On Charged Black Hole in Multidimensional Theory with Ricci-Flat Internal Spaces

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A generalization of the Reissner-Nordstrom solution for the case of n internal Ricci-flat spaces is obtained.

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In Ref. 1 the Schwarzschild solution is generalized for the case of n internal Ricci-flat spaces. It was proved that a horizon in the 4-dimensional section of the metric exists only when the internal space scale factors are constant. In this paper we consider a generalization of the solution[2,3] on the electro-vacuum case.

Let us consider the action

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{4} F_{MN} F^{MN} \right] , \tag{1}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the strength of the electromagnetic field A, R is a scalar curvature of the metric $g_{MN} dx^M \otimes dx^N$ and κ is gravitational constant. The field equations, corresponding to Eq. (1), are

$$\nabla_M F^{MN} = 0 , \tag{2}$$

$$R_{MN} - (1/2) g_{MN} R = \kappa^2 \left[F_{MP} F^P_N - (1/4) g_{MN} F_{PQ} F^{PQ} \right] . \tag{3}$$

Let us consider the D-dimensional manifold

$$M = M_0^{(4)} \times M_1 \times \cdots \times M_n , \tag{4}$$

where $M_0^{(4)}$ is 4-dimensional space-time manifold, M_i are Ricci-flat manifolds with the metrics $g_{(i)}$ and

$$\dim M_i = N_i , \quad D = 4 + \sum_{i=1}^{n} N_i , \quad i = 1, \cdots, n.$$

We are interested in static, spherically symmetric (O(3)-symmetric) solutions of Eqs. (2) and (3) on the manifold (4), and so we consider the following ansatz for the metric

$$\begin{aligned} g \;=\; & -e^{2\beta_{-1}(u)} dt \otimes dt + e^{2 \sum_{\nu=-1}^{n} N_\nu \beta_\nu(u)} du \otimes du \\ & + e^{2\beta_0(u)} d\Omega^2 + \sum_{i=1}^{n} e^{2\beta_i(u)} g_{(i)} , \end{aligned} \tag{5}$$

and for A_μ

$$A_0 = \varphi(u) \ , \quad A_i = 0 \ , \quad i = 1,2,3. \tag{6}$$

In Eq. (5), $d\Omega^2$ is the standard metric on S^2, $N_{-1} = 1$, $N_0 = 2$ and u is a radial variable.

From Eqs. (2), (5) and (6) we have

$$\varphi' = Qe^{2\beta_{-1}} \ , \tag{7}$$

where Q is a constant. Using Eqs. (5) and (7) we find that Eq. (3) is equivalent to the following system of equations

$$\beta''_{-1} = \kappa^2 Q^2 \frac{D-3}{D-2} e^{2\beta_{-1}} \ , \tag{8}$$

$$\sum_{\nu=-1}^{n} [-\beta''_\nu + \beta'_\nu \alpha' - (\beta'_\nu)^2] N_\nu = -\kappa^2 Q^2 \, e^{2\beta_{-1}} \frac{D-3}{D-2} \ , \tag{9}$$

$$1 - e^{2\beta_0 - 2\alpha} \beta''_0 = \frac{\kappa^2 Q^2}{D-2} \, e^{2\beta_0 + 2\beta_{-1} - 2\alpha} \ , \tag{10}$$

$$\beta''_i = \kappa^2 \frac{Q^2}{2-D} \, e^{2\beta_{-1}} \ , \quad i = 1, \cdots, n, \tag{11}$$

where $\alpha = \sum\limits_{\nu=-1}^{n} N_\nu \beta_\nu$. Solving Eqs. (7)–(11), we obtain

$$\begin{aligned} g \ = \ & -f_1(u) dt \otimes dt + (f_1(u))^{1/(3-D)} f_2^2(u) e^{-2\sum\limits_{i=1}^{n} N_i (A_i u + D_i)} du \otimes du \\ & + (f_1(u))^{1/(3-D)} f_2(u) e^{-2\sum\limits_{i=1}^{n} N_i (A_i u + D_i)} d\Omega^2 \\ & + \sum_{i=1}^{n} f_1^{1/(3-D)}(u) e^{2(A_i u + D_i)} g_{(i)} \ , \end{aligned} \tag{12}$$

and

$$\varphi = -\frac{1}{\kappa^2 Q} \left(C_1 \frac{D-2}{D-3} \right)^{1/2} \operatorname{cth} \left[\left(C_1 \frac{D-3}{D-2} \right)^{1/2} (u - u_1) \right] + \varphi_0 \ . \tag{13}$$

In Eq. (12)

$$f_1(u) = C_1 / \kappa^2 Q^2 \operatorname{sh}^2 \left[\left(C_1 \frac{D-3}{D-2} \right)^{1/2} (u - u_1) \right] \ ,$$

$$f_2(u) = C_2 / \operatorname{sh}^2 \left[\sqrt{C_2} \, (u - u_2) \right] \ ,$$

where D_i, φ_0, u_1, u_2 and $Q \neq 0$ are arbitrary constants and the constants C_1, C_2 and A_i obey the following relation

$$2C_2 = C_1 + \left(\sum_{i=1}^{n} N_i A_i \right)^2 + \sum_{i=1}^{n} N_i A_i^2 \ . \tag{14}$$

For $N_i \to 0, D \to 4$ we have the well-known Reissner-Nordstrom solution

$$\begin{aligned} g &= -\left(1 - L/r + \kappa^2 Q^2/2r^2\right) dt \otimes dt \\ &\quad + \left(1 - L/r + \kappa^2 Q^2/2r^2\right)^{-1} dr \otimes dr + r^2 d\Omega^2 , \end{aligned} \tag{15}$$

$$\varphi = Q/r + \text{Const} . \tag{16}$$

In our case ($D > 4$) the Coulomb law (16) is modified by the presence of internal dimensions. The dependence of the potential φ on the radial variable r, where

$$r^2 = f_1^{1/(3-D)}(u) f_2(u) e^{-2\sum_{i=1}^{n} N_i (A_i u + D_i)} d\Omega^2 \tag{17}$$

(see Eq. (12)), may be found by substitution of $u = u(r)$ from Eq. (13).

Note that the solution (12)–(13) may be generalized also on p-adic and $D = d + \sum_{i=1}^{n} N_i$($O(d-1)$-symmetric) cases, as it was done in Ref. 4.

References

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