K Parameter Analysis of the Radio Emission Region of Pulsars *

WU Xin-ji(吴鑫基)[1], K. N. Yu(余君岳)[2], ZHANG Cheng-min(张承民)[3], WEN Xue-shi(温学诗)[4]
[1]*Department of Geophysics, Peking University, and Chinese Academy of Science–Peking University Joint Astrophysical Center, Beijing 100871*
[2]*Department of Physics and Materials Science, City University of Hong Kong, Hong Kong*
[3]*Department of Physics, Hebei University of Technology, Tianjin 300130*
[4] *Beijing Planetarium, Beijing 100044*

(Received 18 May 1998)

Using a new method of multi-frequency of K parameter, we estimate the ratio of altitudes of radio emission region of pulsars at 430 and 1418 MHz. The value of the ratio is 1.34±0.12 and the power law index of altitude-frequency is 0.24±0.08. It is indicated that the lower radio frequencies originate further out and the emission region is radially narrow.

PACS: 97. 60. Gb

From theoretical arguments,[1] the altitude of emission region of pulsars is proportional to $\omega^{-2/3}$. It is impossible to determine directly from observation how far away from the surface of neutron star and what is the relation between the altitude and frequency for the radio emission region. There have been some discussions on the altitudes of the radio emission region of pulsars based on the analysis of observational data for a few pulsars. A radius-to-frequency mapping (RFM) model was assumed by Cordes[2] to explains the systematic increase of the separation between the profile components and the profile widths with decreasing frequency. They obtained the emission radii for frequencies 430 and 1400 MHz to be less than 100−1000 km for 3 pulsars. Phillips[3] used multi-frequency timing measurements of 4 pulsars to search for propagation delays predicted by the RFM model. Blaskiewicz *et al.*[4] used a relativistic model of pulsar polarization to estimate the altitudes of emission region for the frequencies of 430 and 1418 MHz for 12 pulsars. This method is only related to the observation data, but the results do not show the obvious relationship between the altitude and frequency.

Kijak *et al.*[5] presented the results that the emission altitudes depend on both the frequency and the pulsar period based on the dipolar magnetic geometry model. One expects the following relation to hold:

$$\rho = 1.24 s r_6^{1/2} P^{-1/2}, \tag{1}$$

where ρ is the radius of the emission cone, P is the pulsar period, $r_6 = r/10^6$ is the emission altitude and the parameter $s(0 < s \leq 1)$ describes a locus of field lines contributing to the emission observed at the longitude ϕ, where the equality holds for the boundary of Goldreich-Julian polar cap,[6] corresponding to the last open field lines. For the calculation of the altitude r_6, from Eq. (1), we need to know the value of ρ. Statistical researches have established some different relations between ρ and the period P, such as $\rho = 6.5^\circ P^{-1/3}$,[7] $\rho = 5.0^\circ P^{-1/2}$,[8–10] $\rho = 5.8^\circ P^{-1/2}$,[11] $\rho = 7.4^\circ P^{-1/2}$,[12] and $\rho = 9.3^\circ P^{-0.36}$.[13] It is obvious that the different relation of $\rho - P$ will give different estimations of ρ and r_6.

Malov[14] used a method of multi-frequency analysis of the parameter n to discuss the distribution of emission region in the pulsar magnetospheres for 6 pulsars with long periods. The parameter n is the same with parameter Q defined by Wu *et al.*[15,16] It is also similar to the parameter β_n defined by Lyne *et al.*[17] The parameters Q, n, and β_n are useful, but for the estimations of the emission altitudes, they are not very good parameters, because they are not solely determined by observational data.

We develop a new method, named the multi-frequency analysis of parameter K, to estimate the ratio of the emission altitude between different frequencies and the spectral index of the altitudes.

Xu *et al.*[17] and Wu *et al.*[18] defined a new parameter K under the polar cap model to study the evolution of the magnetic inclination and the radius of the emission cone,

$$K = \frac{\sin \Delta\psi}{\sin \Delta\phi} = \frac{\sin \alpha}{\sin \rho}, \tag{2}$$

where $\Delta\psi$ is the half polarization position angle swing, $\Delta\phi$ is the half apparent beam-width, and these are solely observable data, so the K is an observed parameter. The α is the magnetic inclination and ρ is the emission cone radius. The K_0 corresponds to the half full width $\Delta\phi_0$ and half full position swing $\Delta\psi_0$, which is the best parameter for our research. Only for a few pulsars with very high ratio of signal to noise. It is possible to measure the parameters of $\Delta\phi_0$ and $\Delta\psi_0$. The high quality Arecibo measurements of 8 pulsars at 430 and 1418 MHz given by Blaskiewicz *et al.*[4] (i.e., BCW91) can be used to determine the parameter K_0, which has made such multi-frequency analysis worthwhile. Figure 1 shows the relation between K_0 (430)

*Supported by the National Natural Science Foundation of China under Grant No.19773001, the Astronomy Project of Climbing Program of National Committee of Science and Technology of China, and Doctoral Program Foundation of Higher Education of China.
©by the Chinese Physical Society

and K_0 (1418). The correlation coefficient R is 0.99 and the regression equation is

$$K_0(430) = 0.92 \times K_0(1418) - 0.19. \quad (3)$$

The result shows that K_0 is a function of frequency. We have

$$\frac{K_0(430)}{K_0(1418)} = \frac{\sin\rho_0(1418)}{\sin\rho_0(430)}. \quad (4)$$

Combining Eqs. (4) and (1), we have

$$m = \frac{r(430)}{r(1418)} \approx \frac{K_0^2(1418)}{K_0^2(430)}. \quad (5)$$

In Eq. (1), the ρ is the full emission cone radius ρ_0. It is very interesting that the m is solely determined by observed parameters of K_0 and is not related with the relation of $\rho - P$.

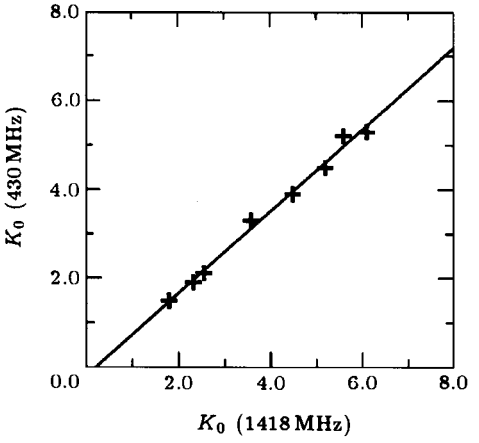

Fig. 1. Relation between K_0(430 MHz) and K_0(1418 MHz).

Table 1. K parameter and the estimations of ratio and spectral index of altitudes of emission region.

PSR	P(s)	f(MHz)	$\Delta\phi$(deg)	$\Delta\psi$(deg)	K	m	ξ
0301+19	1.380	430	20.7	138.9	5.21	1.15	0.12
0301+19	1.380	1418	19.5	144.4	5.60	1.15	0.12
0525+21	3.745	430	25.0	161.1	4.50	1.34	0.25
0525+21	3.745	1418	21.9	161.1	5.20	1.34	0.25
0540+23	0.245	430	50.6	77.8	1.49	1.46	0.32
0540+23	0.245	1418	40.8	77.8	1.80	1.46	0.32
0950+08	0.253	430	51.6	133.3	2.11	1.47	0.32
0950+08	0.253	1418	42.0	133.3	2.56	1.47	0.32
1133+16	1.187	430	16.5	100.0	5.30	1.32	0.23
1133+16	1.187	1418	14.4	100.0	6.10	1.32	0.23
1530+27	1.124	430	20.8	67.6	3.30	1.18	0.14
1530+27	1.124	1418	15.8	59.2	3.59	1.18	0.14
1737+13	0.803	430	29.1	164.4	3.94	1.28	0.21
1737+13	0.803	1418	25.8	172.6	4.50	1.28	0.21
1913+16	0.059	430	62.5	180.0	1.90	1.50	0.34
1913+16	0.059	1418	50.8	180.0	2.33	1.50	0.34

The detailed form of radius to frequency mapping depends on the physics of the emission mechanism, and is written as

$$r_\nu \propto \nu^{-\xi}, \quad (6)$$

where ξ is the spectral index. From Eqs. (5) and (6), we have

$$\frac{r_\nu(1)}{r_\nu(2)} = \left[\frac{\nu(1)}{\nu(2)}\right]^{-\xi}, \quad (7)$$

$$\xi = 1.93 \times \lg\left[\frac{K_0(1418)}{K_0(430)}\right]^2. \quad (8)$$

The multi-frequency analysis of K can directly give the information of the ratio m of emission altitudes between different frequencies and the spectral index ξ. The values of m and ξ are determined by the observational parameter K.

In Table 1, we show the observation parameters and the estimated results. The value of m is 1.34±0.12 from 1.15 to 1.50. This result is similar to that of Kijak *et al.*[5] (i.e., KG97). These results show that the pulsar emission region is radially compact and in agreement with the RFM model. The low frequencies are produced at larger altitudes than the high frequencies. On the contrary, for the relativistic model of BCW91, there are 4 pulsars with $m \leq 1$ in the sample of 8 pulsars, which is in conflict of the RFM model.

Theoretical estimation of the spectral index ξ is 0 given by Barnard *et al.*[19] and 2/3 given by Ruderman *et al.*[1] The method of multi-frequency analysis of the parameter K leads to a conservative upper bound on the spectral index, i.e., 0.34, and the lowest value is 0.14 shown in Table 1. The average value of ξ is 0.24±0.08.

The data of Cordes *et al.*[20] set a much stronger limit, $\xi \leq 0.12$, for the millisecond pulsar (PSR 1937±21). The timing data of Phillips *et al.*[21] lead to a conservative upper bound on the spectral index of $\xi \leq 0.66$.

The new method of multi-frequency of K parameter has special advantage. The ratio of the altitudes for different frequencies and the spectral index are determined by the observational parameter K.

REFERENCES

1 M. A. Ruderman *et al.*, Astrophys. J. 196 (1975) 51.
2 J. M. Cordes, Astrophys. J. 222 (1978) 1006.
3 J. A. Phillips, Astrophys. J. 385 (1992) 282.
4 M. Blaskiewicz *et al.*, Astrophys. J. 370 (1991) 643.
5 J. Kijak and J. A. Gil, Mon. Not. R. Astro. Soc. (1998) in press (KG97).
6 P. Goldreich *et al.*, Astrophys. J. 157 (1969) 869.
7 A. G. Lyne *et al.*, Mon. Not. R. Astro. Soc. 234 (1988) 477.
8 A. D. Kuzmin *et al.*, Sov. Astron. Lett. 10 (1984) 854.
9 A. D. Kuzmin *et al.*, Astrophys. Space Sci. 190 (1992) 209.
10 Wu X. J. and J. A. Gil, Acta Astrophys. 15 (1995) 40 (in Chinese); Chin. Astron. Astrophys. 19 (1995) 156.
11 J. M. Rankin, Astrophys. J. 405 (1993) 285.
12 D. M. Gould, Ph. D Dissertation, Univ. of Manchester of UK, 1994.
13 I. F. Malov, Astrofizika, 24 (1986) 507 (in Russian).
14 I. F. Malov, Sov. Astron. Lett. 17 (1991) 254.
15 Wu X. J. *et al.*, Acta Astron. Sin. 26 (1985) 69 (in Chinese).
16 Wu X. J. *et al.*, Astrophys. Space Sci. 119 (1986) 101.
17 Xu W. and Wu X. J., Astrophys. J., 380 (1991) 550.
18 Wu X. J. and Xu W., *in Proceedings of IAU Colloquium 128, The Magneto-spheric Structure and Emission Mechanisms of Radio Pulsars*, Poland, June 1990, edited by T. H. Hankin, J. M. Rankin and J. A. Gil (Pedagogocal Univ. Press, Zielona Gora, Poland, 1992) p.162.
19 J. J. Barnard and J. Arons, Astrophys. J. 302 (1986) 138.
20 J. M. Cordes *et al.*, Astrophys. J. 277 (1984) L53.
21 J. A. Phillips and A. Wolszczan, in *Low Frequency Astrophysics from Space*, edited by N. Kassim and K. Weiler (Springer, New York, 1990) p. 175.