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Self-Affine Multifractal Spectrum and Levy Stability Index from NA27 Data *

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A self-affine analysis of multiparticle production in pp collisions at 400 GeV/c was performed by using the method of continuously varying scale and the method of the factorial moments of continuous order. The self-affine generalized fractal dimensions and multifractal spectrum have been obtained. The self-affine multifractal spectrum is concave downward with a maximum at $q = 0$, $f(\alpha(0)) = D(0) = 1$. $D(q)$ decreases with increasing q showing that there is self-affine multifractal behaviour in multiparticle production at the 400 GeV/c pp collisions. The Levy index $\mu > 1$ indicates that a non-thermal phase transition may exist in the pp collisions at 400 GeV/c.

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Since Bialas and Peschansky proposed to study nonstatistical fluctuation in multiparticle production by the method of factorial moments $F_q(\delta)$,[1] a large variety of experiments have been performed to search for the anomalous scaling behaviour[2]

$$F_q(\delta) \propto (\delta)^{-\phi_q}, \qquad \delta \to 0\ . \tag{1}$$

The results have shown that the power-law behaviour does not hold exactly for high-energy hadron–hadron collisions. The reason for this is that the usual procedure for calculating higher-dimensional factorial moments is to divide phase space into bins with the same M in each direction. This is called the self-similar analysis. However, phase space in high-energy multiparticle production is anisotropic. Wu and Liu[3] have proposed a new method to calculate the higher-dimensional factorial moments, which is called the self-affine analysis. In the self-affine analysis, different shrinking ratios are taken for different phase-space directions.

Recently, the expected anomalous scaling has been observed successfully in the data of both the π^+p and K^+p collisions at 250 GeV/c from NA22 (Ref. 4) and the pp collisions at 400 GeV/c from NA27 (Ref. 5). The results, however, only show that the second-order factorial moment has power-law behaviour. This is not enough for understanding the properties of the dynamical fluctuations in high-energy multiparticle production. We should study the behaviour of the self-affine fractality. The multifractal dimensions (Renyi dimensions) are defined using the scaling behaviour of the moments.[6] If the anisotropic self-affine fractal object has a definite Hurst exponent, and the shrinkage of phase space is done according to the Hurst exponent, then we can obtain the multifractal dimensions and the multifractal spectrum.

In this letter, we present the results of self-affine analysis for the data of multiparticle production in pp collisions at 400 GeV/c by using the method of continuously varying scale and the method of factorial moments of continuous order.

Hwa[7] has proposed a new method to obtain factorial moments of continuous order $F(q)$ which eliminates statistical fluctuations. The suggestion is to expand the multiplicity distribution P_n in a series of negative binomial distributions:

$$P_n = \sum_{j=0}^{N} a_j P_n^{NB}(k_j, x_j). \tag{2}$$

The number of expansion coefficients a_j is $N+1$, where N is the observed maximum multiplicity. The Monte Carlo simulation indicated that Hwa's method cannot be applied to real experimental data,[8] because the method requires that the expansion penetrates all the experimental points P_n precisely. So, large statistical noises are included in the expansion coefficient a_j. Very large a_j with alternating signs may appear and cause great instability for the results of $F(q)$. So, Zhang and Wang[9] have extended Hwa's method in order to apply it to real experimental data. Considering that the negative binomial distribution can fit the experimental multiplicity distribution well, they have proposed to choose a smaller number of a_j to fit the experimental data by using the maximum likelihood method.

$$\widetilde{P}_n = \sum_{j=0}^{J} a_j P_n^{NB}(k_j, x_j), \tag{3}$$

where $n = 0, 1, \cdots, N$ (N is the maximum observed multiplicity), $J < N$ and

$$P_n^{NB}(k_j, x_j) = \frac{\Gamma(n+k_j)}{\Gamma(n+1)\Gamma(k_j)} \left(\frac{k_j}{k_j + x_j}\right)^{k_j} \left(\frac{x_j}{k_j + x_j}\right)^{n}, \tag{4}$$

where

$$x_j = x(1+\Delta_j), \quad k_j = K(1+\Delta_j), \tag{5}$$

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and

$$x = \langle n \rangle = \sum_{n=0}^{N} n P_n, \tag{6}$$

$$K = (F_2 - 1)^{-1}, \tag{7}$$

$$F_2 = \langle n(n-1) \rangle / x^2, \tag{8}$$

$$\Delta_j = \Delta(-\frac{1}{2} + \frac{j}{J}), \quad j = 1, 2, \cdots, J, \tag{9}$$

where Δ can be set to 0.5 and Δ_j ranges from $-\Delta/2$ to $\Delta/2$ in equal steps. The fit is performed with $N+1$ points P_n. Considering that the event number N_n at large n is very small, the fit should be performed by using the maximum likelihood method. We select a_j so that the following likelihood function L reaches its maximum:

$$L = \frac{N_{\rm ev}!}{\prod_{n=0}^{N} N_n!} \prod_{n=0}^{N} (N_{\rm ev} \widetilde{P}_n)^{N_n}, \tag{10}$$

where $N_{\rm ev}$ is the total number of events, N_n is the number of events with multiplicity n. After getting the expansion coefficients a_j, we can calculate the factorial moments of continuous order $F(q)$ according to the following formulae:

$$F(q) = \frac{f(q)}{(f(1))^q}, \tag{11}$$

where

$$f(q) = \sum_{j=0}^{J} a_j f^{NB}(q, j), \tag{12}$$

and

$$f^{NB}(q, j) = \left[\frac{x_j}{k_j}\right]^q \frac{\Gamma(q + k_j)}{\Gamma(k_j)} \quad . \tag{13}$$

The self-affine analysis has been performed for NA27 data.[5] The obtained Hurst exponent is $H_{\eta\varphi} = 0.74$. In order to obtain the two-dimensional factorial moment of continuous order, different shrinking ratios are taken for different phase-space directions. In order to reduce the effect of non-flat average distribution, the cumulative variables X_η and X_φ are used instead of η and φ.[10] In the $X_\eta(X_\varphi)$ space we divided the region [0,1] into $M_\eta(M_\varphi)$ bins, respectively. The partitioning was taken as $M_\eta = M_\varphi^{H_{\eta\varphi}}$ (M_φ is an integer from 1 to 20; M_η is a non-integer from 1 to 9.18). The $(\eta - \varphi)$ space is divided into $M = M_\eta \times M_\varphi$ bins and the calculation is done in each bin independently. Firstly, the multiplicity distribution $P_{n,m}$ of bin m is calculated for the experimental data. Secondly, $P^{NB}_{n,m}(k_{j,m}, x_{j,m})$ is calculated according to Eqs. (4)–(9), and $a_{j,m}$ is determined by the maximum likelihood method according to Eqs. (3) and (10). The number of a_j is determined according to the maximum multiplicity $N_{\rm max}$ in the bin m. When $N_{\rm max} > 6$, $J = 4$; $N_{\rm max} = 5$ and 6, $J = 3$; otherwise, $J = 2$. Then, $f_m(q)$ is calculated for the bin m according to Eq. (11). Lastly, an average is made over each bin:

$$F(q, M) = \frac{1}{M_t} \sum_{m=1}^{M_t} \frac{f_m(q)}{[f_m(1)]^q} \quad , \tag{14}$$

where $M_t = M_\varphi \times \text{integer}(M_\eta)$. So, we obtained the factorial moments of continuous order (q from -1 to 4 with step 0.2).

The two-dimensional $\ln F(q, M)$ versus $\ln M$ for $q = 2, 3, 4$ are shown in Fig. 1. A better scaling behaviour is clearly seen for the factorial moments of continuous order. We obtain the intermittency exponents $\phi(q)$ by fitting $\ln F(q, M)$ versus $\ln M$ ($M \geq 2$). The intermittency exponents $\phi(q)$ for $q = 2$, 3, 4 are listed in Table 1.

Table 1. Intermittency exponents $\phi(q)$ for q =2, 3 and 4.

q	2	3	4
$\phi(q)$	0.049 ± 0.006	0.136 ± 0.015	0.224 ± 0.023

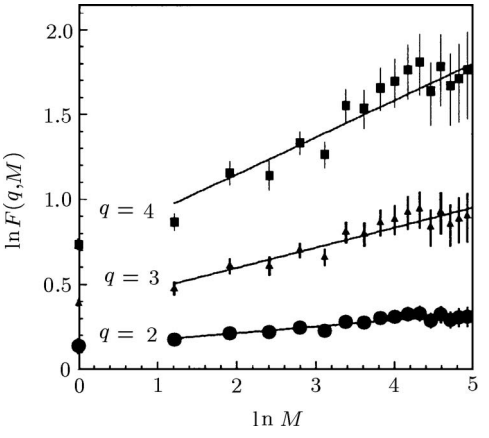

Fig. 1. Two-dimensional $\ln F(q, M)$ versus $\ln M$ for the data obtained using the method of factorial moment of continuous order.

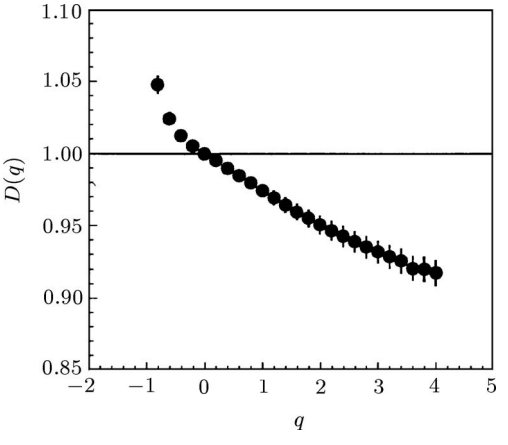

Fig. 2. Multifractal dimension $D(q)$ versus q.

We obtained the intermittency exponents $\phi(q)$ for continuous order q by fitting $\ln F(q,M)$ versus $\ln M$. From $\phi(q)$, we can calculate the multifractal dimension $D(q)$ and multifractal spectrum $f(\alpha)$ of the self-affine fractal using the relations

$$\tau(q) = q - 1 - \phi(q), \quad D(q) = \frac{\tau(q)}{q-1},$$

$$\alpha = \frac{\mathrm{d}\tau(q)}{\mathrm{d}q}, \quad f(\alpha) = q\alpha - \tau(q). \tag{15}$$

$D(q)$ versus q is shown in Fig. 2, in which $D(q)$ decreases with increasing q, showing that there is self-affine multifractal behaviour in multiparticle production at 400 GeV/c pp collisions. The self-affine multifractal spectrum $f(\alpha)$ is shown in Fig. 3. It is a concave downward with a maximum at $q = 0$, $f(\alpha(0)) = D(0) = 1$. This means that multiparticle production at 400 GeV/c pp collisions is a complex self-affine multifractal process.

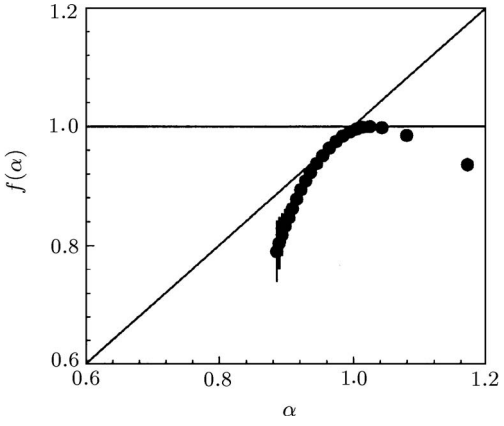

Fig. 3. Characteristics of multifractal spectrum $f(\alpha)$ versus α.

Levy stable laws are introduced to describe intermittency in multiparticle production at high-energy collisions. The Levy index helps in classifying intermittency regimes due to different kinds of phase transitions during the cascading process: (a) $0< \mu < 1$, there is a thermal phase transition (quark–gluon plasma); (b) $\mu > 1$, there is a "non-thermal" phase transition.[11] The problem is how to measure the Levy index. Usually, people use the factorial moment to obtain the intermittency index $\phi(q)$ and then use the formula

$$\phi(q) = \phi(2)\frac{q^\mu - q}{2^\mu - 2} \quad , \tag{16}$$

to obtain the value of μ. In fact, only a few indices $\phi(q)$ of $q > 2$ (e.g. $q = 3, 4, 5$) can be obtained to fit the complicated formula (16). So, the results are not satisfactory. Recently, Hu *et al.*[12] have proposed a method to extract the Levy index from the multifractal spectrum $f(\alpha)$ by using the method of factorial moments of continuous order. From Eqs. (15) and (16) we can obtain the following relationship

$$1 - f(\alpha) \propto (B - \alpha)^{\mu/(\mu-1)} \quad , \qquad \text{for } \alpha < B. \tag{17}$$

where B is the value of α when $f(\alpha) = 1$. From Fig. 3, the value of α for $f(\alpha) = 1$ is found to be $B = 1.0260$. The behaviour of $\ln(1-f(\alpha))$ versus $\ln(B-\alpha)$ is shown in Fig. 4. The linear fit of $\ln(1\text{-}f(\alpha))$ versus $\ln(B-\alpha)$ is performed. The slope $C = 2.0997 \pm 0.0098$ and the Levy index $\mu = C/(C-1) = 1.91 \pm 0.01$ are obtained. The Levy index $\mu > 1$ indicates that a non-thermal phase transition may exist in the pp collisions at 400 GeV/c. This result is consistent with ours.[13]

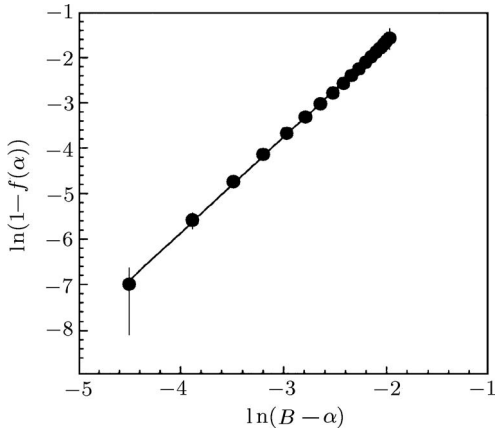

Fig. 4. Linear fit of $\ln(1 - f(\alpha))$ versus $\ln(B - \alpha)$ for $q = 0.2 - 4$.

A self-affine multifractal analysis in pp collisions at 400 GeV/c was performed using the method of factorial moments of continuous order. The self-affine multifractal dimension $D(q)$ and the multifractal spectrum $f(\alpha)$ are presented. The self-affine multifractal spectrum is a concave downward with a maximum at $q = 0$, $f(\alpha(0)) = D(0) = 1$. $D(q)$ decreases with increasing q showing that there is self-affine multifractal behaviour in multiparticle production at 400 GeV/c pp collisions. The Levy index $\mu > 1$ would discard the possibility of the formation of a quark–gluon plasma mixed with a cascading decay.

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