Auto-Bäcklund Transformation and Soliton-Type Solutions of the Generalized Variable-Coefficient Kadomtsev–Petviashvili Equation *

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Using the truncated Painlevé expansion, an auto-Bäcklund transformation and soliton-type solutions of the generalized variable-coefficient Kadomtsev–Petviashvili (GKP) equation are obtained by symbolic computation. Since the cylindrical Korteweg-de Vries (cKdV) equation, the cylindrical KP (cKP) equation and the generalized cKP (GcKP) equation are all special cases of the GKP equation, we can also obtain the corresponding results of these equations.

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In recent years, research directly seeking exact solutions of nonlinear evolution equations (NLEEs) has become more attractive, because of the applications in many important scientific problems and the availability of symbolic computation.[1−6]. There has been considerable interest in the KP equation, which arises in many physical applications including shallow water waves and plasma physics. The generalized variable-coefficient Kadomtsev–Petviashvili (GKP) equations have been proposed in Ref. [7-9], which provides a more realistic model equation than the KP equation itself such as in the canonical and cylindrical cases.

Here we consider the GKP equation in the form[8]

$$(u_t + uu_x + u_{xxx})_x + a(y,t)u_x + b(y,t)u_y + c(y,t)u_{yy} + d(y,t)u_{xy} + e(y,t)u_{xx} = 0, \tag{1}$$

where $a(y,t)$, $b(y,t)$, $c(y,t)$, $d(y,t)$ and $e(y,t)$ are sufficiently differentiable analytic functions.

Equation (1) contains the following important cases:

(1) If $b(y,t) = c(y,t) = d(y,t) = e(y,t) = 0$ and $a(y,t) = \frac{1}{2\,t}$, Eq. (1) becomes the cKdV equation[8,10−12]

$$u_t + uu_x + u_{xxx} + \frac{1}{2\,t}u = 0, \tag{2}$$

(2) If $b(y,t) = d(y,t) = e(y,t) = 0$, $a(y,t) = \frac{1}{2\,t}$, $c(y,t) = \frac{3\sigma^2}{t^2}$, Eq. (1) becomes the cKP equation[8,10,13]

$$(u_t + uu_x + u_{xxx})_x + \frac{1}{2\,t}u_x + \frac{3\sigma^2}{t^2}u_{yy} = 0, \tag{3}$$

where $\sigma^2 = \pm 1$.

(3) If $b(y,t) = 0$, $a(y,t) = \frac{1}{2\,t}$, $c(y,t) = \frac{\alpha^2}{4\,t^2}$, $d(y,t) = R(t)$, $e(y,t) = yS(t) + T(t)$, Eq. (1) becomes the GcKP equation[8,14,15]

$$(u_t + uu_x + u_{xxx})_x + \frac{1}{2t}u_x + \frac{\alpha^2}{4t^2}u_{yy} + R(t)u_{xy} + [y\,S(t) + T(t)]u_{xx} = 0, \tag{4}$$

where $\alpha^2 = \pm 1$, $R(t)$, $S(t)$ and $T(t)$ are arbitrary functions.

(4) If $a(y,t) = \frac{1}{t}$, $b(y,t) = \frac{(3-c^2)^2}{16yt^2}$, $c(y,t) = \frac{(3-c^2)^2}{16t^2}$, $d(y,t) = e(y,t) = 0$, Eq. (1) becomes the spherical KP (SKP) equation [16,17]

$$(u_t + uu_x + u_{xxx})_x + \frac{1}{t}u_x + \frac{(3-c^2)^2}{16yt^2}u_y + \frac{(3-c^2)^2}{16t^2}u_{yy} = 0, \tag{5}$$

where c is a real constant.

In this Letter, we aim at deriving an auto-Bäcklund transformation (BT) as well as special solutions by using the truncated Painlevé expansion in Refs. [18,19]. By leading-order analysis of Eq. (1), we obtain the truncated Painlevé expansion at the constant level term, i.e.

$$u(x,y,t) = \frac{1}{\phi(x,y,t)^2}\big[u_0(x,y,t) + \phi(x,y,t)u_1(x,y,t) + \phi(x,y,t)^2u_2(x,y,t)\big]. \tag{6}$$

Without loss of generality, we assume that $\phi_x(x,y,t) \neq 0$. Putting Eq. (6) into Eq. (1), we make the coefficients of like powers of $\phi(x,y,t)$ to vanish, as the set of Painlevé-Bäcklund equations

$$\phi^{-6}: 10{u_0}^2\phi_x^2 + 120u_0\phi_x^4 = 0 \;\Rightarrow u_0 = -12\phi_x^2, \tag{7}$$

$$\phi^{-5}: 12u_0u_1\phi_x^2 + 24u_1\phi_x^4 - 8u_0\phi_xu_{0,x} - 96\phi_x^3u_{0,x} - 2u_0^2\phi_{xx} - 144u_0\phi_x^2\phi_{xx} = 0 \;\Rightarrow u_1 = 12\phi_{xx}, \tag{8}$$

$$\phi^{-4}: -72c(y,t)\phi_y^2\phi_x^2 - 72\phi_t\phi_x^3 - 72d(y,t)\phi_y\phi_x^3$$

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$$
\begin{aligned}
&- 72e(y,t)\phi_x^4 + 792\phi_x^2\phi_{xx}^2 + 288\phi_x^3\phi_{xxx} \\
&+ 24\phi_x^2(-24\phi_{xx}^2 - 24\phi_x\phi_{xxx}) - 72u_2\phi_x^4 = 0, \\
\phi^{-3}: &24b(y,t)\phi_y\phi_x^2 + 24c(y,t)\phi_{yy}{\phi_x}^2 + 24a(y,t)\phi_x^3 \\
&+ 48\phi_x^3u_{2,x} + 72{\phi_x}^2\phi_{xt} + 96c(y,t)\phi_y\phi_x\phi_{xy} \\
&+ 72d(y,t){\phi_x}^2\phi_{xy} + 24c(y,t){\phi_y}^2\phi_{xx} \\
&+ 72\,\phi_t\phi_x\,\phi_{xx} + 72d(y,t),\phi_y\phi_x\,\phi_{xx} \\
&+ 144e(y,t){\phi_x}^2\phi_{xx} + 144u_2{\phi_x}^2\phi_{xx} \\
&- 72{\phi_{xx}}^3 - 576\phi_x\phi_{xx}\phi_{xxx} + 24{\phi_x}^2\phi_{xxxx} \\
&- 8\phi_x(-72\phi_{xx}\phi_{xxx} - 24\phi_x\phi_{xxxx}) = 0, \qquad (10) \\
\phi^{-2}: &- 24b(y,t)\phi_x\phi_{xy} + d(y,t)u_{0,xy} \\
&+ c(y,t)(-24{\phi_{xy}}^2 - 24\phi_x\phi_{xyy}) \\
&- 12b(y,t)\phi_y\phi_{xx} - 12c(y,t)\phi_{yy}\phi_{xx} \\
&- 36a(y,t)\phi_x\phi_{xx} - 72\phi_x u_{2,x}\phi_{xx} \\
&- 36\phi_{xt}\phi_{xx} - 12d(y,t)\phi_{xy}\phi_{xx} \\
&- 12e(y,t){\phi_{xx}}^2 - 12u_2{\phi_{xx}}^2 - 12{\phi_x}^2u_{2,xx} \\
&- 36\phi_x\phi_{xxt} - 24c(y,t)\phi_y\phi_{xxy} \\
&- 12d(y,t)\phi_x\phi_{xxy} - 12\phi_t\phi_{xxx} \\
&- 12d(y,t)\phi_y\phi_{xxx} - 24e(y,t)\phi_x\phi_{xxx} \\
&- 24u_2\phi_x\phi_{xxx} + 24{\phi_{xxx}}^2 \\
&+ e(y,t)(-24{\phi_{xx}}^2 - 24\phi_x\phi_{xxx}) \\
&+ u_2(-24{\phi_{xx}}^2 - 24\phi_x\phi_{xxx}) \\
&- 36\phi_{xx}\phi_{xxxx} - 72\phi_x\phi_{xxxxx} = 0, \qquad (11) \\
\phi^{-1}: &12\phi_{xx}u_{2,xx} + 12b(y,t)\phi_{xxy} + 12c(y,t)\phi_{xxyy} \\
&+ 12a(y,t)\phi_{xxx} + 24u_{2,x}\phi_{xxx} + 12\phi_{xxxt} \\
&+ 12d(y,t)\phi_{xxxy} + 12e(y,t)\phi_{xxxx} \\
&+ 12u_2\phi_{xxxx} + 12\phi_{xxxxxx} = 0, \qquad (12) \\
\phi^0: &b(y,t)u_{2,y} + c(y,t)u_{2,yy} + a(y,t)u_{2,x} \\
&+ u_{2,x}^2 + u_{2,xt} + d(y,t)u_{2,xy} \\
&+ e(y,t)\,u_{2,xx} + u_2\,u_{2,xx} + u_{2,xxxx} = 0. \qquad (13)
\end{aligned}
$$

The set of Eqs. (6)–(13) forms an auto-BT to Eq. (1), if the set is consistent (or solvable).

For the soliton-type solutions, we substitute into the auto-BT a trial solution with

$$\phi(x,y,t) = 1 + e^{i\,\varphi(x,y,t)}, \qquad (14)$$

where $\varphi(x,y,t)$ could be complex, and equate separately to zero the real terms and the coefficients of the imaginary terms in Eq. (9), we obtain

$$
\begin{aligned}
&\phi^{-4}: \varphi_{xx} = 0 \\
&\Rightarrow \varphi(x,y,t) = A_1(y,t)\,x + A_2(y,t). \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
\phi^{-4}\,real: &\, c(y,t){\varphi_y}^2 + \varphi_x[\varphi_t + d(y,t)\varphi_y \\
&+ \varphi_x[e(y,t) + u_2 - \varphi_x^2]] = 0 \\
\Rightarrow &\, u_2(x,y,t) = [-e(y,t)A_1^2 + A_1^4 \\
&- A_1(xA_{1,t} + A_{2,t}) - d(y,t)A_1(xA_{1,y} \\
&+ A_{2,y}) - c(y,t)(xA_{1,y} + A_{2,y})^2]/A_1^2. \qquad (16)
\end{aligned}
$$

Putting $\varphi(x,y,t)$, $u_2(x,y,t)$ into Eq. (10) and making the coefficient of the x power vanish, we obtain

$$
\begin{aligned}
&\phi^{-3}, x: b(y,t)A_{1,y} + c(y,t)A_{1,yy} = 0, \qquad (17) \\
&\phi^{-3}, x^0: a(y,t)A_1 + A_{1,t} + d(y,t)A_{1,y} \\
&\quad + b(y,t)A_{2,y} + c(y,t)A_{2,yy} = 0, \qquad (18)
\end{aligned}
$$

then, Eqs. (11) and (12) equate to zero naturally. Combining Eqs. (13) and (16)–(18), We discuss the solutions as follows:

Case 1: Assuming that $A_{1,y} = 0$ and $A_{2,yy} = 0$, we have

$$
\begin{aligned}
&a(y,t) = \frac{-b(y,t)\,f_2(t) - A_1'(t)}{A_1(t)}, \qquad (19) \\
&A_1(y,t) = A_1(t), \qquad (20) \\
&A_2(y,t) = f_1(t) + y f_2(t), \qquad (21) \\
&u_2(x,y,t) = -e(y,t) + A_1(t)^2 - \frac{d(y,t)f_2(t)}{A_1(t)} \\
&\quad - \frac{c(y,t)f_2(t)^2}{A_1(t)^2} - \frac{x\,A_1'(t)}{A_1(t)} - \frac{f_1'(t)}{A_1(t)} - \frac{y\,f_2'(t)}{A_1(t)}. \qquad (22)
\end{aligned}
$$

with

$$
\begin{aligned}
&- b(y,t)f_2(t)A_1'(t) - 3A_1'(t)^2 \\
&+ b(y,t)A_1(t)f_2'(t) + A_1(t)A_1''(t) \\
&+ b(y,t)f_2(t)^2c_y(y,t) + b(y,t)A_1(t)f_2(t)d_y(y,t) \\
&+ b(y,t)A_1(t)^2e_y(y,t) + c(y,t)f_2(t)^2c_{yy}(y,t) \\
&+ c(y,t)A_1(t)f_2(t)d_{yy}(y,t) \\
&+ c(y,t)A_1(t)^2e_{yy}(y,t) = 0. \qquad (23)
\end{aligned}
$$

Then, the solution of Eq. (1) is

$$
\begin{aligned}
u^{(1)}(x,y,t) = &-3A_1(t)^2\mathrm{Sech}^2\Big[\frac{i\varphi(x,y,t)}{2}\Big] \\
&- e(y,t) + A_1(t)^2 - \frac{d(y,t)f_2(t)}{A_1(t)} - \frac{c(y,t)f_2(t)^2}{A_1(t)^2} \\
&- \frac{xA_1'(t)}{A_1(t)} - \frac{f_1'(t)}{A_1(t)} - \frac{y\,f_2'(t)}{A_1(t)}, \\
\varphi(x,y,t) = &\, A_1(t)x + f_1(t) + yf_2(t). \qquad (24)
\end{aligned}
$$

where $f_1(t)$ and $f_2(t)$ are arbitrary functions, and $A_1(t)$ satisfies Eq. (23).

Especially, in virtue of Eqs. (19)–(24), a family of soliton-type solutions of Eq. (2) are obtained as

$$
\begin{aligned}
u^{(2)}(x,t) = &-3\,A_1(t)^2\mathrm{Sech}^2\Big[\frac{i\varphi(x,t)}{2}\Big] + A_1(t)^2 \\
&- \frac{x\,A_1'(t)}{A_1(t)} - \frac{f_1'(t)}{A_1(t)}, \\
\varphi(x,t) = &\, A_1(t)\,x + f_1(t). \qquad (25)
\end{aligned}
$$

The solutions of Eq. (2) found previously in Refs. [11,12] are the special cases of solution (24) and this kind of solution can be shown as in Fig. 1.

For Eq. (5), we also can obtain a family of soliton-type solutions. However, since they are the same as solutions presented by the method in Ref. [20], we do not show them here in detail.

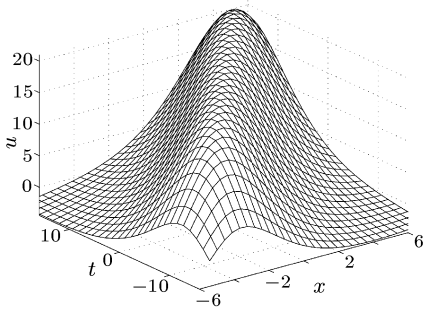

Fig. 1. Soliton-type solution of Eq. (2) with $A_1(t) = i$, $f_1(t) = it^2$.

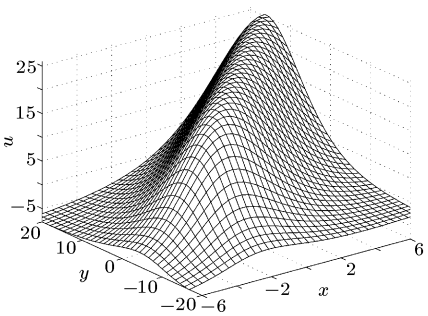

Fig. 2. Soliton-type solution of Eq. (3) with $A_1(t) = i$, $f_1(t) = it^2$, $f_2(t) = it$, $\sigma^2 = 1$, $t = 2$.

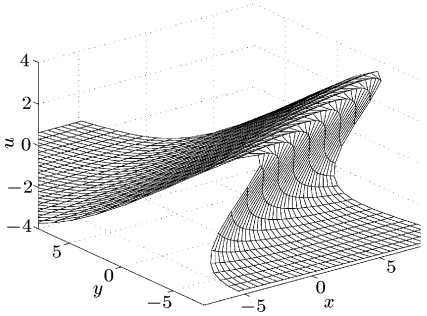

Fig. 3. Soliton-type solution of Eq. (4) with $A_1(t) = i$, $f_1(t) = it^2$, $f_2(t) = it$, $R(t) = S(t) = T(t) = 1$, $t = 2$.

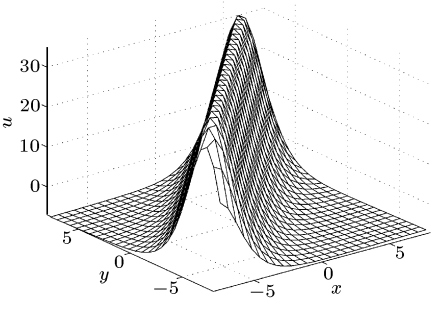

Fig. 4. Solitary wave solution of Eq. (1) with $d(t) = n_1(t) = c(t) = 1$, $z(t) = it^3$, $n_2(t) = m_1(t) = 0$, $k_1 = -2$, $a(t) = 2$, $g(t) = 2i$, $t = 1$.

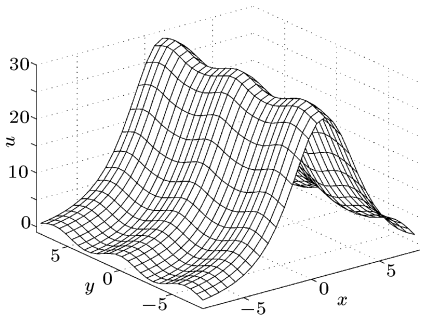

Fig. 5. Interaction between two regular soliton-like solutions. The parameters used here are $a(t) = d(t) = n_2(t) = 0$, $c(t) = t$, $m_1(t) = g(t) = I$, $z(t) = -i\sin t$, $y = 0$.

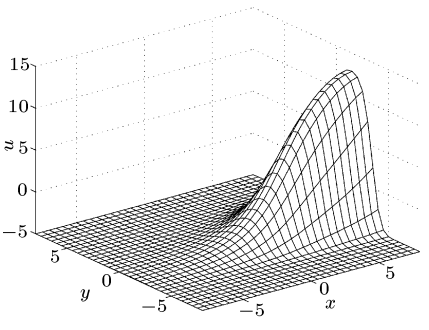

Fig. 6. This picture shows the soliton-like structure at a given time $t = -2$. The parameters used here are $d(t) = n_1(t) = c(t) = a(t) = 1$, $z(t) = it$,$g(t) = m_1(t) = i$, $k_1 = -1/2$.

Case 2: Assuming that $A_1(y,t) = m(t)y + n(t)$, we obtain

$$e(y,t) = C_3(t) + yC_4(t) + \frac{e^4\int a(t)dt\, y^2\left[2a(t)^2 + a'(t)\right]}{2c_3}, \tag{26}$$

$$c(t) = \frac{c_3}{e^4\int a(t)dt}, \tag{27}$$

$$b(y,t) = 0, c(y,t) = c(t),$$

$$d(y,t) = d(t), a(y,t) = a(t), \tag{28}$$

$$A_2(y,t) = C_1(t) + yC_2(t) - \frac{1}{6c(t)}\big\{y^2[3d(t)m(t) + a(t)[ym(t) + 3n(t)] + ym'(t) + 3n'(t)]\big\}. \tag{29}$$

We can obtain the solution of Eq. (1) as follows:

$$u^{(3)}(x,y,t) = -3[m(t)y + n(t)]^2\mathrm{Sech}^2\Big[\frac{i\varphi(x,y,t)}{2}\Big] + u_2(x,y,t),$$

$$\varphi(x,y,t) = [m(t)y + n(t)]x + C_1(t) + yC_2(t) - \frac{1}{6c(t)}\big\{y^2\big[3d(t)m(t) + a(t)[ym(t) + 3n(t)] + ym'(t) + 3n'(t)\big]\big\}, \tag{30}$$

where c_2 and c_3 are integration constants; $u_2(x,y,t)$ satisfies Eq. (16); $m(t)$, $n(t)$, $C_1(t)$, $C_2(t)$, $C_3(t)$ and $C_4(t)$ are arbitrary functions.

According to case 2, a class of corresponding soliton-type solutions of Eq. (3) are given by

$$u^{(4)}(x,y,t) = -3A_1(t)^2\mathrm{Sech}^2\Big[\frac{i\varphi(x,y,t)}{2}\Big] + A_1(t)^2 - \frac{3\sigma^2 f_2(t)^2}{t^2A_1(t)^2} - \frac{xA_1'(t)}{A_1(t)} - \frac{f_1'(t)}{A_1(t)} - \frac{yf_2'(t)}{A_1(t)},$$

$$\varphi(x,y,t) = A_1(t)x + f_1(t) + yf_2(t), \tag{31}$$

which can be shown in Fig. 2, and

$$u^{(5)}(x,y,t) = -3[m(t)y + n(t)]^2\mathrm{Sech}^2\Big[\frac{i\varphi(x,y,t)}{2}\Big] + u_2(x,y,t),$$

$$\varphi(x,y,t) = [m(t)y + n(t)]x + C_1(t) + yC_2(t) - \frac{1}{18\sigma^2}\big\{t^2y^2\big[[ym(t) + 3n(t)]/(2t) + ym'(t) + 3n'(t)\big]\big\}, \tag{32}$$

with $u_2(x,y,t)$ satisfies Eq. (16). Solutions (32) is different from previous solutions in Refs. [10,13].

Similarly, by (26)-(30), the corresponding solutions of Eq. (4) can be given by

$$u^{(6)}(x,y,t) = -3A_1(t)^2\mathrm{Sech}^2\Big[\frac{i\varphi(x,y,t)}{2}\Big] - [yS(t) + T(t)] + A_1(t)^2 - \frac{R(t)f_2(t)}{A_1(t)} - \frac{3\sigma^2 f_2(t)^2}{t^2A_1(t)^2} - \frac{xA_1'(t)}{A_1(t)} - \frac{f_1'(t)}{A_1(t)} - \frac{yf_2'(t)}{A_1(t)},$$

$$\varphi(x,y,t) = A_1(t)x + f_1(t) + yf_2(t), \tag{33}$$

which is shown in Fig. 3, and

$$\begin{aligned} u^{(7)}(x,y,t) =& -3[m(t)y+n(t)]^2\mathrm{Sech}^2\Big[\frac{i\varphi(x,y,t)}{2}\Big] \\ &+ u_2(x,y,t), \\ \varphi(x,y,t) =& [m(t)y+n(t)]x + C_1(t) + yC_2(t) \\ &- \frac{1}{3\alpha^2}\big\{2t^2y^2[3R(t)m(t)/(2t) \\ &+ [ym(t)+3n(t)] + ym'(t) + 3n'(t)]\big\}, \end{aligned} \tag{34}$$

with $u_2(x,y,t)$ satisfying Eq. (16). These solutions cannot be obtained by the method in Refs. [14,15].

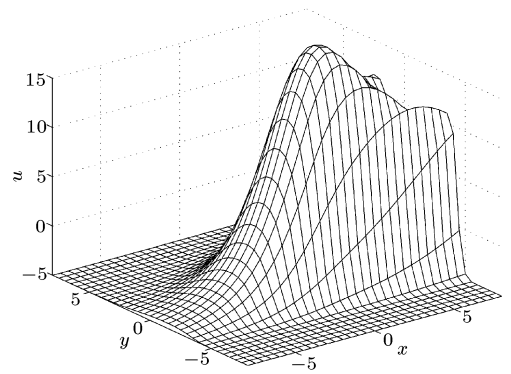

Fig. 7. The parameters are the same as Fig. 6 except that $t = 1$.

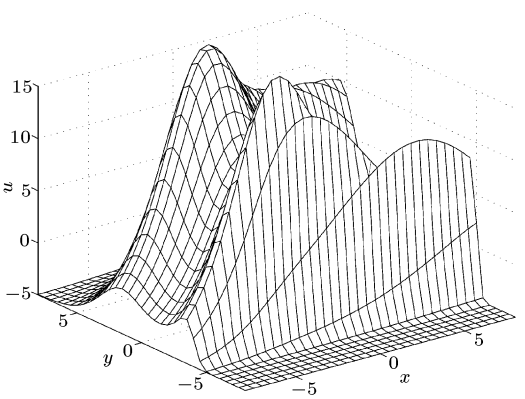

Fig. 8. The parameters are the same as Fig. 6 except that $t = 5$.

Case 3: Assuming that $A_1(y,t) = g(t)$, we obtain

$$\begin{aligned} e(y,t) =& -\frac{e^{2yk_1}c(t)m_1(t)^2}{g(t)^2} + n_1(t) + \frac{e^{yk_1}n_2(t)}{k_1} \\ &+ \frac{1}{c(t)^2g(t)^2k_1}\big\{y[-c(t)g(t)^2a'(t) \\ &+ a(t)g(t)^2c'(t) + g(t)c'(t)g'(t) - 2c(t)g'(t)^2] \big\}, \end{aligned} \tag{35}$$

$$\begin{aligned} &b(y,t) = -k_1c(y,t) = -k_1c(t), \\ &d(y,t) = d(t),\ a(y,t) = a(t), \end{aligned} \tag{36}$$

$$A_2(y,t) = \frac{a(t)g(t)+g'(t)}{c(t)k_1}y + \frac{e^{yk_1}m_1(t)}{k_1} + z(t). \tag{37}$$

Combining all the terms, we can obtain the solution of Eq. (1) to be

$$\begin{aligned} u^{(8)}(x,y,t) =& -3g(t)^2\mathrm{Sech}^2\Big[\frac{i\varphi(x,y,t)}{2}\Big] \\ &+ u_2(x,y,t), \\ \varphi(x,y,t) =& g(t)x + \frac{a(t)g(t)+g'(t)}{c(t)k_1}y \\ &+ \frac{e^{yk_1}m_1(t)}{k_1} + z(t), \end{aligned} \tag{38}$$

where k_1 is an integration constant; $u_2(x,y,t)$ satisfies Eq. (16); $z(t)$, $m_1(t)$, $g(t)$, $n_1(t)$ and $n_2(t)$ are arbitrary functions. A kind of solution of Eq. (38) is shown in Fig. 4.

Compared to some relative investigation made before, the novel feature of this study is that the equation considered is more general. More solutions are obtained at present. For example, the soliton-like solutions in Ref. [21] are only special cases of solution (30).

In Fig. 5, we present the picture of solution (38) for some parameters values while we choose some values of y. In Figs. 6–8, we present the picture of solution (38), which shows the soliton-like structure of solution (38).

In summary, we have obtained an auto-BT and some new soliton-typed solutions to Eq. (1) and also present the corresponding results of Eqs. (2)–(4), respectively.

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