CHIN.PHYS.LETT. Vol.12,No.8(1995)509

Quantum Field Theory of the Universe in the Induced Gravity*

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(Received 2 February 1995)

The quantum field theory on the universe in the induced theory of gravity is discussed in detail. After the wave function of the universes was regarded as a universe field operator in the minisuperspace, not only the difficulty in a probabilistic interpretation in the quantum cosmology can be circumvented, but also the conclusion of the multiple production of universes is obtained. We find the distribution of these universes created from 'nothing' is the planck distribution.

PACS: 98. 80. Dr

Since the exploration by Hartle, Hawking and Vilenkin on the quantum production of universes under the frame of quantum gravity, the quantum cosmology has been passed three stages. They are in order: 1) the single universe quantum theory, 2) the wormhole organization and the variation of spatial topology, 3) quantum theory of multiple universe system, i. e. so-called quantum field theory of universe or third quantization. Apparently, it is quite important for us to study the property of the universe at the moment ($\sim 10^{-43}$ s) in order to understand the universe soundly.

In Refs. 1–3, the basic idea of the third quantization (i. e. the quantum field theory of the universe) has been expounded. In this paper, we would like to use the procedure similar to an ordinary quantum field theory in curved space-time, and to regard the wave function of the universes as a universe field operator in corresponding minisuperspace. It turns out that not only the difficulty about the probabilistic interpretation of the wave function of the universes encountered in quantum cosmology can be circumvented, but also the conclusion of the multiple production from 'nothing' of the universe is obtained.

Hosoya *et al.*[2] have discussed isotropic homogeneous universe with a real scalar field by means of this procedure. Here we discuss the third quantization in the indecuced theory of gravity. The Wheeler-De Witt equation has been solved in the vicinity of vacuum of Higgs field. The metric of minisuperspace model is

$$ds^2 = -dt^2 + a^2(t)d\Omega_3^2 \ , \tag{1}$$

where $d\Omega_3^2$ is a unit three-sphere metric.

* Supported by the National Natural Science Foundation of China.

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Let

$$\alpha = a\phi\ , \qquad \beta = \ln\phi\ , \qquad d\eta = \frac{\mathrm{i}dt}{a}\ . \tag{2}$$

In the vicinity of vacuum of Higgs field, consider an action of the induced theory of gravity[4]

$$S_{\mathrm{E}} = -\frac{1}{2}\int[\alpha'^2 - c_0^2\alpha^2\beta'^2 + \alpha^2]d\eta\ , \tag{3}$$

where $c_0^2 = (1+6\varepsilon)/(6\varepsilon)$. Then the classical Hamiltonian is

$$H = \frac{1}{2}(\alpha'^2 - \alpha^2 - c_0^2\alpha^2\beta'^2)\ , \tag{4}$$

and the constraint of classical Hamiltonian is

$$H = 0\ . \tag{5}$$

Under this constraint, the Weeler-De Witt equation can be obtained

$$\Big[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - e^{4x}\Big]\psi(x,y) = 0\ , \tag{6}$$

where $x = \ln\alpha$, and $y = c_0\beta$. The special solution of Eq. (6) can be written as

$$u_p(x,y) = N_0 e^{\mathrm{i}py} Z_\nu(\frac{ie^x}{2})\ , \tag{7}$$

where $\nu = \frac{\mathrm{i}}{2}|p|$.

The general solution can be expressed[2] as

$$\psi(x,y) = \sum_p \Big(c_p u_p(x,y) + c_p^+ u_p^*(x,y)\Big)\ , \tag{8}$$

and the normalization condition can be expressed as

$$\mathrm{i}\int d\phi\Big(u_p^*\frac{\partial}{\partial x}u_q - u_q\frac{\partial}{\partial y}u_p^*\Big) = \delta_{pq}\ . \tag{9}$$

In the minisuperspace, the factor x plays the role of time while y does that of space. The commutation law is

$$\Big[\frac{\partial\psi(x,y)}{\partial x}, \frac{\partial\psi(x,y')}{\partial x}\Big] = 0\ , \tag{10}$$

$$\Big[\frac{\partial\psi(x,y)}{\partial x}, \psi(x,y')\Big] = -\mathrm{i}\delta(y-y')\ , \tag{11}$$

$$[\psi(x,y), \psi(x,y')] = 0\ . \tag{12}$$

The commutation relation between the annihilation and the creation operators should be

$$[c_p, c_q^+] = \delta_{pq}\ , \qquad [c_p, c_q] = [c_p^+, c_q^+] = 0\ . \tag{13}$$

The universal Fock space is measured by the vector

$$\{c_{p_1}^+, c_{p_2}^+, \quad |0\rangle\}\ . \tag{14}$$

For all $p \in R$, definition of the ground state is

$$c_p|0\rangle = 0 \ . \tag{15}$$

From Eq. (6), we can define the incoming field and the outgoing field and calculate their Bogoliubov transformation coefficients. When $\alpha \to 0$, that is $x \to -\infty$, we have incoming wave function

$$u_p(x,y) \longrightarrow \frac{1}{\sqrt{2|p|}} e^{\mathrm{i}(py-|p|x)} \ . \tag{16}$$

For Eq. (6), incoming field can be written as

$$u_p^{\mathrm{in}}(x,y) = N_1 \exp(\mathrm{i}py) J_\nu(z) \ , \tag{17}$$

where $z = (\mathrm{i}/2)e^{2x}$, N_1 is a normalization constant and can be deduced from Eq. (9) as follows

$$N_1 = (\frac{\pi}{4})\Big(\sinh(\frac{\pi|p|}{2})\Big)^{-\frac{1}{2}} \exp(\frac{\pi|p|}{4}) \ . \tag{18}$$

Obviously Eq. (16) satisfies Eq. (17), and the universal wave function expressed by $\{u_p^{\mathrm{in}}\}$ can be written as

$$\psi(x,y) = \sum_p \Big(c_p^{\mathrm{in}} u_p^{\mathrm{in}}(x,y) + c_p^{\mathrm{in}^*} u_p^{\mathrm{in}^*}(x,y)\Big) \ . \tag{19}$$

The incoming vacuum state can be defined as $c_p^{\mathrm{in}}|0,\mathrm{in}\rangle = 0$ for all $p \in R$, and $|0\rangle$ means nothing. On the other hand, the positive frequency outgoing wave function can be given

$$u_p^{\mathrm{out}}(x,y) = N_2 \exp(\mathrm{i}py) H_\nu^{(1)}(z) \ , \tag{20}$$

where N_2 is a normalization constant and can be deduced from Eq. (9) as follows

$$N_2 = (\frac{\pi}{4})^{\frac{1}{2}}\Big(\cosh(\frac{\pi|p|}{2})\Big)^{-\frac{1}{2}} \ . \tag{21}$$

The wave function of universe can be written as

$$\psi(x,y) = \sum_p \{c_p^{\mathrm{out}} u_p^{\mathrm{out}}(x,y) + c_p^{\mathrm{out}^+} u_p^{\mathrm{out}^*}(x,y)\} \ , \tag{22}$$

and the outgoing vacuum state can be defined as

$$c_p^{\mathrm{out}}|0,\mathrm{out}\rangle = 0 \quad \text{for all } p \in R \ , \tag{23}$$

when $\alpha \to +\infty$, i.e. $x \to +\infty$, we have

$$u_p^{\mathrm{out}}(x,y) \longrightarrow \exp(\mathrm{i}py)\exp(-x - \frac{1}{2}e^{2x}) \ . \tag{24}$$

Then outgoing field function can be given from $\{u_p^{\mathrm{in}}\}$. Let

$$u_p^{\mathrm{out}}(x,y) = \sum_q \Big(c_1(p,q) u_q^{\mathrm{in}}(x,y) + c_2(p,q) u_q^{\mathrm{in}^*}(x,y)\Big) \ , \tag{25}$$

where $c_1(p,q), c_2(p,q)$ are the Bogoliubov transformation coefficients among $u_p^{\rm out}$, $u_p^{\rm in}$ and $u_q^{\rm in}$. Utilizing following relation

$$H_\nu^{(1)}(z) = \frac{\exp(-i\pi\nu)J_\nu(z) - J_{-\nu}(z)}{-i\sin(\pi\nu)} , \tag{26}$$

we have

$$c_1(p,q) = -\frac{i}{[1-\exp(-2\pi|p|)]^{\frac{1}{2}}}\delta_{p,q} , \tag{27}$$

$$c_2(p,q) = \frac{i}{[\exp(2\pi|p|)-1]^{\frac{1}{2}}}\delta_{p,-q} , \tag{28}$$

they must satisfy the condition of probability conservation

$$\sum_q [|c_1(p,q)|^2 - |c_2(p,q)|^2] = 1 . \tag{29}$$

From Eqs. (19), (22) and (25), we can give out the relationships between $\{c^{\rm in},\ c^{\rm in^+}\}$ and $\{c^{\rm out},\ c^{\rm out^+}\}$ as

$$c_p^{\rm in}u_p^{\rm in} = \sum_q \left(c_1(p,q)c_p^{\rm out} + c_2(p,q)c^{\rm out^+}\right)u_p^{\rm in} , \tag{30}$$

$$c_p^{\rm in}u_p^{\rm in^*} = \sum_q \left(c_2(p,q)c^{\rm out} + c_1(p,q)c^{\rm out^+}\right)u_p^{\rm in^*} . \tag{31}$$

It follows from Eqs. (30) and (31)

$$c_p^{\rm out^+} = \frac{1}{bb^* - aa^*}(c_p^{\rm in}u_p^{\rm in}b^* - c_p^{\rm in}u_p^{\rm in^*}a) , \tag{32}$$

$$c_p^{\rm out} = \frac{1}{bb^* - aa^*}(c_p^{\rm in}u_p^{\rm in^*}b - c_p^{\rm in}u_p^{\rm in}a^*) , \tag{33}$$

where $a = \sum_q c_1(p,q)u_p^{\rm in}$, $b = \sum_q c_2(p,q)u_q^{\rm in}$. (34)

From this, we calculated the average number of the universe which is produced from 'nothing' (the in-vacuum $|0,\ {\rm in}\rangle$) as follows

$$N_p = \langle 0,{\rm in}|c_p^{+{\rm out}}, c_p^{\rm out}|0,{\rm in}\rangle , \tag{34}$$

we have

$$N_p = \frac{1}{\exp(2\pi|p|)-1} . \tag{35}$$

It is the Planck distribution. Thus the distribution equation (36) of created universes represents essentially the average number of these universes which are labeled by p and have the energy density $E = p^2/(12\varepsilon a^2\phi^2)$.

The author thanks Professor Liu Liao and Professor Zhao Zheng for their help to this work.

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