Radiatively Inefficient Accretion Flows with Outflow *

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We construct a family of self-similar solutions to describe radiatively inefficient accretion flows with outflow. We show that outflow has important effects on the dynamical structure of accretion flows. In particular, we prove that outflow can be an effective mechanism of removing the released gravitational energy of accreted gas.

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Outflows in the form of winds or jets are observed in many astrophysical systems such as active galactic nuclei, close binaries, and young stellar objects. All these systems are believed to be powered by accretion processes. The generally accepted idea is that outflows are originated from the accreted matter. However, it remains unclear as how some part of accreted materials (inflows) can be transferred into outflows. On the other hand, in many systems containing accreting galactic and extragalactic black holes, the accretion flow seems to be radiatively inefficient, since the luminosity of these systems is much lower than that predicted by the standard Shakura–Sunyaev disc model.[1] The most famous model of radiatively inefficient accretion flows (RIAFs) is the advection-dominated accretion flow (ADAF). In this model, most of the released gravitational energy of accreted matter is advected inwards in the form of internal (thermal) energy and is finally absorbed by the black hole, rather than being radiated away. Reference [2] first noticed that in its self-similar solution of ADAF the Bernoulli constant has positive values, and suggested that this implies the possibility of outflow formation, since the gas with a net positive energy is able to escape to infinity. However, the ADAF solution of Ref. [2] itself is inconsistent with the case of any outflow, because it assumed a constant mass accretion rate and did not allow for any mass loss of the accretion flow. Later, Ref. [3] proposed another model of RIAFs, called the adiabatic inflow-outflow solutions (ADIOS), in which a large fraction of the accreted matter is driven away in the form of outflow because of the positive energy, and the mass accretion rate is no longer a constant and is varying with the radius accordingly. Reference [4] criticized ADIOS model and showed that, different from the case of self-similar solutions, in global solutions of accretion flows with low viscosity the Bernoulli function (not a constant) is negative; and that even a positive Bernoulli function is only a necessary and not a sufficient condition for outflow formation. It should be stressed that both Refs. [3,4] were only for the low viscosity case, and the question still remains whether purely hydrodynamic accretion flows with high viscosity are able to produce outflows, or outflow formation must invoke non-hydrodynamic factors such as magnetic fields, radiation, etc. Still, another model of RIAFs is called the convection-dominated accretion flow (CDAF).[5] It was also realized by Ref. [2] that ADAFs are possible to be convectively unstable in the radial direction because of the inward increase of the entropy of accreted gas. In the CDAF model, the radiative inefficiency of accretion flows is due to neither advection nor outflow; this is because that convection makes an extremely small radial velocity and an extremely small accretion rate of the flow.

In a previous paper we presented a model of RIAFs with outflow.[6] The model can be regarded as an extension of the ADIOS model. In this Letter, we continue to investigate the effect of outflow on the dynamics of accretion flows. We construct a family of solutions for RIAFs in the self-similar form and find that the ADAF, ADIOS, CDAF and our model can be recovered. Therefore the comparisons among these models can be made.

RIAFs are described by a set of height-averaged equations based on the conservation laws of mass, radial momentum, angular momentum, and energy (e.g. Ref. [2]):

$$\dot{M}(R) = 4\pi RH\rho|v|, \tag{1}$$

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$$v\frac{dv}{dR}=(\Omega^2-\Omega_K^2)R-\frac{1}{\rho}\frac{d}{dR}(\rho c_s^2), \tag{2}$$

$$\frac{d(\Omega R^2)}{dR}=\frac{1}{RH\rho v}\frac{d}{dR}\Big(\mu R^3 H\rho\frac{d\Omega}{dR}\Big), \tag{3}$$

$$(3+3\varepsilon)H\rho v\frac{dc_s^2}{dR}-2Hvc_s^2\frac{d\rho}{dR}=2f\mu R^2H\rho\Big(\frac{d\Omega}{dR}\Big)^2, \tag{4}$$

where $\dot{M}$ is the mass accretion rate, R is the radius, H is the half vertical scale height defined as $H=c_s/\Omega_K$, ρ is the density, v is the radial velocity, Ω is the angular velocity, $\Omega_K=(GM/R^3)^{1/2}$ is the Keplerian angular velocity with M being the mass of central black hole, c_s is the isothermal sound speed, $\mu=\alpha c_sH$ is the kinematic viscosity coefficient with a being the dimensionless viscosity parameter, $\varepsilon\equiv(5/3-\gamma)(\gamma-1)$ with γ being the ratio of specific heats, and f is the advection parameter, in the extreme limit of no radiative cooling $f=1$, while in the opposite limit of very efficient cooling $f=0$. Note that Eqs. (2)–(4) are the same as in Ref. [2], but Eq. (1) is not. The requirement of constant $\dot{M}$ for the ADAF and CDAF solutions is relaxed here in order to allow for outflow.

We search for self-similar solutions of Eqs. (1)–(4) with scale laws similar to those in Ref. [2] except that we must allow for mass loss,

$$\begin{aligned}\rho&=c_1R^{-n},\\ v&=c_2R\Omega_K,\\ \Omega&=c_3\Omega_K,\\ c_s&=c_4R\Omega_K,\end{aligned} \tag{5}$$

where c_1, c_2, c_3, and c_4 are the constants to be determined, the exponent n measures the degree of mass loss due to outflow (as discussed in the following). Accordingly, the specific total energy e (or called the Bernoulli function, which is not constant along a fluid stream for a viscous flow) can be scaled as

$$\begin{aligned}e&\equiv\frac{1}{2}[v^2+(R\Omega)^2]+\frac{\gamma c_s^2}{\gamma-1}-\frac{GM}{R}\\ &=\Big[\frac{1}{2}(c_2^2+c_3^2)+\frac{\gamma c_4^2}{\gamma-1}-1\Big](R\Omega_K)^2.\end{aligned} \tag{6}$$

Substituting Eqs. (5) into Eqs. (1)–(4), we obtain the following four algebraic equations for c_1, c_2, c_3, and c_4,

$$\begin{aligned}&\dot{M}(R)=4\pi\sqrt{GM}c_1|c_2|c_4R^{3/2-n},\\ &\frac{c_2^2}{2}+c_3^2+c_4^2(n+1)-1=0,\\ &\frac{c_2}{2}-\frac{3}{2}c_4^2\alpha(n-2)=0,\\ &-9c_3^2f\alpha+c_2[4n-6(\varepsilon+1)]=0,\end{aligned} \tag{7}$$

which can be easily solved as

$$\begin{aligned}c_1&=\frac{\dot{M}_{\rm out}}{4\pi\sqrt{GM}R_{\rm out}^{3/2-n}|c_2|c_4},\\ c_2&=\frac{1}{\alpha}(-A-\sqrt{A^2+2\alpha^2}),\\ c_3^2&=\frac{1}{\alpha^2}(-AB-B\sqrt{A^2+2\alpha^2}),\\ c_4^2&=\frac{1}{\alpha^2(n+1)}[-A^2+AB-(A-B)\sqrt{A^2+2\alpha^2}],\end{aligned} \tag{8}$$

where

$$\begin{aligned}A&\equiv\frac{4n-6(\varepsilon+1)}{9f}+\frac{n+1}{3(n-2)},\\ B&\equiv\frac{4n-6(\varepsilon+1)}{9f},\end{aligned}$$

$R_{\rm out}$ is some outer radius of the flow, and $\dot{M}_{\rm out}$ is the mass accretion rate at $R_{\rm out}$.

The first formula of Eqs. (7) presents that $\dot{M}\propto R^{3/2-n}$ and is similar to $\dot{M}\propto R^p$ in Ref. [3] if we write $p=3/2-n$. As argued in Ref. [3], the value of p should be restricted as $0<p<1$ (i.e. $1/2<n<3/2$) in order to ensure $\dot{M}$ to decrease with decreasing radius, while the released energy can still increase. The limit $p=0$ ($n=3/2$) corresponds to a constant $\dot{M}$ and is the case of the ADAF model.[2] The opposite limit $p=1$ ($n=1/2$) gives $\rho\propto R^{-1/2}$ and is just the $1/2$ law of radial density distribution of the CDAF model.[5] No outflow occurs in these two models. The value of p between these two limits measures the strength of outflow, and a larger p denotes a stronger outflow, since $\dot{M}$ is to decrease more rapidly with the decreasing radius. The ADIOS model is for $0<p<1$, i.e. with outflow. Therefore, all these three well known models can be regarded as special classes of our solution family of Eqs. (5) and (8). What is new here is that the ADIOS model is restricted to the low viscosity case ($\alpha\sim0.01$), while we use our solutions to describe RIAFs with high viscosity ($\alpha\sim0.1$) as well, since this is a more plausible case for outflow formation.

Figure 1 shows the radial velocity v (with a negative sign), angular velocity Ω, sound speed c_s, and total energy e as functions of the outflow parameter p, with $\gamma=1.4259$ and $f=0.9$. The solid and dashed lines are for $\alpha=0.3$ and $\alpha=0.01$, respectively. It is seen that in the low viscosity case v is very small, $|v|\ll R\Omega$, which is just the assumption of the ADIOS model; while in the high viscosity case v becomes comparable with the rotational velocity, $|v|\ll R\Omega$, and this is reasonable because the inward motion of accreted gas is caused by viscosity. As p increases, Ω increases, while c_s (i.e. the internal energy) and e decrease. This means that more internal energy is removed out from the accreted gas by a stronger outflow,

rather than being advected inward towards the central black hole as in the ADAF model. The result that e decreases with increasing p confirms the connotative idea of Ref. [3], in which the mass outflow diminishes the total energy (thus diminishes the capability of further escape) of accreted gas.

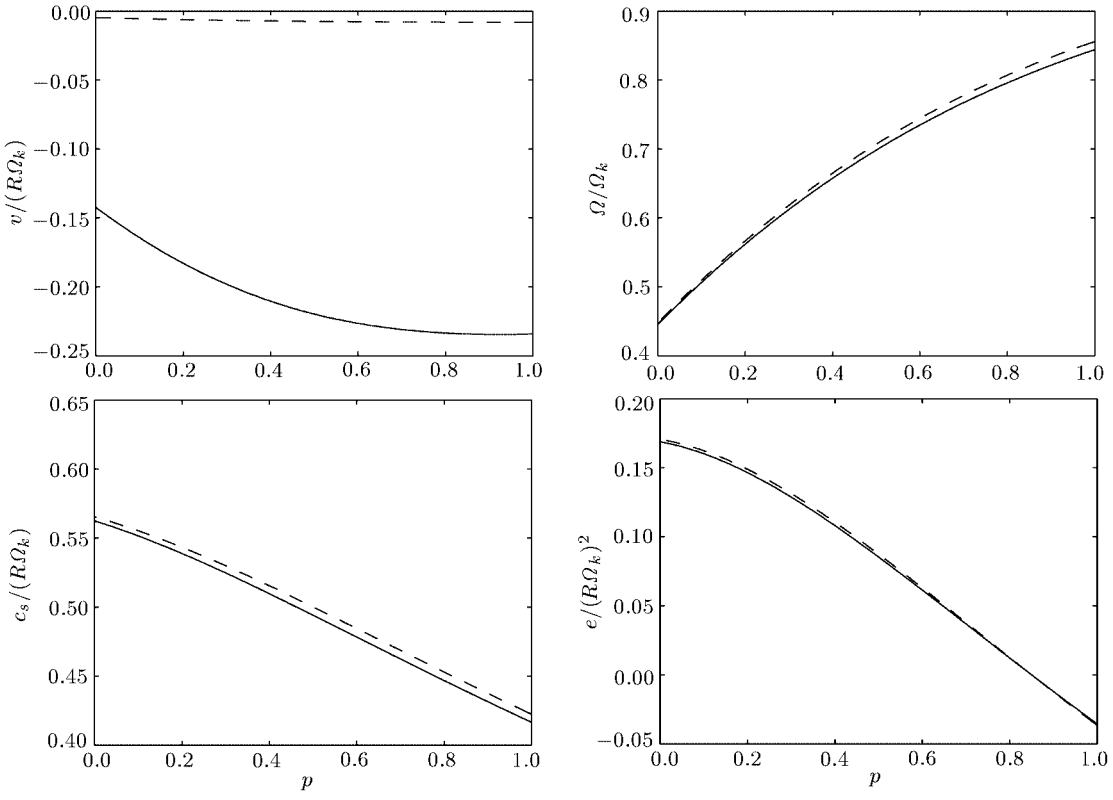


Fig. 1. The radial velocity v, angular velocity Ω, sound speed c_s, and specific total energy e as functions of the outflow parameter p, with the ratio of specific heats $\gamma = 1.4259$ and the advection parameter $f = 0.9$. The solid and dashed lines are for the viscosity parameter $\alpha = 0.3$ and $\alpha = 0.01$, respectively.

Figure 1 also indicates that the influence of viscosity on v is strong, but that on Ω, c_s, and e is very weak. Showing the above properties, we linearize Eqs. (8), keeping only the first order of α/A and neglecting all higher orders (both $\alpha = 0.01$ and 0.3 satisfy $\alpha/A < 1$), and obtain the following approximate formulae,

$$\begin{aligned}
&\frac{v}{R\Omega_K} \approx \frac{\alpha}{A},\\
&\frac{\Omega}{\Omega_K} \approx \sqrt{\frac{B}{A}},\\
&\frac{c_s}{R\Omega_K} \approx \sqrt{\frac{A-B}{A(n+1)}},\\
&\frac{e}{(R\Omega_K)^2} \approx \frac{B}{2A} + \frac{\gamma(A-B)}{(\gamma-1)A(n+1)} - 1. \qquad (9)
\end{aligned}$$

It is seen that v is directly proportional to α, but α does not appear in the expressions of Ω, c_s, and e, i.e. viscosity affects these three quantities only on higher orders.

We now investigate the effect of outflow on energy release of accreted gas. Consider a radial interval of the accretion flow between an inner radius $R_{\rm in}$ and an outer radius $R_{\rm out}$. The energy released within this interval is

$$\Delta E_n = \dot{M}_{\rm out}\Delta\Phi - \int_{R_{\rm in}}^{R_{\rm out}} d\dot{M}\delta\Phi, \qquad (10)$$

where $\dot{M}_{\rm out}$ is given by the first formula of Eqs. (8); and $d\dot{M}$, $\Delta\Phi$, and $\delta\Phi$ are the differential mass accretion rate at R, the difference of gravitational potential energy within the radial interval $[R_{\rm in}, R_{\rm out}]$, and the difference of potential energy within $[R_{\rm in}, R]$, and are expressed as

$$\begin{aligned}
&d\dot{M} = 4\pi\sqrt{GM}c_1|c_2|c_4(3/2-n)R^{1/2-n}dR,\\
&\Delta\Phi = -GM\Big(\frac{1}{R_{\rm out}} - \frac{1}{R_{\rm in}}\Big),\\
&\delta\Phi = -GM\Big(\frac{1}{R} - \frac{1}{R_{\rm in}}\Big),
\end{aligned}$$

respectively. This released energy is a function of n, i.e. it is related to the strength of outflow. If there is no outflow ($n = 3/2$, the ADAF model), then $d\dot{M} = 0$, and the released energy is

$$\Delta E_{ADAF} = \dot{M}_{\rm out}\Delta\Phi. \qquad (11)$$

The energy carried away by outflow should be the sum of kinetic energy and internal energy of the gas. Within the same radial interval $[R_{\rm in}, R_{\rm out}]$ this lost energy can be written as

$$Q^-_{\rm outflow} = \int_{R_{\rm in}}^{R_{\rm out}} \left[\frac{1}{2}(\Omega^2 R^2 + v^2) + \frac{\gamma c_s^2}{\gamma - 1}\right] \frac{d\dot{M}}{dR} dR. \tag{12}$$

Combining Eqs. (10)–(12), we have the following two ratios:

$$\frac{\Delta E_n}{\Delta E_{ADAF}} = 1 + \frac{\int_{R_{\rm in}}^{R_{\rm out}} \left(\frac{1}{R} - \frac{1}{R_{\rm in}}\right)\left(\frac{3}{2} - n\right) R^{1/2-n} dR}{R_{\rm out}^{3/2-n}\left(\frac{1}{R_{\rm in}} - \frac{1}{R_{\rm out}}\right)}, \tag{13}$$

$$\frac{Q^-_{\rm outflow}}{\Delta E_{ADAF}} = \frac{\frac{3/2-n}{1/2-n}\left[\frac{1}{2}(c_2^2 + c_3^2) + \frac{\gamma c_4^2}{\gamma - 1}\right](R_{\rm out}^{1/2-n} - R_{\rm in}^{1/2-n})}{R_{\rm out}^{3/2-n}\left(\frac{1}{R_{\rm in}} - \frac{1}{R_{\rm out}}\right)}. \tag{14}$$

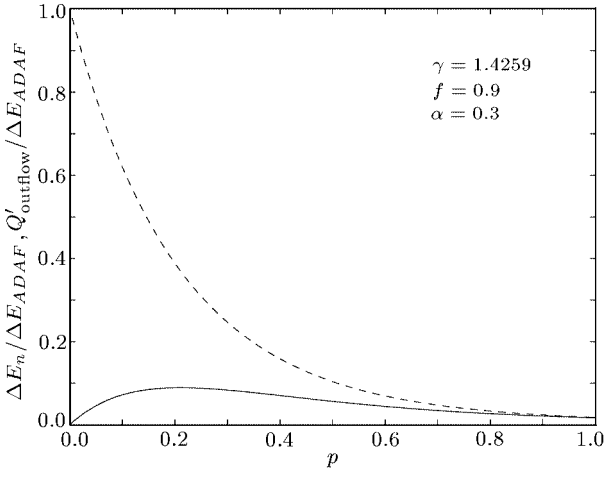


Fig. 2. Ratios $\Delta E_n/\Delta E_{ADAF}$ (dashed line) and $Q^-_{\rm outflow}/\Delta E_{ADAF}$ (solid line) for $\alpha = 0.3$, $R_{\rm in} = 3R_g$, $R_{\rm out} = 1000R_g$, also with $\gamma = 1.4259$ and $f = 0.9$.

It is seen that the ratio $\Delta E_n/\Delta E_{ADAF}$ is independent of α, but the ratio $Q^-_{\rm outflow}$ is related to a via c_2, c_3, and c_4. Figure 2 shows these two ratios as functions of $p(\pi 3/2 - n)$ for $\alpha = 0.3$, $R_{\rm in} = 3R_g$ ($R_g = 2GM/c^2$ is the gravitational radius), $R_{\rm out} = 1000R_g$, again with $\gamma = 1.4259$ and $f = 0.9$. The important information drawn from the figure is that in the weak outflow case ($p < 0.5$) the energy carried away by outflow $Q_{\rm outflow}$ is much smaller than the released energy ΔE_n, i.e. most of the released energy is advected inwards with accreted gas; while in the strong outflow case ($p > 0.5$), $Q_{\rm outflow}$ becomes comparable with ΔE_n, and when the outflow is very strong ($p > 0.8$) almost all of the released energy is removed away by outflow.

Figure 2 proves that outflow can indeed be an effective cooling mechanism of accretion flow. The observed low luminosity of many astrophysical systems indicates that radiative cooling in the accretion flow in these systems is inefficient (RIAFs). The released gravitational potential energy of accreted gas has been suggested to be either advected inwards to the black hole (the ADAF model), or transformed outwards by convection (the CDAF model). As an alternative, we have shown that outflow can remove the released energy very efficiently, and the basic idea of the ADIOS model is further confirmed here, especially for the high viscosity case. In view of the fact that outflows have often been observed in accretion-powered astrophysical systems, our results should have important implications.

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