Controllable Optical Bistability and Multistability in a Four-Level Atomic System with Closed-Loop Configuration *

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We theoretically investigate optical bistability (OB) and multistability (OM) behaviour of a closed-loop configuration atomic system driven by a degenerate coupling field and a degenerate probe field inside a unidirectional ring cavity. It is found that the OB and OM behaviour can be controlled by adjusting the intensity and the frequency detuning of the coupling field, respectively. Interestingly, our numerical results show that it is easy to realize the transition from OB to OM or vice versa by adjusting the intensity of the coupling field under a appropriate frequency detuning. The effect of the atomic cooperation parameter on the OB behaviour is also discussed.

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Optical bistability (OB) in two-level atomic systems has been extensively studied experimentally and theoretically owing to its wide applications such as in optical transistors, memory elements and all optical switches.[1,2] The experimental demonstration of OB in two-level alkali atoms confined in an optical resonator makes the application more reliable.[2] The OB of three-level atomic systems confined in an optical ring cavity has also been theoretically studied[3] and experimentally demonstrated.[4] Harshawardhan and Agarwal have demonstrated a scheme that can decrease substantially the threshold required for a bistable device using field-induced transparency and quantum interference effect in Ref. [5]. The influence of the phase fluctuation effect [6,7] and the effect of the squeezed state field[8,9] on OB have subsequently been studied. Chen *et al.*[10] also studied the effect of squeezed vacuum field on OB and found that the OB behaviour could also appear for small cooperation parameter due to the presence of squeezed vacuum field.

In recent years, the effect of spontaneously generated coherence (SGC) in a general three-level atomic system has been widely studied such as amplification without population inversion,[11] quantum interference in a multilayer dielectric medium,[12] the disappearance of the dark state due to SGC,[13] and the enhanced index of refraction without absorption.[14,15] The OB behaviour in the three-level atomic system can also be affected by the SGC effect as shown in Refs. [16,17]. For example, Cheng *et al.*[18] investigated the OB behaviour of a nearly equispaced ladder-type three-level atomic system considering the SGC effect and showed that the OM can be observed by adjusting the relative phase between two applied fields. The OB behaviour of a four-level system in a ring cavity driven by two coherent laser fields have also been studied in Ref. [19]. It is shown that the difference between the two frequency detunings of the incident and control fields from the corresponding atomic transition frequencies can affect the OB behaviour of atomic media. In this Letter, we present a new scheme of a closed-loop configuration atomic system driven by two degenerate laser fields inside a unidirectional ring cavity for OB and OM. Interestingly, we can easily realize the switch between OB and OM without needing the SGC effect just by adjusting the intensity of the coupling field under an appropriate frequency detuning.

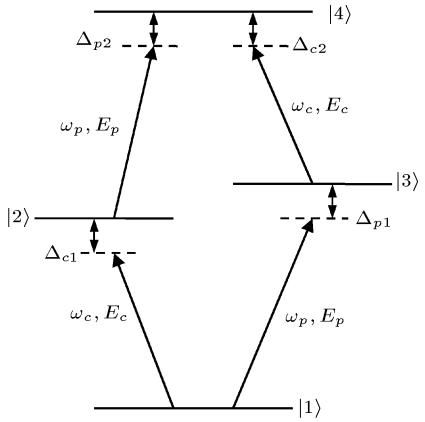

Fig. 1. Schematic diagram of a four-level atomic system with closed-loop configuration.

Consider a four-level atomic system (see Fig. 1) with two lower states $|1\rangle$ and $|2\rangle$, and two excited states $|3\rangle$ and $|4\rangle$. The transition $|1\rangle \leftrightarrow |2\rangle$ (with resonant frequency ω_{21}) and $|3\rangle \leftrightarrow |4\rangle$ (with resonant frequency ω_{43}) are driven simultaneously by a degen-

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erate coupling field of angular frequency ω_c. A degenerate probe field of angular frequency ω_p is applied to drive simultaneously the transition $|1\rangle \leftrightarrow |3\rangle$ (with resonant frequency ω_{31}) and $|2\rangle \leftrightarrow |4\rangle$ (with resonant frequency ω_{42}). One real atomic model for this scheme is ^{87}Rb atoms of nuclear spin $I = 3/2$. We can choose $|5S_{1/2}, F = 1, m_F = 1\rangle$, $|5P_{1/2}, F = 1, m_F = 0\rangle$, $|5P_{3/2}, F = 1, m_F = 1\rangle$, and $|5D_{3/2}, F = 2, m_F = 0\rangle$ as the states $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$ and apply a right-hand σ^- polarized laser and a linearly π polarized laser as the coupling field and the probe field, respectively. $\Omega_{p1} = d_{13}E_p/2\hbar$, $\Omega_{p2} = d_{24}E_p/2\hbar$, $\Omega_{c1} = d_{12}E_c/2\hbar$, and $\Omega_{c2} = d_{34}E_c/2\hbar$ are the one-half Rabi frequencies of the two coherent fields, where E_p and E_c denote the amplitude of the two degenerate field, respectively; $d_{ij} = \vec{d}_{ij} \cdot \boldsymbol{e}_L$ ($\boldsymbol{e}_L$ is the polarization unite vector of the laser field) denote the dipole moments for the transition $|i\rangle \leftrightarrow |j\rangle$ and they are regarded as real parameters in this study. $\Delta_{p1} = \omega_{31} - \omega_p$, $\Delta_{p2} = \omega_{42} - \omega_p$, $\Delta_{c1} = \omega_{21} - \omega_c$, and $\Delta_{c2} = \omega_{43} - \omega_c$ are the frequency detunings of the coupling and probe fields from the corresponding two-level transition (see Fig. 1). Then it is apparent that the four frequency detuning satisfy the relation $\Delta_{c1} + \Delta_{p2} = \Delta_{p1} + \Delta_{c2}$. The decay rates from the states $|4\rangle$ to $|2\rangle$, $|4\rangle$ to $|3\rangle$, $|2\rangle$ to $|1\rangle$, and $|3\rangle$ to $|1\rangle$ are γ_{42}, γ_{43}, γ_{21}, and γ_{31}, respectively. In the interaction picture and the rotating-wave approximation, the interaction Hamiltonian of our system can be written as[20]

$$\begin{aligned} H_I =& \Delta_{p1}|3\rangle\langle 3| + \Delta_{c1}|2\rangle\langle 2| + (\Delta_{c1} \\ &+ \Delta_{p2})|4\rangle\langle 4| - (\Omega_{p1}|3\rangle\langle 1| \\ &+ \Omega_{p2}|4\rangle\langle 2| + \Omega_{c1}|2\rangle\langle 1| + \Omega_{c2}|4\rangle\langle 3| + \text{H.c.}). \end{aligned} \tag{1}$$

By the standard approach, the density-matrix equations can be derived as follows:

$$\begin{aligned} \dot{\rho}_{11} =& \gamma_{21}\rho_{22} + \gamma_{31}\rho_{33} + i\Omega_{c1}^*\rho_{21} + i\Omega_{p1}^*\rho_{31} \\ &- i\Omega_{c1}\rho_{12} - i\Omega_{p1}\rho_{13}, \end{aligned} \tag{2a}$$

$$\begin{aligned} \dot{\rho}_{22} =& \gamma_{42}\rho_{44} - \gamma_{21}\rho_{22} + i\Omega_{c1}\rho_{12} + i\Omega_{p2}^*\rho_{42} \\ &- i\Omega_{c1}\rho_{21} - i\Omega_{p2}\rho_{24}, \end{aligned} \tag{2b}$$

$$\begin{aligned} \dot{\rho}_{33} =& \gamma_{43}\rho_{44} - \gamma_{31}\rho_{22} + i\Omega_{c2}^*\rho_{43} + i\Omega_{p1}\rho_{13} \\ &- i\Omega_{c2}\rho_{34} - i\Omega_{p1}^*\rho_{31}, \end{aligned} \tag{2c}$$

$$\begin{aligned} \dot{\rho}_{44} =& -(\gamma_{43} + \gamma_{42})\rho_{44} + i\Omega_{p2}\rho_{24} + i\Omega_{c2}\rho_{34} \\ &- i\Omega_{p2}^*\rho_{42} - i\Omega_{c2}^*\rho_{43}, \end{aligned} \tag{2d}$$

$$\begin{aligned} \dot{\rho}_{12} =& (-\gamma_{21}/2 + i\Delta_{c1})\rho_{12} + i\Omega_{c1}^*(\rho_{22} - \rho_{11}) \\ &+ i\Omega_{p1}^*\rho_{32} - i\Omega_{p2}\rho_{14}, \end{aligned} \tag{2e}$$

$$\begin{aligned} \dot{\rho}_{13} =& (-\gamma_{31}/2 + i\Delta_{p1})\rho_{13} + i\Omega_{p1}^*(\rho_{33} - \rho_{11}) \\ &+ i\Omega_{c1}^*\rho_{23} - i\Omega_{c2}\rho_{14}, \end{aligned} \tag{2f}$$

$$\begin{aligned} \dot{\rho}_{14} =& [-(\gamma_{43} + \gamma_{42})/2 + i(\Delta_{c1} + \Delta_{p2})]\rho_{14} \\ &+ i\Omega_{c1}^*\rho_{24} + i\Omega_{p1}^*\rho_{34} - i\Omega_{p2}^*\rho_{12} - i\Omega_{c2}^*\rho_{13}, \end{aligned} \tag{2g}$$

$$\begin{aligned} \dot{\rho}_{23} =& [-(\gamma_{21} + \gamma_{31})/2 + i(\Delta_{p1} - \Delta_{c1})]\rho_{23} + i\Omega_{c1}\rho_{13} \\ &+ i\Omega_{p2}^*\rho_{43} - i\Omega_{c2}\rho_{24} - i\Omega_{p1}^*\rho_{21}, \end{aligned} \tag{2h}$$

$$\begin{aligned} \dot{\rho}_{24} =& [-(\gamma_{43} + \gamma_{42} + \gamma_{21})/2 + i\Delta_{p2}]\rho_{24} \\ &+ i\Omega_{p2}^*(\rho_{44} - \rho_{22}) + i\Omega_{c1}\rho_{14} - i\Omega_{c2}^*\rho_{23}, \end{aligned} \tag{2i}$$

$$\begin{aligned} \dot{\rho}_{34} =& [-(\gamma_{43} + \gamma_{42} + \gamma_{31})/2 + i\Delta_{c2}]\rho_{34} \\ &+ i\Omega_{c2}^*(\rho_{44} - \rho_{33}) + i\Omega_{p1}\rho_{14} - i\Omega_{p2}^*\rho_{32}, \end{aligned} \tag{2j}$$

along with the equations of their complex conjugates $\rho_{ij} = \rho_{ji}^*$ and $\sum_{j=1}^{4}\rho_{jj} = 1$.

Now we consider a medium of length L composed by the above described atomic system immersed in unidirectional ring cavity as shown in Fig. 1 of Ref. [5] but with the notation change of $E_1 \to E_c$ and $E_2 \to E_p$ and adopt a standard model[1] for studying OB, in which the mirrors M_3 and M_4 are perfect reflectors, while the intensity reflection and transmission coefficients of mirrors M_1 and M_2 are R and T, respectively ($R + T = 1$). The total electromagnetic field can be written as

$$E = E_p e^{-i\omega_p t} + E_c e^{-i\omega_c t} + \text{c.c.}, \tag{3}$$

where E_p is the amplitude of the probe field, which circulate in the ring cavity and E_c is the amplitude of the coupling field that does not circulate in the ring cavity. Then under slowly varying envelope approximation, the dynamic equation of the probe field is described by

$$\frac{\partial E_p}{\partial t} + c\frac{\partial E_p}{\partial z} = i\frac{\omega_p}{2\varepsilon_0}P(\omega_p), \tag{4}$$

where $P(\omega_p)$ is the induced polarization $P(\omega_p) = N(d_{13}\rho_{13} + d_{24}\rho_{24})$, in which N is the atomic number density of the medium. For a perfectly tuned cavity, in the steady state limit, the boundary conditions between the incident field E_p^I and the transmitted field E_p^T are[21]

$$E(0) = \sqrt{T}E_p^I + RE_p(L), \tag{5a}$$

$$E_p^T = \sqrt{T}E_p(L). \tag{5b}$$

Under the mean-field limit[22] and associating with the boundary conditions Eqs. (5) and the Maxwell equation (4), we drive the mean-field steady state equation of the transmitted field,

$$y = 2x - iC[\rho_{13} + (d_{24}/d_{13})\rho_{24}], \tag{6}$$

where $C = LN\omega_p d_{13}^2/\hbar c\varepsilon_0 T$ is the atomic cooperation parameter, $x = d_{13}E_p^T/\sqrt{T}$, and $y = d_{13}E_p^I/\sqrt{T}$. In general, the dipole moments can satisfy $d_{13} = cd_{24}$ and $d_{12} = sd_{34}$ (s and c are real), for the sake of simplification we choose $s = c = 1$ in the following computation. Therefore the Rabi frequencies satisfy the relation $\Omega_{p1} = \Omega_{p2} = G_p$ and $\Omega_{c1} = \Omega_{c2} = G_c$,

where G_p and G_c are regarded as real parameters. By setting the time derivatives $\dot{\rho}_{ij} = 0$ $(i, j = 1 - 4)$, we obtain the corresponding steady-state density matrix, together with the field equation (6) then we can obtain the steady-state solutions. In the following numerical calculation, note that all involving parameters are reduced to dimensionless units by scaling γ and select the system parameters as $\gamma_{42} = \gamma_{43} = \gamma$, $\gamma_{21} = \gamma_{31} = 1.2\gamma$, $\omega_{21} - \omega_{43} = \omega_{31} - \omega_{42} = 7\gamma$, then $\Delta_{c1} - \Delta_{c2} = \Delta_{p1} - \Delta_{p2} = 7\gamma$.

We give a few numerical results for the steady state of the output field intensity $|x|$ versus the input field intensity $|y|$ with different corresponding parameters, as illustrated in Figs. 2–5. In Figs. 2, we show that the influence of the coupling field intensity G_c and the cooperative parameter C on the atomic OB behaviour, respectively. It is easy to see from Fig. 2(a) that the bistable threshold are dramatically augmented as enhancing the intensity of the coupling field. The reason can be qualitatively explained as follows. The coupling field applied to the transitions $|1\rangle \leftrightarrow |2\rangle$ and $|3\rangle \leftrightarrow |4\rangle$ can modify the absorption of the probe field between $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$ and the Kerr nonlinearity of the atomic medium, which lead to the change of the OB behaviour, as shown in Fig. 2(a). In Fig. 2(b), we display the behaviour of OB for different values of the atomic cooperation parameter C. It is shown that the threshold value becomes larger and the bistable hysteresis cycle becomes wider as the value of the atomic cooperation parameter increases. When the cooperation parameter less than 25, the OB will be disappearance, which can be used to control the behaviour of OB. From the term $C = LN\omega_p d_{13}^2/\hbar c\varepsilon_0 T$, we can see that the cooperation parameter C is proportional to atomic number density N. Thus the raise of the threshold intensity can be explained as the enhancement in the absorption of the sample as the number density of atoms increases.

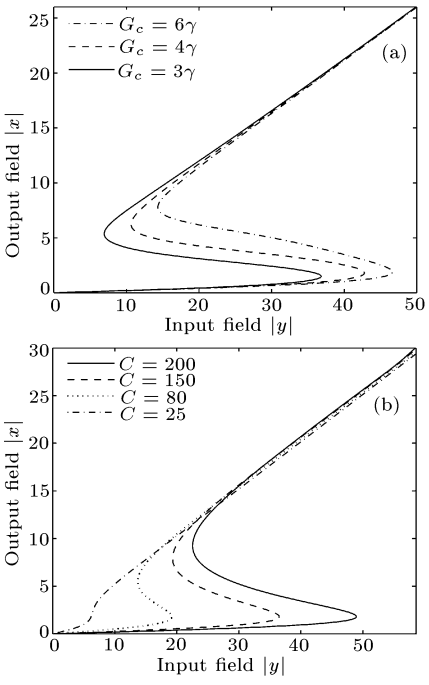

Fig. 2. Plots of the input-output field curves for different values of G_c ((a) $C = 200$) and for different values of C ((b) $G_c = 30\gamma$). The other parameters are chosen as $\gamma_{42} = \gamma_{43} = \gamma$, $\gamma_{21} = \gamma_{31} = 1.2\gamma$, $\Delta_{p1} = \Delta_{c1} = 0$, and $\Delta_{p2} = \Delta_{c2} = -7\gamma$.

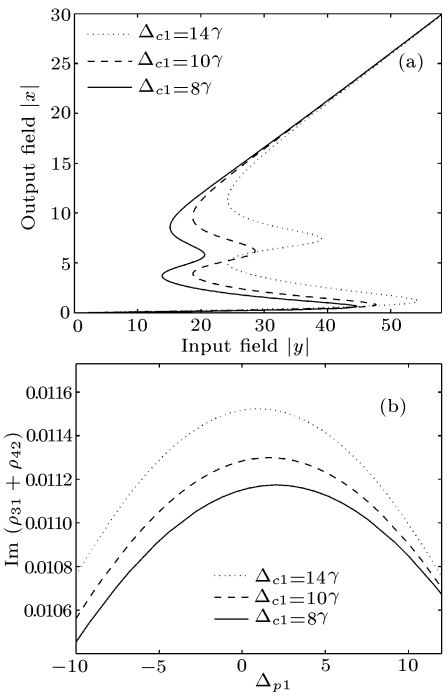

Fig. 3. Plots of the input-output field curves ((a) $\Delta_{p1} = 3\gamma$) and the absorption curve ((b) $G_p = 25\gamma$) of the probe field for different values of Δ_{c1}. The other parameters are $\gamma_{42} = \gamma_{43} = \gamma$, $\gamma_{21} = \gamma_{31} = 1.2\gamma$, $C = 200$, $\Delta_{c2} = \Delta_{c1} - 7\gamma$, $\Delta_{p2} = \Delta_{p1} - 7\gamma$, and $G_c = 5\gamma$.

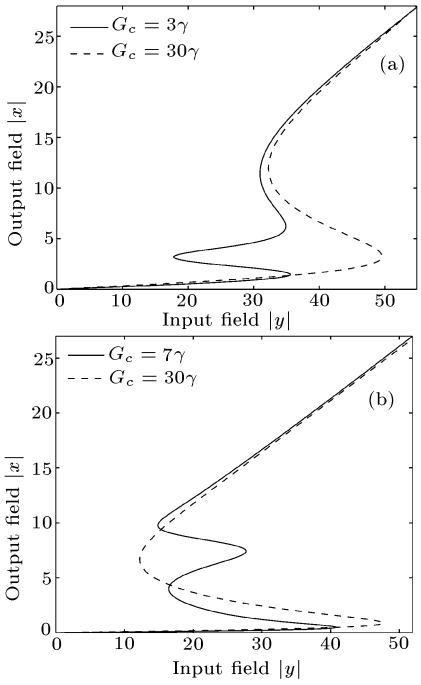

Fig. 4. Plots of the input–output field curves for different values of G_c: (a) considering two photon resonance and choosing $\Delta_{c1} = -\Delta_{p2} = 10\gamma$, then $\Delta_{p1} = -\Delta_{c2} = 3\gamma$; (b) considering common detunings $\Delta_{c1} = -5\gamma$, $\Delta_{p1} = 2\gamma$, then $\Delta_{c2} = -12\gamma$, $\Delta_{p2} = -5\gamma$. The other parameters values are chosen as $\gamma_{42} = \gamma_{43} = \gamma$, $\gamma_{21} = \gamma_{31} = 1.2\gamma$, and $C = 200$.

In Fig. 3(a), we plot the input-output field curves for different values of the frequency detuning of the coupling field, i.e. Δ_{c1} ($\Delta_{c2} = \Delta_{c1} - 7$). The other parameters are $\gamma_{42} = \gamma_{43} = \gamma$, $\gamma_{21} = \gamma_{31} = 1.2\gamma$, $\Delta_{p1} = 3\gamma$, $\Delta_{p2} = \Delta_{p1} - 7\gamma$, $C = 200$, and $G_c = 5\gamma$. It can be easily seen that decreasing the frequency detuning of the coupling field can lead to a reduction of the multistable threshold. In order to explain such phenomena, we study the absorption of the probe field for $\Delta_{c1} = 14\gamma$ (dotted line), $\Delta_{c1} = 10\gamma$ (dashed line),

and $\Delta_{c1} = 8\gamma$ (solid line) in Fig. 3(b). From this figure, we can see that the absorption is reduced with the decrease of the frequency detuning of the coupling field at $\Delta_{p1} = 3\gamma$. Thus the multistable threshold becomes smaller as the frequency detuning of the coupling field decreases.

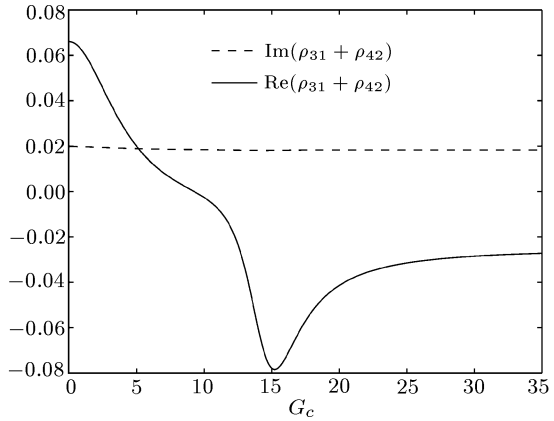

Fig. 5. Plot of the absorption $[\mathrm{Im}(\rho_{31}+\rho_{42})]$ and dispersion $[\mathrm{Re}(\rho_{31}+\rho_{42})]$ versus the coupling field intensity G_c. The parameters are the same as those in Fig. 4(b) except for $G_p = 15\gamma$.

In Fig. 4(a), we show the numerical results for steady state of the output field amplitude $|x|$ as a function of the input field amplitude $|y|$ for different values of the intensity of the coupling field G_c under two-photon resonance ($\Delta_{c1}+\Delta_{p2} = \Delta_{p1}+\Delta_{c2} = 0$). It is shown that the multistable behaviour disappears as the intensity of the coupling field increases. The same results can also be obtained under common frequency detuning of the two applied laser fields as shown in Fig. 4(b). The parameters are chosen as $\Delta_{c1} = -5\gamma$, $\Delta_{p1} = 2\gamma$, then $\Delta_{c2} = -12\gamma$, $\Delta_{p1} = -5\gamma$, which satisfy the condition $\Delta_{c1} - \Delta_{c2} = \Delta_{p1} - \Delta_{p2} = 7\gamma$. A qualitative explanation for this phenomena is that too large intensity of the coupling field can lead to a reduction in the Kerr nonlinearity of the atomic medium, which makes the disappearance of the OM behaviour. In order to gain deeper insight into this phenomena, we plot the absorption $[\mathrm{Im}(\rho_{31}+\rho_{42})]$ and dispersion $[\mathrm{Re}(\rho_{31}+\rho_{42})]$ versus the coupling field intensity G_c in Fig. 5. Then we find that the change of the dispersion associated with the Kerr nonlinearity of the atomic medium is much larger than the absorption. Thus the influence of the absorption on the OB behaviour can be neglected in the process of adjusting the value of G_c. According to the above discussion, we can realize the switch between the OB and OM just by tuning the intensity of the coupling field in our scheme.

In conclusion, we have analysed in detail the OB and OM behaviour of a closed-loop configuration atomic system driven by a degenerate coupling field and a degenerate probe field inside a unidirectional ring cavity. It is clearly shown that the coupling field intensity and the cooperation parameter can affect the OB behaviour dramatically, which can be used to control the change of the OB behaviour. We also observe the OM behaviour in a proper range of the frequency detuning. More importantly, we present a new means for realizing the switch between the OB and OM by adjusting the intensity of the coupling field.

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