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Ferromagnetism in an Electron Gas Obeying Fractional Statistics

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We study an ideal electron gas obeying fractional statistics. The parameters for exclusion statistics are given by the strength of the repulsive interactions. In some parameter regions, it is found the system shows itinerant ferromagnetism.

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It is known that particles with some special interactions may obey fractional statistics in lower dimensions. For example, in the fractional quantum Hall effect, quasi-particles carry fractional chargers, which may be described by Chern–Simons theory.[1] In this theory, as well as others, the number of spatial dimensions plays a crucial role because fractional statistics is defined on the basis of monodromy properties of wavefunction. Recently, Haldane[2] proposed a definition of fractional statistics independent of the spatial dimensions. It is based on the state counting of many-body systems and can be considered as a generalization of the Pauli exclusion principle. Imagine that there is an N-particle system with species α_i, $i = 1, 2, \cdots, N$. Let D_{α_i} be the dimension of such a basis which may be changed when a particle is added into this space without changing the boundary and the size of the system. Haldane introduced the constants statistical interaction $g_{\alpha\beta}$ through the following relation:

$$\Delta D_\alpha = -\sum_\beta g_{\alpha\beta}\Delta N_\beta, \tag{1}$$

where N_α is the particle number in the α-th space and $g_{\alpha\beta}$ is a constant independent of N_α. Obviously, D_α denotes the number of rest states in this space. The special case of $g_{\alpha\beta} = 0$ and $g_{\alpha\beta} = \delta_{\alpha\beta}$ correspond to free bosons and free fermions, respectively. The thermodynamics of this model have been studied in detail by several authors.[3−7]

In this letter, we study an electron gas system obeying such statistics. The main observation of this letter is that the system may show itinerant ferromagnetism in some parameter region. To show the result clearly, we give a brief review of the fractional statistics. Integrating Eq.(1) with N_α, we obtain

$$D_\alpha = \frac{G_\alpha}{(2\pi)^3} - \sum_\beta g_{\alpha\beta}N_\beta, \tag{2}$$

where G_α denotes the phase volume without occupation in three dimensions. Thus we assume that G_α is irrelated to the spin orientation. That is to say, $G_\alpha = G_\beta$. The total number of microsocopic states with N_α is given by

$$W = \prod_\alpha \frac{(D_\alpha + N_\alpha)!}{N_\alpha! D_\alpha!}. \tag{3}$$

For convenience, we introduce the notation $g_{\uparrow\uparrow} = g$ and $g_{\uparrow\downarrow} = f$. The system is called the generalized ideal gas, if the total energy is given by

$$E = \sum_\alpha N_\alpha \epsilon_\alpha \tag{4}$$

with the one particle energy ϵ_α. Here ϵ_α is independent of the distribution $\rho_\alpha = N_\alpha/G_\alpha$. The distribution functions of the particles can been obtained by minimizing the thermodynamic potential $\Omega = E - TS - \sum_\alpha \mu_\alpha N_\alpha$ with respect to ρ_α. With the identity $S = \ln W$, the equation $\delta\Omega/\delta\rho_\alpha = 0$ gives

$$(1+\omega_\alpha)\prod_\beta \left(\frac{\omega_\beta}{1+\omega_\beta}\right)^{g_{\alpha\beta}} = \exp\left(\frac{\epsilon_\alpha - \mu_\alpha}{T}\right), \tag{5}$$

with

$$\omega_\alpha = \rho^*_\alpha/\rho_\alpha, \tag{6}$$

where μ_α is the chemical potential and $\rho^*_\alpha = D_\alpha/G_\alpha$ is the distribution function of the dual particle (hole) of α, ρ^*_α is related to ρ_α by

$$\rho^*_\alpha + \sum_\beta g_{\alpha\beta}\rho_\beta = \frac{1}{(2\pi)^3}, \tag{7}$$

which is obtained by dividing both sides of Eq.(2) by G_α. The above expression can also be rewritten as $\sum_\beta (\delta_{\alpha\beta}\omega_\alpha + g_{\alpha\beta})\rho_\beta = 1/(2\pi)^3$. Its expression is

$$\begin{aligned}\rho_\uparrow &= \frac{1}{(2\pi)^3}\frac{g - f + \omega_\downarrow}{(g+\omega_\downarrow)(g+\omega_\uparrow) - f^2},\\ \rho_\downarrow &= \frac{1}{(2\pi)^3}\frac{g - f + \omega_\uparrow}{(g+\omega_\downarrow)(g+\omega_\uparrow) - f^2}.\end{aligned} \tag{8}$$

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The thermodynamic potential has also a simple form:

$$\Omega = -T\sum_{\alpha} G_{\alpha}\ln[1+\omega_{\alpha}^{-1}]. \tag{9}$$

Now we discuss the properties at zero temperature. We apply a small magnetic field to the free particle system. Then $\epsilon_{0\uparrow}(k) = k^2/2 - \mu_{\rm B}H$ and $\epsilon_{0\downarrow}(k) = k^2/2 + \mu_{\rm B}H$, where $\mu_{\rm B}$ is the Bohr magneton and H is the external magnetic field. The $g > f$ case has been studied extensively as expected, such as a system that has a paramagnetic ground state as that of the ideal electron gas. Now a question arises, i.e., what happens for $g < f$? To answer this question, let us check the ground state.

(1) If all electron spin is polarized up in the ground state, and $|k_{\uparrow}| < \mu_{\rm F}$, then $\rho_{\uparrow} = 1/[(2\pi)^3 g]$ and $\epsilon_{\uparrow} = \epsilon_{0\uparrow}/g$, $\epsilon_{\downarrow} = \epsilon_{0\downarrow} - f\epsilon_{0\uparrow}/g$.

$$\begin{aligned} N &= \frac{4\pi V}{h^3}\int_{-\mu H}^{\mu_{\rm F}}\sqrt{2(\epsilon+\mu H)}\rho_{\uparrow}{\rm d}\epsilon \\ &= \frac{\sqrt{2}V}{3\pi^2 g h^3}\sqrt{(\mu_{\rm F}+\mu H)^3}. \end{aligned} \tag{10}$$

For $H = 0$, the ground-state energy is given by

$$E_1 = \frac{3N}{5}\left(\frac{3\pi^2 g h^3}{\sqrt{2}V}\right)^{2/3}. \tag{11}$$

(2) If the ground state is still paramagnetic, there should be 'generalized Fermi levels' $\mu_{{\rm F}\alpha}$ for $\alpha = \uparrow, \downarrow$ which satisfies $\epsilon_{\alpha}(k) < 0$ for $|k_{\alpha}| < \mu_{\alpha}$ and $\epsilon_{\alpha}(k) > 0$ for $|k_{\alpha}| > \mu_{\alpha}$, and $\rho_{\alpha}(k) = 0$ for $|k_{\alpha}| > \mu_{\alpha}$.

$$\begin{aligned} N &= N_{\uparrow} + N_{\downarrow} \\ &= \frac{4\pi V}{h^3}\left(\int_{-\mu H}^{\mu_{\rm F\uparrow}}\sqrt{2(\epsilon_{\uparrow}+\mu H)}\rho_{\uparrow}{\rm d}\epsilon_{\uparrow}\right. \\ &\quad \left. + \int_{\mu H}^{\mu_{\rm F\downarrow}}\sqrt{2(\epsilon_{\downarrow}+\mu H)}\rho_{\downarrow}{\rm d}\epsilon_{\downarrow}\right) \\ &= \frac{\sqrt{2}V}{3\pi^2(f+g)h^3}\left[\sqrt{(\mu_{\rm F\uparrow}+\mu H)^3}\right. \\ &\quad \left. + \sqrt{(\mu_{\rm F\downarrow}+\mu H)^3}\right], \end{aligned} \tag{12}$$

and the ground-state energy is:

$$E_2 = \frac{3N}{5}\left(\frac{3\pi^2(f+g)h^3}{2\sqrt{2}V}\right)^{2/3}. \tag{13}$$

Comparing Eqs. (11) and (13) we conclude that: (a) if $g > f$, the ground state is paramagnetic as expected; (b) if $f > g$, a ferromagnetic ground state is realized. While $f = g$ is a quantum critical point at which the quantum phase transition between the paramagnetic phase and ferromagnetic phase occurs. Careful calculations show that there is no phase transition at finite temperature for $f > g$. To our knowledge, this is the first time itinerant ferromagnetism has been obtained in an ideal fractional electron gas.

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