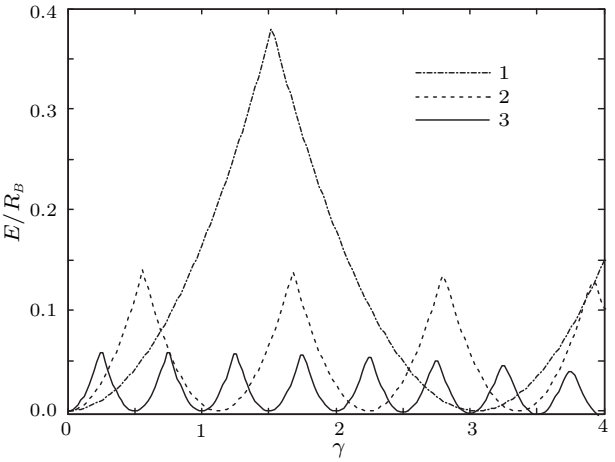


Fig. 2. Ground state energy of an electron located on an EQR as a function of magnetic field with $a = 2.0000 \times 10^{-4}$. The labels 1, 2, and 3 correspond to parameter $\xi = 9.0, 9.5, 9.9035$, respectively.

In order to better understand the difference between the EQR and the CQR, in Fig. 2, we display the variation of the ground state energy as a function of the magnetic field with $a = 2.0000 \times 10^{-4}$, when the parameters ξ are taken to be 9.9035, 9.5, and 9.0, respectively. As is expected the ground state energy exhibits AB oscillations which are periodic in flux. The amplitudes of the oscillations of the ground state energy of the EQR depend on complex interplay of the geometrical confinement and the magnetic-field confinement. It is interesting to note that our results of the EQR are in excellent agreement with the results of the CQR with $R = 2.0a_0^*$, when the parameters ξ is taken to be 9.9035 (corresponding to the limit case of CQR). It is clearly observed that the maximum values of the ground state energy are $0.0567R_B$, and the oscillation periods are 0.5γ. All those agree with the accurate results of Ref. [25]. In addition, the periods and the amplitudes of the AB oscillations become larger for smaller parameter ξ. This is due to the fact that the effect of the magnetic field on the ground state energy becomes weaker for the flatter EQR, which is equivalent to increasing the geometrical confinement and the electron can not occupy a larger free space. On the other hand, this can be explained by the fact that the geometrical confinement confines the electron more strongly than the magnetic confinement. High magnetic field reduces the electron cyclotron radius, increases the effect on the ground state energy, especially in the weak geometrical confinement. It is concluded that a flatter EQR significantly can enhance

the maximum of the ground state energy and the periods of oscillation. A distorted QR has a structural defect, which may occur in fabricated, grown, or field-effect QRs. Many QRs are elliptical, so the observed experiment results will be different from the theoretical calculation of the CQR. Our theoretical results are in good agreement with the experimental results by adjusting the parameters ξ and a.

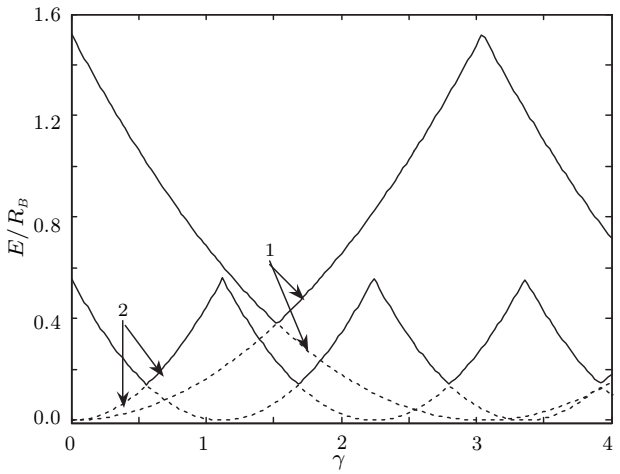


Fig. 3. The energy of the ground state (dashed line) and the first excited state (solid lines) of an electron located on an EQR as a function of the magnetic field with $a = 2.0000 \times 10^{-4}$. The labels 1 and 2 correspond to parameter $\xi = 9.0, 9.5$, respectively.

The energy of the ground state and the first excited state of an electron located on an EQR as a function of the magnetic field for several parameters ξ are presented in Fig. 3. The AB oscillation can also be observed in the first excited state. As is expected, the different curvature of the EQR leads to occurrences of the AB oscillations with different periods and the amplitudes. It is well known that the energy levels of the CQR are intersecting parabolas. In distorted rings, gaps will be open at the points of intersection of the parabolas. The effect is qualitatively similar to the effect of disorder. The effect of curvature on the electronic energy in EQR with an electron becomes more intensive than the effect of the magnetic confinement. These results can be obtained by using an exact numerical diagonalization. The electronic energy oscillations in an EQR have a sawtooth form, when parameters ξ is taken as 9.0. The distortion of the EQR produces a smoothing of the oscillations due to the opening of energy gaps at the intersection points. The curvature of the EQR changes the electronic energy dependence of the magnetic field.

In Fig. 4 we display the behaviour of ground state energy as a function of the curvature ξ in the EQR with $a = 2.0000 \times 10^{-4}$ and $\gamma = 3.0$. With the increasing ξ, the ground state energy is more insensitive to the variation of ξ for small curvature, while the ground state energy increases sharply until a maximum value is obtained for large curvature, and then it begins to drop quickly. It should be noted that a very small value in curvature has produced a quenching of the AB oscillation of the ground state energy. This is due to the fact that the wider regions of the unflattering EQRs correspond to a weaker effective geometrical potential for the electron. In addition, the electron is strongly confined by the geometrical confinement for small ξ, while the contribution of the magnetic field to the ground state energy can be neglected. In practice, it is not a feasible task to control the width of a QR with a high accuracy. However, in some cases, a partial compensation between curvature and width might be attained. On the other hand, measurable variations in the width should noticeably affect the electronic states, and hiding the interesting effects of the curvature. Therefore, the effect of the curvature of the EQR on the electronic states is more dramatic for non-width rings, which is the focus of this work.

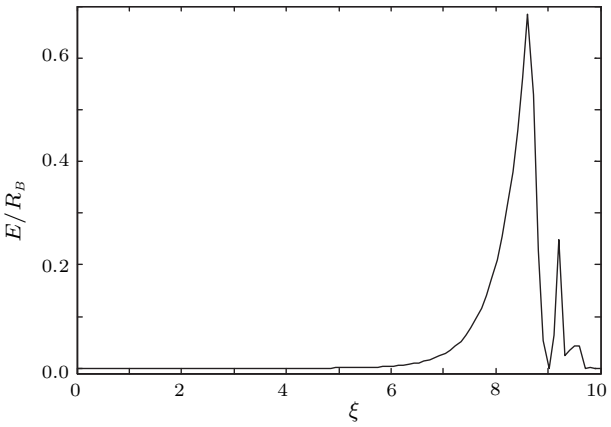


Fig. 4. Variation of the ground state energy of an electron located on an EQR as a function of the curvature ξ with $\gamma = 3.0$.

In summary, we have presented the ground and the first excited state energies of an electron located on an EQR for different curvature in the presence of a uniform magnetic field by using an exact numerical diagonalization procedure. In the limiting case of a CQR, it is worth remarking that our results are in excellent agreement with those in Ref. [24]. By calculating the effect of the magnetic field on the ground and the first excited state energies for several curvature ξ, the typical AB oscillations of the energy levels are clearly observed. The AB effect demonstrates that the electromagnetic potentials are the fundamental quantities in quantum mechanics. For small curvature ξ, larger amplitudes and periods of the oscillation can be obtained because of stronger geometrical confinement. This is due to the fact that the geometrical confinement confines the electron more strongly than the magnetic confinement. Additionally, the result is

useful for deeper understanding of the effects of EQRs on the electronic spectrum.

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