Theoretical Investigation of the Exotic Structure of the Mirror Nuclei ^{17}Ne and ^{17}N *

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The exotic structures of the ground state of the mirror nuclei ^{17}Ne and ^{17}N are investigated by means of the asymptotic normalization coefficient (ANC) method to explore the role of the Coulomb interaction. The probabilities of a valence nucleon outside the binding-potential are $P = 56.69 \pm^{2.98}_{7.46}$ % for ^{17}Ne and $P = 45.51 \pm^{2.32}_{5.81}$ % for ^{17}N. The rms radii are $\langle r^2\rangle^{1/2} = 5.06\pm^{0.11}_{0.30}$ fm and $\langle r^2\rangle^{1/2} = 4.24\pm^{0.06}_{0.16}$ fm, respectively. The results obtained are nearly independent of the potential parameters. According to the halo occurrence conditions, it is suggested that ^{17}Ne is a two-proton halo and ^{17}N is a two-neutron skin. Moreover, two effects of the Coulomb interaction on the exotic structure are analysed. From the present results, the exotic structure of the nucleus in the proton-rich side is more obvious than that of its mirror nucleus because of the Coulomb interaction.

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Since mirror nuclei have a similar structure and nuclear forces are independent of the charge, the difference between the mirror nuclei may come from the Coulomb interaction. Therefore, the study of mirror nuclei is a good way to explore the role of the Coulomb interaction. In recent years, the ground state of mirror nuclei ^{17}Ne and ^{17}N were studied using some experimental and theoretical methods. The large β-decay asymmetry between ^{17}Ne and ^{17}N has been measured, which suggests an re-ordering orbital for the valence protons in ^{17}Ne.[1,2] Ozawa *et al.*[3] measured the interaction cross sections for $A = 17$ isotopes and their result shows that there is a large enhancement for ^{17}Ne. The momentum distribution and the cross section for the two-proton removal from ^{17}Ne have been measured by Kanungo *et al.*[4] The result tells us that there is a much narrower momentum width in ^{17}Ne than the normal nucleus. The narrower momentum distribution was analysed as a possible evidence of two-proton halo in ^{17}Ne in Ref. [5]. Zhukov[6] and Grigorenko *et al.*[7] suggested a two-proton halo structure for ^{17}Ne based on the Faddeev model wavefunctions. Zhang *et al.*[8] investigated the structures of ^{17}Ne and ^{17}N by using the relativistic mean-field approach and obtained the results that there is a two-proton halo in ^{17}Ne and a two-neutron skin in ^{17}N for their ground states. However, there is still some different opinions on these issues. Millener [9] explained the β-decay asymmetry successfully and suggested no halo formation in these nuclei. Also no halo is suggested by Fortune and Sherr[10] after analysing the Coulomb energy of ^{17}Ne which requires the d orbital configuration to dominate for the last two nucleons. Since the situation about the halo formation of ^{17}Ne is not very clear, it is meaningful for us to investigate the exotic structures of ^{17}Ne and ^{17}N by means of the asymptotic normalization coefficient (ANC) method in this Letter.

Since it is difficult to probe halo nuclei in experiments and the theoretical analysis is sometimes dependent on the model parameter, we select a proper experimental observable which is nearly parameter-independent for the accuracy. In this study, the probabilities of a valence nucleon outside the binding-potential and the rms radii are extracted by the nuclear ANC approach[11,12] for the mirror nuclei ^{17}Ne and ^{17}N. Because the nuclear ANC is proven to be nearly model-independent,[12,13] the probabilities and the rms radii obtained by this means are two proper quantities for the study on nuclear exotic structure.

In the ground state of ^{17}Ne, the separation energy of the last two-proton ($S_{2p} = 0.94$ MeV) is much smaller than its neighbouring nuclei. In Ref. [4], three different models are used to interpret the longitudinal momentum distribution, the two-proton removal cross section and the interaction cross section. All the data taken together provide a consistent understanding that the structure of ^{17}Ne is a ^{15}O core plus two

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uncorrelated protons and a 65–100% s-wave probability and 0–35% d-wave probability of the two valence protons.[4,5] Therefore, the two protons can be treated as identical which means that each has a separation energy S_p equal to half the two-proton separation energy S_{2p}. For Borromean nucleus, the same choice was used in Refs. [14,15]. According to the above analysis, ^{17}Ne can be treated as the mixing of s and d configurations in the single-particle model with the ANC method. Because a couple of mirror nuclei are believed to have the same configuration in general, the same treatment can be carried out to ^{17}N, which consists of a ^{15}N core plus two uncorrelated neutrons.

Now, let us take ^{17}Ne as an example to explain the calculation procedure. In the single-particle model, the normalized single-particle radial wavefunction $\phi_{nlj}(r)$ in the bound state $(n\,l\,j)$ is calculated with the potential $V(r) = V_N(r) + V_C(r)$, where the nuclear potential $V_N(r)$ is chosen as the Woods–Saxon potential $V_N(r) = V_0\Big\{1 + \exp\Big(\dfrac{r - r_0 A^{1/3}}{a_0}\Big)\Big\}^{-1}$ and $V_C(r)$ is the Coulomb potential. Here the potential depth V_0 of the Woods–Saxon potential is fitted with the proton separation energy, r_0 and a_0 are the radius and diffuseness parameter, respectively. For the halo state whose wavefunction extends beyond the range of the nuclear force, the asymptotic behaviour of single-particle radial wavefunction $\phi_{nlj}(r)$ can be expressed as follows:[11,13,16]

$$\phi_{nlj}(r) \xrightarrow{r \succ R} b_{lj}\frac{W_{-\eta,l+1/2}(2kr)}{r}, \tag{1}$$

where R is the binding-potential radius derived from the measured mean-square radius of the core, i.e.[12,17]

$$R = \sqrt{\frac{5}{3}(\langle r_c^2\rangle + 4)}, \tag{2}$$

b_{lj} is the single-particle ANC, $W_{-\eta,l+1/2}(2kr)$ is the Whittaker function, η is the Sommerfeld parameter for the corresponding bound state, $k = \sqrt{2\mu S_p}/\hbar$ is the wave number, μ is the reduced mass of the system, S_p is one-proton separation energy which is half of S_{2p}. Therefore, the single-particle ANC (b_{lj}) can be obtained from Eq. (1) and the nuclear ANC (C^B_{Aplj}) can be obtained from the equation[11,13,16]

$$C^B_{Aplj} = b_{lj}[S^{(sp)}_{lj}]^{1/2}, \tag{3}$$

where C^B_{Aplj} is the nuclear ANC, $S^{(sp)}_{lj}$ is the single-particle spectroscopic factor. In our study, the single-particle spectroscopic factor for the $2s_{1/2}$ and $1d_{5/2}$ states are extracted as $S_s = 0.9\pm^{0.1}_{0.25}$ and $S_d = 0.1\pm^{0.25}_{0.1}$,[4,5] respectively; and $r_0 = 1.27$ fm and $a_0 = 0.67$ fm were offered by Kanungo *et al.*[4,5] and used to calculate b_{lj}. From Eq. (3) the nuclear ANC is obtained to be $8.68\pm^{0.47}_{1.30}$.

The probability of a valence nucleon outside the binding-potential and the rms radius can be calculated by

$$P(R, r_0, a_0) = \frac{\int_R^\infty r^2\phi^2_{nlj}(r)dr}{\int_0^\infty r^2\phi^2_{nlj}(r)dr}, \tag{4}$$

$$\langle r^2\rangle^{1/2} = \Big[\int_0^\infty r^4\phi^2_{nlj}(r)dr\Big]^{1/2}. \tag{5}$$

Because the single-particle radial wave-function $\phi_{nlj}(r)$ is dependent on the potential parameters r_0 and a_0, and the probability and the rms radius calculated with Eqs. (4) and (5) are both dependent on these parameters.

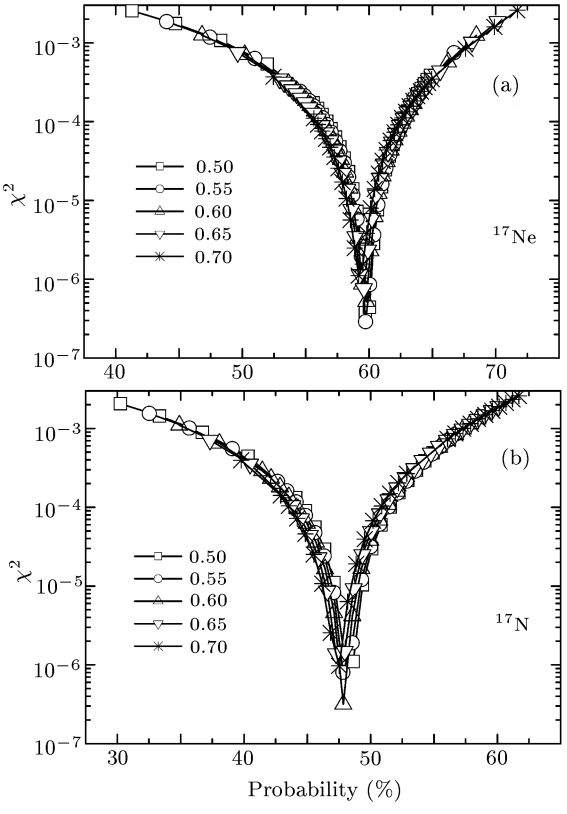


Fig. 1. Dependences of χ^2 on the probability of the valence nucleon outside the binding potential for (a) ^{17}Ne and (b) ^{17}N with different a_0 values.

Although the single-particle ANC b_{lj} obtained by Eq. (1) is parameter-dependent, the nuclear ANC C^B_{Aplj} is proven to be almost parameter-independent.[12,13] Therefore, we can search for the suitable potential parameters in such a way that makes the quantity[12,17]

$$\chi^2 = \sum_{r_i=6\,\mathrm{fm}}^{24\,\mathrm{fm}}\Big\{[S^{(sp)}_{lj}]^{1/2}\phi_{nlj}(r_i) - C^B_{Aplj}\frac{W_{-\eta,l+1/2}(2kr_i)}{r_i}\Big\}^2 \tag{6}$$

minimal. When χ^2 becomes minimal, the asymptotic behaviour can be satisfied by means of Eq. (3). The limit of integral is chosen according to the fact that $r \geq 6$ fm is the asymptotic region and the contribution of $r \geq 24$ fm can be neglected. In the calculation, we choose the diffuseness parameter a_0 as 0.50, 0.55, 0.60, 0.65, 0.70 fm, and for each a_0 the radius parameter r_0 varies around 1.25 fm in a small step. The quantities $\phi_{nlj}(r)$, P, $\langle r^2\rangle^{1/2}$ and χ^2 can be calculated with each pair of r_0 and a_0. For each pair of r_0 and a_0, the χ^2 values versus P and $\langle r^2\rangle^{1/2}$ are plotted as shown in Figs. 1 and 2 for $S_s = 0.9$, respectively.

It can be seen from Figs. 1 and 2 that there is only a small difference in the values of the probability P and the rms radius $\langle r^2\rangle^{1/2}$ for the five minima χ^2, which means that the probability and the rms radius obtained in this way are nearly independent of r_0 and a_0. We can calculate the average values of the probability and the rms radius by

$$P = \sum w_i P(R, r_{0i}, a_{0i}), \tag{7}$$

$$\langle r^2\rangle^{1/2} = \sum w_i \langle r^2\rangle_i^{1/2}, \tag{8}$$

where $w_i = \dfrac{[\chi_i^2(r_{0i}, a_{0i})]^{-1}}{\sum[\chi_i^2(r_{0i}, a_{0i})]^{-1}}$ and the sum includes the five minimum χ^2 for different a_0. In this way, the average values of the probability and the rms radius for the s configuration $P_s = 59.67\%$ and $\langle r_s^2\rangle^{1/2} = 5.17$ fm and for the d configuration $P_d = 29.84\%$ and $\langle r_d^2\rangle^{1/2} = 3.88$ fm are obtained, respectively. For the ground state of ^{17}Ne, which contains s and d configuration mixing, the wavefunction can be written as $\psi(r) = \sqrt{S_s}\phi_s(r) + \sqrt{S_d}\phi_d(r)$, where $\phi_s(r)$ and $\phi_d(r)$ are the single-particle radial wavefunctions of s and d configuration, respectively. Therefore, the probability and the rms radius are calculated by

$$P = S_s P_s + S_d P_d, \tag{9}$$

$$\langle r^2\rangle^{1/2} = (S_s\langle r_s^2\rangle + S_d\langle r_d^2\rangle)^{1/2}. \tag{10}$$

The same calculation procedure is used to the ground state of ^{17}N, and the results for ^{17}Ne and ^{17}N are summarized in Table 1. According to the conditions for the halo state occurrence $P \geq 50\%$ and $\langle r^2\rangle^{1/2}/r_c^2 \geq 2$,[17,19] a two-proton halo in ^{17}Ne and a two-neutron skin in ^{17}N are suggested.

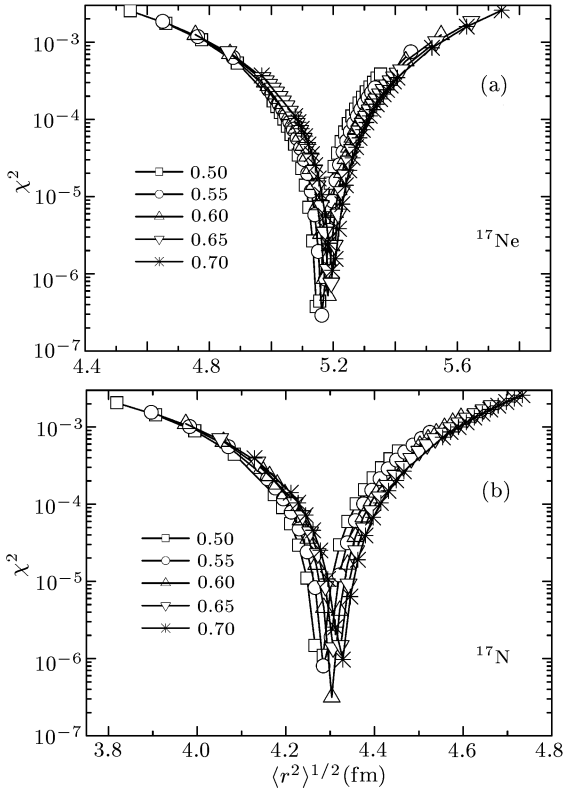


Fig. 2. Dependences of χ^2 on the rms radius of the valence nucleon for (a) ^{17}Ne and (b) ^{17}N with different a_0 values.

Table 1. S_{2p}, ANC, $\langle r^2\rangle^{1/2}$, and P for the ground states of ^{17}Ne and ^{17}N.

Nucleus	S_{2p} (MeV)	nlj	$S_{lj}^{(sp)}$	C_{Aplj}^{B} (fm$^{-1/2}$)	r_c (fm)	$\langle r_l^2\rangle^{1/2}$ (fm)	P_l (%)	$\langle r^2\rangle^{1/2}$ (fm)	P (%)
^{17}Ne	0.94	$2s_{1/2}$	$0.9\pm^{0.1}_{0.25}$	$8.68\pm^{0.47}_{1.30}$	2.44[13]	5.17	59.67	$5.06\pm^{0.11}_{0.30}$	$56.69\pm^{2.98}_{7.46}$
		$1d_{5/2}$	$0.1\pm^{0.25}_{0.1}$	$0.35\pm^{0.30}_{0.35}$	2.44[13]	3.88	29.84		
^{17}N	8.37	$2s_{1/2}$	$0.9\pm^{0.1}_{0.25}$	$3.83\pm^{0.21}_{0.58}$	2.42[13]	4.30	47.83	$4.24\pm^{0.06}_{0.16}$	$45.51\pm^{2.32}_{5.81}$
		$1d_{5/2}$	$0.1\pm^{0.25}_{0.1}$	$0.32\pm^{0.27}_{0.32}$	2.42[13]	3.62	24.59		

Since ^{17}Ne and ^{17}N are a couple of mirror nuclei, the difference between the structures of ^{17}Ne and ^{17}N is caused by the Coulomb interaction. It can be seen from Table 1 that the differences of the probabilities and the rms radii between ^{17}Ne and ^{17}N are $11.18\pm^{0.66}_{1.65}\%$ and $0.82\pm^{0.05}_{0.14}$ fm, respectively. This result shows that the Coulomb interaction plays an important role on the exotic formation of the mirror nuclei. One of the effects of the Coulomb interaction is that it makes the energy level in ^{17}Ne shift closer to the Fermi level, which means that the two-proton is easier to extend far from the nuclear potential. In order to explore the contribution of the energy level shifted to the exotic formation, we assume that the binding energy of ^{17}N is the same as ^{17}Ne. The probability and the rms radius for ^{17}N are calculated, and the results $P = 75.46\pm^{3.84}_{9.60}\%$ and $\langle r^2\rangle^{1/2} = 7.25\pm^{0.24}_{0.65}$ fm are obtained. They imply that the energy level shifted makes P and $\langle r^2\rangle^{1/2}$ increase $29.95\pm^{1.52}_{3.79}\%$ and $3.01\pm^{0.18}_{0.49}$ fm, respectively. On the

other hand, the Coulomb barrier of the protons hinders the exotic formation. The probability and the rms radius of ^{17}Ne are less than the results obtained by neglecting the effect of the Coulomb barrier. Therefore, the Coulomb barrier makes P and $\langle r^2\rangle^{1/2}$ decrease $18.77\pm^{0.86}_{2.14}\%$ and $2.19\pm^{0.13}_{0.35}$ fm, respectively. It can be obviously seen that two effects of the Coulomb interaction are opposite on the probabilities and the rms radii.

In summary, the exotic structures of a couple of mirror nuclei ^{17}Ne and ^{17}N are investigated with the asymptotic normalization coefficient (ANC) method in order to study the effect of the Coulomb interaction on the halo formation. The probabilities of a valence nucleon outside the binding-potential and the rms radii are calculated for the first time. Our results about two-proton halo in ^{17}Ne are almost consistent with the results of Kanungo *et al.*,[4,5] Zhukov *et al.*,[6] Grigorenko *et al.*,[20] and Zhang *et al.*,[8] but still are inconsistent with the results of Millener,[9] and Fortune and Sherr.[10] Since our results are obtained by the ANC method which is proven to be parameter-independent, they should be more reliable. Moreover, two effects of the Coulomb interaction on the exotic structure are both analysed in this study. The result shows that the exotic structure of the nucleus in the proton-rich side is more obvious than that of its mirror nucleus because of the Coulomb interaction.

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