Magic Wavelength for Caesium Transition Line $6S_{1/2} - 6P_{3/2}$ *

ZHENG Yu-Nan(郑羽南), ZHOU Xiao-Ji(周小计)**, CHEN Jing-Biao(陈景标), CHEN Xu-Zong(陈徐宗)
School of Electronics Engineering and Computer Science, Peking University, Beijing 100871

(Received 23 January 2006)

We investigate the magic wavelengths of the trapping laser for $6S_{1/2} - 6P_{3/2}$ of the Cs atom in a region where the optical shift between two different states can be eliminated. For fine levels and linear polarized laser they are 930.4 nm and 937.2 nm. The magic wavelengths range from 927.7 nm to 945.0 nm for circle-polarized perturbing laser. Effects of nuclear spin, the hyper-fine Zeeman levels, and the polarization of the light, which generate different magic wavelengths, are further discussed.

PACS: 03. 67. Lx, 06. 20. − f

Optical trap with a far off-resonant laser is a very useful tool for confinement of cold atoms. The optical shift needs to be avoided in precisely controlled trapping atoms. The wavelength of the trapping laser should be tuned into a region where the optical shift between two different states can be eliminated. The wavelength λ is the so-called 'magic' wavelength.[1,2] Various experimental schemes of optical clock based on alkaline-earth and Yb atoms with an uncertainty of better than 2×10^{-17} are proposed,[3−5] where the optical shift of the ground state and the excited state for the clock transition is carefully analysed in optical lattices. On the other hand, the optical shifts of the ground state for alkaline atoms such as Cs and Rb are also studied.[6] However, the $6S_{1/2} - 6P_{3/2}$ transition between the ground state and the exited state of Cs is used for cooling and trapping, which needs to be considered in the cavity quantum electrodynamics (QED)[7] and addressable optical lattice[8] for quantum computation and communication. In this Letter, the calculations of the Stark shifts of the hyper-fine Fm_F levels of the ground state $6S_{1/2}$ and the excited state $6P_{1/2}$ of Cs are presented and the circle-polarized laser is further discussed.

From the second-order perturbation theory the energy shift $U_i(\omega, p, m_i)$ of atomic state i with energy E_i and Zeeman level m_i, which is induced by a perturbing laser field with frequency $\nu = \omega/2\pi$, polarization p, and irradiance I can be expressed as $U_i(\omega, p, m_i) = -\alpha_i(\omega, p, m_i)I/2\epsilon_0 c$ with the induced polarizability α_i. The polarizability can be calculated by summing up the contributions from all dipole transitions from the desired state i to levels k with the respective Einstein coefficients A_{ki} (spontaneous emission rate for $E_k > E_i$), Zeeman levels m' and transition frequencies $\nu_{ik} = \omega_{ik}/2\pi$,[1]

$$\alpha_i = 6\pi c^3 \epsilon_0 \sum_{k,m'} \frac{A_{ki}(2J_k+1)}{\omega_{ik}^2(\omega_{ik}^2 - \omega^2)} \begin{pmatrix} J_i & 1 & J_k \\ m_i & p & -m' \end{pmatrix}^2 . \quad (1)$$

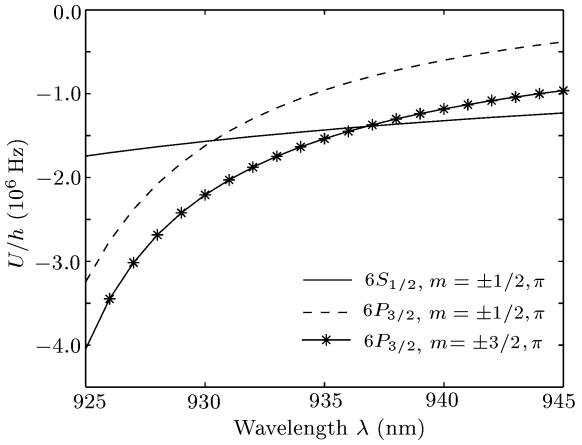


Fig. 1. The ac Stark shift U for a laser irradiance of 10 kW/cm^2. With linear polarization light π, the Zeeman levels of $m = \pm 1/2$ show the same shift, so do $m = \pm 3/2$. The crossing points of lines at 930.4 nm and 937.2 nm shows the frequency shift vanishing points of the inter-combination line, i.e. the so-called magic wavelength.

The expression in large parentheses denotes a $3J$ symbol. It describes the selection rules and relative strength of the transition depending on the involved angular momenta J, their projection m, and the polarization p. The transition frequencies and A_{ki} coefficients are taken from the data collection of Kurucz and Bell.[9] Values of $3J$ symbols can also be looked up via website (Ref. [10]). In Fig. 1, the function $U_i(\omega, p, m_i)$ is depicted as a function of wavelength λ for different polarizations, Zeeman levels, selected levels and wavelengths. One observes that at the so-called magic

* Supported by the National Key Basic Research and Development Programme of China under Grant No 2005CB3724500, and the National Natural Science Foundation of China under Grant Nos 60271003, 10574005 and 60490280.
** Email: xjzhou@pku.edu.cn
©2006 Chinese Physical Society and IOP Publishing Ltd

wavelength (for proper zeeman sublevel and polarization of the perturbing field) the shifts of $6S_{1/2}$ and $6P_{3/2}$ states are the same, so that no frequency shift of the intercombination line is observed. This occurs for linear polarization light π and $m = \pm 1/2$ Zeeman sublevel of $6P_{3/2}$ level at 930.4 nm, and 937.2 nm for the $m = \pm 3/2$ level. At 930.4 nm, the light shift ν changes with the perturbing wavelength λ as $d\nu/d\lambda = 3.9 \times 10^5$ Hz/nm and 1.7×10^5 Hz/nm for the $6S_{1/2} m = \pm 1/2$ and $6P_{3/2}$ $m = \pm 1/2$ states, respectively, while $d\nu/d\lambda = 2.2 \times 10^4$ Hz/nm and 7.0×10^4 Hz/nm for the $6S_{1/2}$ $m = \pm 1/2$ and $6P_{3/2}$ $m = \pm 3/2$ states at 937.2 nm.

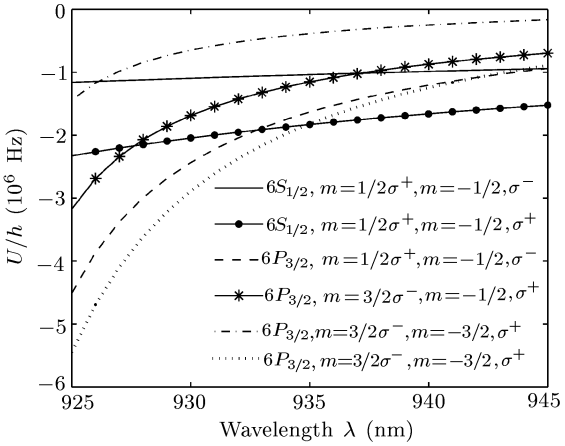


Fig. 2. The ac Stark shift U for a circle-polarized laser light with irradiance of 10 kW/cm^2. Some combinations of Zeeman levels and polarizations show the same shift. The crossing points of lines shows the vanishing frequency shift of the intercombination line.

The ac Stark shift of the $6P_{3/2}$ level is dominated in the depicted wavelength interval by transitions $6P - 7S$ (1469 nm), $6P - 6D$ (917 nm) and $6P - 8S$ (794 nm). Due to a vanishing value of the 3J symbol in Eq. (1), some combinations of Zeeman levels and polarizations of $6P_{3/2}$ do not couple to the $7S_{1/2}$ level by the 1469 nm transition. For example, in the case of π polarization, the $m = \pm 3/2$ sublevels only couple to $m' = \pm 3/2$ sublevels, which do not exist in the $7S_{1/2}$ level. The 917 nm, 794 nm, and 1469 nm transitions are of special interests in the prediction of magic wavelengths because they are not too far detuned and their influences on the frequency shift are strong (see Eq. (1), it has an $\omega_{ik}^2 - \omega^2$ factor in the denominator). The transition wavelength 1469 nm is larger than the magic wavelengths. Thus, this line introduces negative contributions to $\alpha_i(\omega, p, m_i)$ in Eq. (1). Contrarily, the transition wavelengths 917 nm and 794 nm are smaller than the magic wavelengths and they introduce positive contributions to α_i. The three lines together have crucial influence on the slope of the curves in Fig. 1.

When the magic wavelengths of Sr and Ca are discussed, the polarization of laser is of less importance because the ac Stark shifts for $m = 0$ sublevels are the same as any polarization. However, the influence of circle-polarized laser light on Cs states is worth studying. Figure 2 shows the ac Stark shift for different combinations of levels and polarizations as a function of the perturbing wavelength λ. There are four possible combinations between Zeeman levels and two polarizations for the upper level $6P_{3/2}$. The base level $6S_{1/2}$ also has two combinations, one is $m = 1/2$, $p = \sigma^+$ and $m = -1/2$, $p = \sigma^-$. Its magic wavelength for the $6P$ $m = \pm 1/2$, $p = \sigma^\pm$; $m = \pm 1/2$, $p = \sigma^\mp$; $m = \pm 3/2$, $p = \sigma^\pm$; $m = \pm 3/2$, $p = \sigma^\mp$ are 945 nm, 937.2 nm, 944 nm, 926.2 nm, respectively. The other is $m = 1/2$, $p = \sigma^-$ and $m = -1/2$, $p = \sigma^+$, its magic wavelength for the $6P$ $m = \pm 1/2$, $p = \sigma^\pm$; $m = \pm 1/2$, $p = \sigma^\mp$; $m = \pm 3/2$, $p = \sigma^\pm$ correspond are 932.8 nm, 927.7 nm, 935 nm, respectively. Table 1 shows the ac Stark shift difference of the two levels ($\delta U = U_{6P_{3/2}} - U_{6S_{1/2}}$) and slope of the shift difference at the previous magic wavelength for linear polarized laser. The slope k shows how fast the frequency shift difference changes with the perturbing laser light wavelength.

Table 1. The ac Stark shift of the intercombination line with the σ polarized line for laser irradiance of 10 kW/cm^2.

$5S_{1/2}$ (m,p)	$(\frac{1}{2},\sigma^+)$; $(-\frac{1}{2},\sigma^-)$		$(\frac{1}{2},\sigma^-)$; $(-\frac{1}{2},\sigma^+)$	
$6P_{3/2}$ (m,p)	ΔU (MHz)	k (MHz/nm)	ΔU (MHz)	k (MHz/nm)
$(\frac{1}{2},\sigma^+)$; $(-\frac{1}{2},\sigma^-)$	−1.2554	0.22	−3.2002	1.34
$(\frac{1}{2},\sigma^-)$; $(-\frac{1}{2},\sigma^+)$	−0.5372	0.14	0.3981	0.11
$(\frac{3}{2},\sigma^+)$; $(-\frac{3}{2},\sigma^-)$	−0.5369	0.11	0.2042	0.08
$(\frac{3}{2},\sigma^-)$; $(-\frac{3}{2},\sigma^+)$	0.6949	0.02	1.4360	0.01

ΔU is the ac Stark shift difference of the intercombination line ($U_{6P_{3/2}}$-$U_{6S_{1/2}}$) at the magic wavelength of linear laser light (930.4 nm for $6P_{3/2}$ $m = \pm 1/2$, 937.2 nm for $6P_{3/2}$ $m = \pm 3/2$), k is the slope of the shift difference at that point. It shows how fast the frequency shift difference changes with the perturbing laser light wavelength.

As the nuclear spin of most alkaline-earth metal elements (for example ^{88}Sr and ^{40}Ca) is zero, the hyperfine levels need not be taken into account. Nevertheless, the nuclear spin for ^{133}Cs is 7/2, so it might arise

some modifications on the ac Stark Shift of the levels. Equation (1) should also be modified when nuclear spin I is considered. The energy shift $U_F(\omega, p, m_F)$ of atomic state i with hyper-fine level F and Zeeman sublevel m_F, which is induced by a perturbing laser field with frequency $\nu = \omega/2\pi$, polarization p, and irradiance I can be expressed as $U_F(\omega, p, m_F) = -\alpha_i(\omega, p, m_F)I/2\epsilon_0 c$ with the induced polarizability α_i. We still sum up the contributions from all dipole transitions from state i to upper state k (including all possible F' of upper levels k). Here α_i can be expressed as

$$\alpha_i = 6\pi c^3 \epsilon_0 \sum_{k,F'} \frac{A_{ki}(2J_k+1)(2F_i+1)(2F_k+1)}{\omega_{ik}^2(\omega_{ik}^2-\omega^2)} \times \begin{pmatrix} F_i & 1 & F_k \\ m_F & p & -m'_F \end{pmatrix}^2 \begin{Bmatrix} J_i & J_k & 1 \\ F_k & F & I \end{Bmatrix}^2. \quad (2)$$

The coefficients in round and curly brackets are the $3J$ and $6J$[11] symbols, respectively. They describe the selection rules and relative strength of the transitions depending on the angular momenta, their projections m_F, the nuclear spin I and the polarization $p = 0, \pm 1$ which stands for π and $\sigma^{\pm}$ transition of related states. The primed quantum numbers refer to the excited states $n'P_{J'}$ coupled to the ground state by allowed electric dipole transitions. The transition frequencies and A_{ki} coefficients we used in calculation are also taken from data collection of Kurucz and Bell,[9] but the transition frequencies are modified according to the hyper-fine level splitting. Figure 3 shows the stark shift of hyper-fine levels of $6S_{1/2}$ and $6P_{3/2}$ with a linear laser and the magic wavelength shifts with hyper-fine level F from 936.9 nm to 938.1 nm.

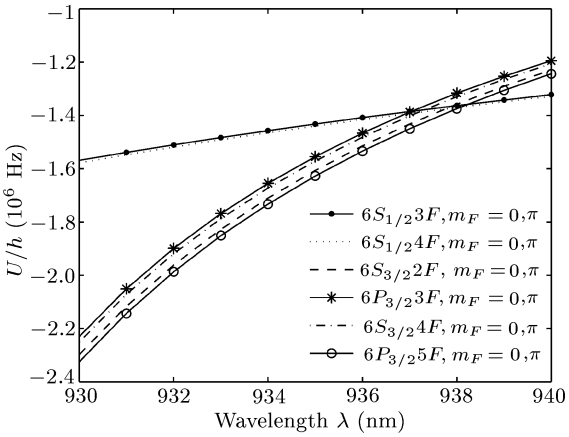


Fig. 3. The ac Stark shift U for Cs super-fine levels with laser irradiance of 10 kW/cm^2. The shift difference between the $6S_{1/2}$ $F = 3$ and $F = 4$ levels is much smaller than those of the $6P_{3/2}$ state.

We calculate the frequency shift of all possible m_F sublevels of state $6S_{1/2}F = 4$ with linear laser light. We found that the error of each m_F sublevel is comparatively small and the error of the magic wavelength is less than 0.1 nm. However, the stark shift of the excited state $6P_{3/2}F = 4$ changes apparently between all m_F sublevels, and magic wavelength change from 937.3 nm ($m_F = 0$) to 938.0 nm ($m_F = \pm 4$).

Of all the Zeeman sublevels, the $m_F = 4$ sublevel is paid special intention to because it is a trapping state usually used for trapping and cooling. In our theoretical calculation, the magic wavelength for $6S_{1/2}$ $m_F = 4$ to $6P_{3/2}$ $m_F = 4$ is 938.0 nm. However, the wavelength should change when circle-polarized laser light is introduced. The situation differs between σ^+ and σ^-. According to the selection rules, $6P_{3/2}$ does not couple to $7S_{1/2}$ or $8S_{1/2}$ states with σ^+ laser light, while with σ^- laser it can couple to.

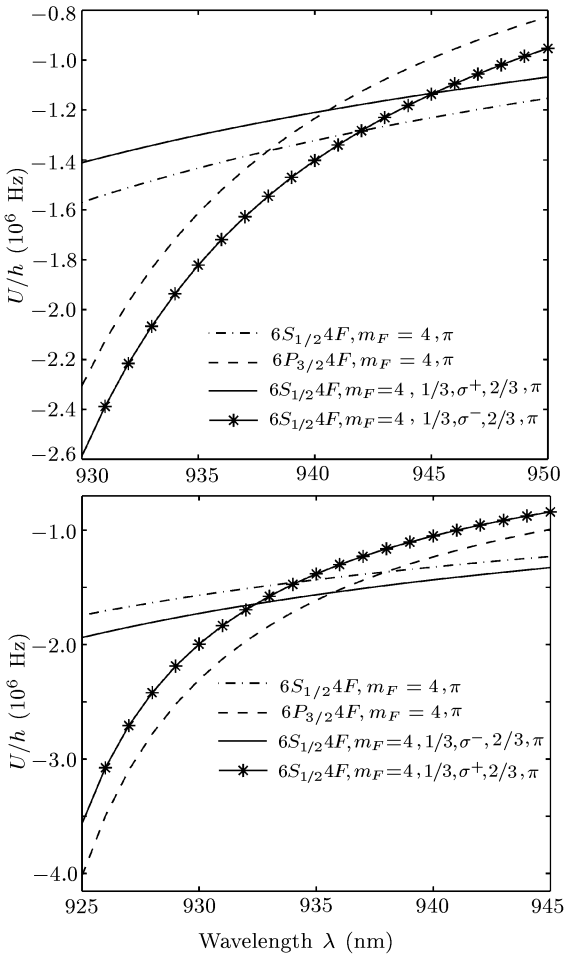


Fig. 4. The ac Stark shift U of $6S_{1/2} - 6P_{3/2}$ for 1/3 circle-polarized and 2/3 linear laser light with the irradiance of 10 kW/cm^2.

Furthermore, we discuss a three-dimensional optical lattice constructing with two directions of linear-polarized light and one direction of circle-polarized light. The total light shift is the summation of 1/3 the shift of circle-polarized and 2/3 shift of the lin-

ear light, as is shown in Fig. 4. The previous magic wavelengths near 937 nm is modified according the polarization of perturbing laser. When the polarization is σ^+ and σ^-, the magic wavelength is 945.0 nm and 932.4 nm, respectively. The slope of the shift difference $k = \Delta U/\Delta\lambda$ ($\Delta U = U_{6P_{3/2}} - U_{6S_{1/2}}$) are 0.030 MHz/nm and 0.086 MHz/nm for σ^+ and σ^-, respectively. Other portions of circle-polarized laser light can be calculated in the same way and still bring about some modification to the frequency shift vanishing points.

In summary, we have calculated the magic wavelength for Cs with linear and circle-polarized laser for the transition $6S_{1/2} - 6P_{3/2}$, which is of interest for trapping and cooling in optical trap and lattice. Different from the optical shift calculating in the earth–alkalic atoms that are often used in the optical clock such as Sr and Ca where the nuclear spin $I = 0$, the m_F Zeeman sublevels and polarization of laser light result in different magic wavelengths and they are carefully calculated. Magic wavelength nearly at 937 nm is found for the transition $6S_{1/2} - 6P_{3/2}$, and some situations used in trapping and cooling are studied, which is beneficial to the study of the quantum information and precise control and measurements.

References

[1] Degenhardt C, Stoehr H, Sterr U, Riehle F and Lisdat C 2004 *Phys. Rev.* A **70** 023414
[2] Katori H, Takamoto M, Pal'chikov V G and Ovsiannikov V D 2003 *Phys. Rev. Lett.* **91** 173005
[3] Takamoto M, Hong F L, Higashi R and Katori H 2005 *Nature* **435** 321
[4] Tao H, Cramer C, Nagourney W and Fortson E N 2005 *Phys. Rev. Lett.* **94** 050801
[5] Santra R, Arimondo E, Ido T, Greene C H and Ye J 2005 *Phys. Rev. Lett.* **94** 173002
[6] Zhou X J, Chen X Z and Chen J B 2005 ArXiv: physics 0512244
[7] McKeever J, Buck J R, Boozer A D, Kuzmich A, Nagerl H C, Stamper-Kurn D M and Kimble H J 2003 *Phys. Rev. Lett.* **90** 133602
[8] Vala J, Thapliyal A V, Myrgren S, Vazirani U, Weiss D S and Whaley K B 2005 *Phys. Rev.* A **71** 032324
[9] Kurucz R L and Bell B 1995 Atomic Line Data
[10] http://plasma-gate.weizmann.ac.il/cgi-bin/calculate_369j.sh
[11] Edmonds A R 1957 *Angular Momentum in Quantum Mechanics* (Princeton: Princeton University Press)