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Total Quantum Statistical Entropy of Reissner–Nordstrom Black Holes: Scalar Field Case

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The total quantum statistical entropy of Reissner–Nordstrom (RN) black holes is evaluated. The spacetime of the black holes is divided into three regions—region 1, ($r > r_o$); region 2, ($r_o > r > r_i$); andregion 3, ($r_i > r > 0$)—where r_o is the radius of the outer event horizon, and r_i is the radius of the inner event horizon. The total quantum statistical entropy of RN black holes is $S = S_1 + S_2 + S_3$, where S_i ($i = 1, 2, 3$) is the entropy, contributed by region S_i ($i = 1, 2, 3$). The detailed calculation shows that $S_2 \approx 0$. $S_1 = (\pi^2/45)[k_b A_o/\epsilon^2\beta^3]$, $S_3 = -(\pi^2/45)[k_b A_i/\epsilon^2\beta^3]$, where A_o and A_i are, respectively, the area of the outer and inner event horizons. Thus, as r_i approaches r_o, in the extreme case the total quantum statistical entropy of RN black holes approaches zero.

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The Hawking temperature of Reissner–Nordstrom (RN) black holes approaches zero as a non-extreme black hole approaches an extreme black hole. According to the Nernst theorem, the entropy of the extreme black hole must be zero. Some authors have achieved the nonzero entropy of RN black holes.[1−3]

Hawking *et al.*[4] have obtained the following results: (a) the entropy of an extreme RN black hole is zero, and (b) the entropy changes discontinuously in the extreme limit. Gibbons and Kallosh[5] concluded that the entropy of an extreme $U(1)$ black hole and other extreme black holes is zero. Using thermodynamic formalism, Zhao *et al.*[6] also arrived at the conclusion that the entropy of an extreme Kerr–Newman black hole is zero. However, they should have introduced the assumption that entropy contributed by the inner event horizon must be negative. Besides this, another problem arises, i.e. one cannot know the information inside the outer event horizon according to classical gravitational theory. There is no such problem in our model and theory; from the Hawking radiation of particle current and the continuation of the particle current we can know or detect the geometric structure and physical nature or property of regions 3 and 2.

In this letter, we calculate in detail the total quantum statistical entropy of the RN black holes by using the brick wall model,[7,8] and we obtain the formulae of the total free energy and total entropy. As the inner event horizon r_i approaches the outer event horizon r_o, i.e. as a non-extreme black hole approaches an extreme black hole the total entropy approaches zero.

The metric of an RN black hole is

$$\mathrm{d}s^2 = -B(r)c^2\mathrm{d}t^2 + \frac{1}{B(r)}\mathrm{d}r^2 + r^2\mathrm{d}\omega_2^2, \qquad (1)$$

where

$$B(r) = 1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{c^4 r^2} = \frac{1}{r^2}(r - r_o)(r - r_i), \quad (2a)$$

and

$$r_o = \frac{1}{c^2}[GM + \sqrt{G^2M^2 - GQ^2}],$$
$$r_i = \frac{1}{c^2}[GM - \sqrt{G^2M^2 - GQ^2}]. \qquad (2b)$$

The Klein–Gordon equation of a massless scalar particle is

$$\frac{1}{\sqrt{-g}}\partial_\alpha[\sqrt{-g}g^{\alpha\beta}\partial_\beta\Phi] = 0. \qquad (3)$$

In the following, we calculate the entropy S_1, S_2, S_3 contributed by the regions $1, 2, 3$.

(1) Region 1 ($r > r_o$) :

In the neighbourhood of the outer event horizon

$$B(r) = \frac{1}{r^2}(r - r_o)(r - r_i) \approx \frac{1}{r_o^2}(r_o - r_i)(r - r_o)$$
$$= p(r_o)(r - r_o). \qquad (4)$$

Taking another approximation[8]

$$r^2\mathrm{d}\Omega_2^2 \approx \mathrm{d}y^2 + \mathrm{d}z^2, \qquad (5)$$

where y, z are constrained in $-L/2 \le y, z \le L/2$, and $l_p \ll L \ll r_o$, and $l_p = (G\hbar)^{1/2}$ is the Planck length.

In considering the above approximations, metric (1) can be written as

$$\mathrm{d}s^2 = -p(r_o)(r - r_o)c^2\mathrm{d}t^2 + \frac{1}{p(r_o)(r - r_o)}\mathrm{d}r^2$$
$$+ \mathrm{d}y^2 + \mathrm{d}z^2. \qquad (6a)$$

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Dividing each term of Eq. (6a) by $l_{\rm p}^2$, then we obtain the dimensionless metric as Eq. (6a) except that the first term does not involve the factor c^2

$$\mathrm{d}s^2 = -p(r_{\rm o})(r-r_{\rm o})\mathrm{d}t^2 + \frac{1}{p_{\rm o}(r-r_{\rm o})}\mathrm{d}r^2 + \mathrm{d}y^2 + \mathrm{d}z^2. \tag{6b}$$

Setting $\mathrm{d}r/\sqrt{p(r_{\rm o})(r-r_{\rm o})} = \mathrm{d}x$, or $2\sqrt{r-r_{\rm o}}/\sqrt{p(r_{\rm o})} = x$, then Eq. (6b) becomes

$$\begin{aligned}\mathrm{d}s^2 &= -\frac{1}{4}p^2(r_{\rm o})x^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \\ &= -x^2\mathrm{d}T^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2,\end{aligned} \tag{7}$$

where T is the proper time.

Setting the testing solution of Eq. (3) by

$$\Phi = \exp(-\mathrm{i}\omega T)f(r)\exp\left[\mathrm{i}(k_1 y + k_2 z)\right]. \tag{8}$$

and substituting Eq. (8) into Eq. (3), we obtain

$$\frac{\omega^2}{x^2} - k^2 + \frac{1}{fx}\frac{\partial f}{\partial x} + \frac{1}{f}\frac{\partial^2 f}{\partial x^2} = 0, \tag{9}$$

where $k^2 = k_y^2 + k_z^2$.

Using the Wentzel–Kramers–Brillouin (WKB) approximation to solve Eq. (9), we can set

$$f \approx \exp(\mathrm{i}s). \tag{10}$$

From the real part of the derived equation from Eqs. (9) and (10), we obtain

$$p_x = \frac{\partial s}{\partial x} = \sqrt{\frac{\omega^2}{x^2} - k^2}. \tag{11}$$

The radical wavenumber is

$$\begin{aligned}n_{\rm r} &= \frac{1}{\pi}\int_{\epsilon}^{\sigma} p_x \mathrm{d}x = \frac{1}{\pi}\int_{\epsilon}^{\sigma}\mathrm{d}x\sqrt{\frac{\omega^2}{x^2} - k^2} \\ &= \frac{1}{\pi}\Big[-\sqrt{\omega^2 - k^2\epsilon^2} + \omega\ln\frac{k\epsilon}{\omega - \sqrt{\omega^2 - k^2\epsilon^2}}\Big],\end{aligned} \tag{12}$$

where ϵ is the cut-off factor which is used to avoid the infinite divergence in the integral of Eq. (12), and $\sigma = \omega/k$. The quantum states of energy less than or equal to ω are

$$\begin{aligned}g(\omega) &= (\frac{L}{2\pi})^2\int a n_{\rm r} k\mathrm{d}k \\ &= \frac{a}{\pi}(\frac{L}{2\pi})^2\int_0^{\omega/\epsilon} k\mathrm{d}k\Big[-\sqrt{\omega^2 - k^2\epsilon^2} \\ &\quad + \omega\ln\frac{k\epsilon}{\omega - \sqrt{\omega^2 - k^2\epsilon^2}}\Big],\end{aligned} \tag{13}$$

where $(L/2\pi)^2$ is the state density of two-dimensional wave vector space and $a = 3\pi^{3/2}/\Gamma[5/2] = 4\pi$.

The free energy corresponding to the outer event horizon area $\Delta A = L^2$ of the RN black hole is

$$\begin{aligned}\Delta F &= \frac{1}{\beta}\sum_n \ln(1 - \mathrm{e}^{-\beta\omega}) \\ &= \frac{1}{\beta}\int_0^{\infty}\mathrm{d}\omega\frac{\partial g(\omega)}{\partial\omega}\ln(1 - \mathrm{e}^{-\beta\omega}) \\ &= -\frac{1}{\pi}\frac{\Delta A}{(2\pi)^2}\frac{a}{2\epsilon^2}\frac{\zeta(4)}{\beta^4},\end{aligned} \tag{14}$$

where $\zeta(x)$ is the Riemann zeta function.

The entropy corresponding to ΔA is

$$\Delta S = k_{\rm b}\beta^2\frac{\partial F}{\partial\beta} = \frac{k_{\rm b}}{\pi}\frac{\Delta A}{(2\pi)^2}\frac{2a}{\epsilon^2}\frac{\zeta(4)}{\beta^3}, \tag{15}$$

where $k_{\rm b}$ is the Boltzmann constant.

The entropy corresponding to event horizon $r_{\rm o}$, or contributed by region 1, is

$$S_1 = \frac{\pi^2}{45}k_{\rm b}\frac{A_{\rm o}}{\epsilon^2\beta^3}. \tag{16}$$

(2) Region 2 $(r_{\rm i} < r < r_{\rm o})$:

In region 2, $B(r)$ changes sign

$$\begin{aligned}B(r) &= 1 - \frac{2GM}{c^2}r + \frac{GQ^2}{c^4 r^2} = \frac{1}{r^2}(r - r_{\rm o})(r - r_{\rm i}) \\ &\approx -\frac{1}{r_{\rm o}^2}(r_{\rm o} - r_{\rm i})(r_{\rm o} - r) = -p(r_{\rm o})(r_{\rm o} - r).\end{aligned} \tag{17}$$

In the inner side of event horizon $r_{\rm o}$ the metric Eq. (1) may be written approximately as

$$\begin{aligned}\mathrm{d}s^2 &= p(r_{\rm o})(r_{\rm o} - r)c^2\mathrm{d}t^2 \\ &\quad - \frac{1}{p(r_{\rm o})(r_{\rm o} - r)}\mathrm{d}r^2 + \mathrm{d}y^2 + \mathrm{d}z^2.\end{aligned} \tag{18}$$

We set

$$\mathrm{d}x = -\frac{r\mathrm{d}r}{[(r - r_{\rm i})(r_{\rm o} - r)]^{1/2}} \approx -\frac{r_{\rm o}\mathrm{d}r}{[(r_{\rm o} - r_{\rm i})(r_{\rm o} - r)]^{1/2}} \tag{19a}$$

or

$$x = \frac{2r_{\rm o}(r_{\rm o} - r)^{1/2}}{(r_{\rm o} - r_{\rm i})^{1/2}}. \tag{19b}$$

In considering the above approximation, the metric (1) can be written as

$$\mathrm{d}s^2 = x^2\mathrm{d}T^2 - \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2, \tag{20}$$

where $\mathrm{d}T = (1/2r_{\rm o}^2)(r_{\rm o} - r_{\rm i})\mathrm{d}t$, and T is the proper time.

Letting the testing solution of Eq. (3) be

$$\Phi = \mathrm{e}^{\mathrm{i}\omega T}f(r)\mathrm{e}^{\mathrm{i}(k_y y + k_z z)} \tag{21}$$

and substituting Eqs. (21) and (20) into Eq. (3), we can derive

$$-\frac{\omega^2}{x^2}-x^2-\frac{1}{fx}\frac{\partial f}{\partial x}-\frac{1}{f}\frac{\partial^2 f}{\partial x^2}=0. \tag{22}$$

Using the WKB approximation to solving Eq. (22), we can set

$$f\approx \mathrm{e}^{\mathrm{i}s}. \tag{23}$$

From the real part of the derived equation, we obtain

$$-\frac{\omega^2}{x^2}-k^2+\left(\frac{\partial s}{\partial x}\right)^2=0. \tag{24}$$

Note that the coordinate parameters $\{t,r,\theta,\phi\}$ or $\{T,x,y,z\}$ are the real parameters; the energy momentum parameters $\{\omega,p_r,k_y,k_z\}$ and $p_r=\partial s/\partial x$ are not real at all, due to the constrained Eq. (24). The solutions of Eq. (24) are divided into two cases: (1) ω and p_r are pure imaginary, k_y and k_z are real; (2) ω, p_r or s are real, k_y and k_z are pure imaginary.

Case (1) is the time decay wavefunction. Case (2) implies imaginary angular momentum, since k_y, k_z are related to angular momentum (see Eq. (5)).

For both cases, there is no static wavefunction or static state in region 2, so the state number n_r, the free energy F_2 and the entropy S_2 are zero.

The very important role played by region 2 is that it can produce particle–hole pairs under the action of electrostatic and gravitational fields. The particle goes through the outer event horizon into region 1, and the hole goes through the inner event horizon into region 3. This process is equal to the particle going in the positive direction of radical coordinate r through the inner event horizon into region 2.

(3) Region 3 $(0<r<r_{\mathrm{i}})$:

$$\begin{aligned}B(r)&=\frac{1}{r^2}(r-r_{\mathrm{o}})(r-r_{\mathrm{i}})\\&=\frac{1}{r^2}(r_{\mathrm{o}}-r)(r_{\mathrm{i}}-r)\approx\frac{1}{r_{\mathrm{i}}}(r_{\mathrm{o}}-r_{\mathrm{i}})(r_{\mathrm{i}}-r).\end{aligned} \tag{25}$$

$B(r)$ does not change its sign, i.e. the sign of the metric Eq. (1) in region 3 is the same as region 1. However, the physical situation is very different. If the particle energy level direction or coordinate points upwards, then the hole energy level direction or coordinate must point downwards. In other words, the energy level coordinate changes its sign, so the free energy ought to change its sign.

$$\begin{aligned}F_3&=\frac{1}{\beta}\sum_n\ln(1-\mathrm{e}^{\beta\overline{\omega}_n})=\frac{1}{\beta}\int_{-\infty}^{0}\mathrm{d}\overline{\omega}\frac{\partial g}{\partial\overline{\omega}}\ln(1-\mathrm{e}^{\beta\overline{\omega}})\\&=\frac{1}{\beta}\int_{\infty}^{0}[-\mathrm{d}\omega][-\frac{\partial g}{\partial\omega}]\ln(1-\mathrm{e}^{-\beta\omega})\\&=-\frac{1}{\beta}\int_0^{\infty}\mathrm{d}\omega\frac{\partial g}{\partial\omega}\ln(1-\mathrm{e}^{-\beta\omega})=\frac{A_{\mathrm{i}}}{(2\pi)^2}\frac{2}{\epsilon^2}\frac{\zeta(4)}{\beta^4}.\end{aligned} \tag{26}$$

Note that $\overline{\omega}$ is negative in the original filled band. The entropy contributed by region 3 is

$$S_3=-\frac{\pi^2}{45}k_{\mathrm{b}}\frac{A_{\mathrm{i}}}{\epsilon^2\beta^3}. \tag{27}$$

The total entropy of the RN black hole is

$$S=S_1+S_2+S_3=\frac{\pi^2}{45}k_{\mathrm{b}}\frac{1}{\epsilon^2\beta^3}(A_{\mathrm{o}}-A_{\mathrm{i}}). \tag{28}$$

When r_{i} approaches r_{o} the total entropy of the RN black hole approaches zero.

In the strict extreme state, $G^2M^2=GQ^2$ and $B(r)=\left(1-GM/c^2r\right)^2$, the two event horizons coincide.

Instead of using the approximation Eq. (5), we solve the Laplace operator on a two-dimensional sphere. The eigenvalue with a given l value is $l(l+1)$.

We set the test solution of Eq. (3) as

$$\Phi(T,x,\theta,\varphi)=\mathrm{e}^{\mathrm{i}\omega T}f(x)Y(\theta,\varphi). \tag{29}$$

Substituting Eq. (29) into Eq. (3) we obtain

$$\frac{\omega^2}{x^2}-\frac{l(l+1)}{r^2}+\frac{1}{fx}\frac{\partial f}{\partial x}+\frac{1}{f}\frac{\partial^2 f}{\partial x^2}=0. \tag{30}$$

Using the WKB approximation to solve Eq. (30) and setting

$$f\approx\exp(\mathrm{i}s), \tag{31}$$

we obtain

$$\begin{aligned}p_{\mathrm{r}}&=\frac{\partial s}{\partial x}=\sqrt{\frac{\omega^2}{x^2}-\frac{l(l+1)}{r^2}}.\\&\approx\sqrt{\frac{\omega^2}{x^2}-\frac{l^2}{r_{\mathrm{o}}^2}}=\sqrt{\frac{\omega^2}{2}x^2-k^2},\end{aligned} \tag{32}$$

where $k^2=l^2/r_{\mathrm{o}}^2$.

The radical wavenumber is

$$\begin{aligned}n_{\mathrm{r}}&=\frac{1}{\pi}\int_{\epsilon}^{\sigma}p_{\mathrm{r}}\mathrm{d}x=\frac{1}{\pi}\int_{\epsilon}^{\sigma}\mathrm{d}x\sqrt{\frac{\omega^2}{x^2}-k^2}\\&=\frac{1}{\pi}\left[-\sqrt{\omega^2-k^2\epsilon^2}+\omega\ln\frac{k\epsilon}{\omega-\sqrt{\omega^2-k^2\epsilon^2}}\right],\end{aligned} \tag{33}$$

where ϵ is the cut-off factor which is used to avoid the infinite divergence in the integral of Eq. (33), and $\sigma=\omega/k=r_{\mathrm{o}}\omega/l$.

The quantum states of energy less than or equal to ω are

$$
\begin{aligned}
g(\omega) &= \int 2n_{\rm r} l{\rm d}l = 2r_{\rm o}^2 \int n_{\rm r} k{\rm d}k \\
&= \frac{2r_{\rm o}^2}{\pi} \int_0^{\omega/\epsilon} k{\rm d}k \Big[- \sqrt{\omega^2 - k^2\epsilon^2} \\
&\quad + \omega \ln \frac{k\epsilon}{\omega - \sqrt{\omega^2 - k^2\epsilon^2}} \Big] \\
&= \frac{2r_{\rm o}^2}{\pi\epsilon^2} \int_0^{\omega} \rho{\rm d}\rho \left[-\sqrt{\omega^2 - \rho^2} + \omega \ln \frac{\rho}{\omega - \sqrt{\omega^2 - \rho^2}} \right] \\
&= \frac{A_{\rm o}}{2\pi^2\epsilon^2} \left[-\frac{1}{3}\omega^3 + \frac{1}{2}\omega^3 \right] = \frac{A_{\rm o}}{12\pi^2\epsilon^2}\omega^3. \qquad (34)
\end{aligned}
$$

The free energy of region 1 is

$$
\begin{aligned}
F_1 &= \frac{1}{\beta} \sum_n \ln((1 - {\rm e}^{-\beta\omega_n}) \\
&= \frac{1}{\beta} \int_0^{\infty} {\rm d}\omega \frac{\partial g(\omega)}{\partial \omega} \ln(1 - {\rm e}^{-\beta\omega}) \\
&= - \int_0^{\infty} {\rm d}\omega g(\omega) \frac{1}{{\rm e}^{\beta\omega} - 1} \\
&= - \frac{A_{\rm o}}{12\pi^2\epsilon^2} \int_0^{\infty} {\rm d}\omega \omega^3 \frac{1}{{\rm e}^{\beta\omega} - 1} \\
&= - \frac{A_{\rm o}}{12\pi^2\epsilon^2\beta^4} \int_0^{\infty} {\rm d}\eta \eta^3 \frac{1}{{\rm e}^{\eta} - 1} \\
&= - \frac{A_{\rm o}}{12\pi^2\epsilon^2\beta^4} 3!\zeta(4) = - \frac{A_{\rm o}}{2\pi^2\epsilon^2\beta^4} \frac{\pi^4}{90} \\
&= - \frac{\pi^2 A_{\rm o}}{180\epsilon^2\beta^4}. \qquad (35)
\end{aligned}
$$

The entropy of region 1 is

$$
S_1 = k_{\rm b}\beta^2 \frac{\partial F_1}{\partial \beta} = \frac{\pi^2}{45} k_{\rm b} \frac{A_{\rm o}}{\epsilon^2\beta^3}. \qquad (36)
$$

The results (16) and (36) show that approximation (5) is equivalent to approximation (32).

Summary and discussion:

(1) In this letter, the total quantum statistical entropy of RN black hole is calculated in detail. The entropy $S_1 = (\pi^2/45)k_{\rm b}A_{\rm o}/\epsilon^2\beta^3$ is contributed by region 1, the entropy $S_2 = 0$ is contributed by region 2, and the entropy $S_3 = -(\pi^2/45)k_{\rm b}A_{\rm o}/\epsilon^2\beta^3$ is contributed by region 3. The total quantum statistical entropy of the RN black hole is $S = S_1 + S_2 + S_3 = S_1 + S_3$.

(2) (i) The entropy S_1 is not the total entropy. Some authors think that S_1 is the total entropy of the RN black hole. (ii) The reasons why S_3 is negative are explained in detail.

(3) The free energy F and entropy S are the statistical quantities of the RN black hole. They are also equilibrium quantities. The Hawking temperature T is a semi-equilibrium quantity. On one hand, T is determined by the mass M and charge Q of the RN black hole, with M and Q being the equilibrium quantities. On the other hand, T determines the particle current density of Hawking radiation; the radiation is a non-equilibrium process.

(4) There is a similar situation in the conduction of high resistance solids or isolated crystals, e.g. the Si–SiO_2–Si system, the current density is $j = nev$, where n is the density of electric charge, e is the charge of electron or hole, and v is the mean velocity of the electric charge. The current density formula is valid only for weak electric fields, n is the thermal equilibrium quantity and v is a semi-equilibrium quantity. On one hand, v is partially determined by the equilibrium distribution function. On the other hand, v also is determined by the small deviation of the equilibrium distribution function which is a non-equilibrium quantity.

(5) Hawking radiation is very different from Vaidya radiation. In Hawking radiation case: (i) the mass M and charge Q of an RN black hole is independent of the time parameter t; (ii) the radiation particle current density and energy momentum tensor are neglected small; (iii) Hawking radiation is a quantum process or effect; (iv) the Hawking temperature T is a thermodynamic quantity in black hole thermodynamics. In Vaidya radiation case: (i) the mass $M(t)$ and charge $Q(t)$ of a Reissner–Nordstrom–Vaidya black hole are dependent on the time parameter t; (ii) the radiation particle current density and energy momentum tensor are dependent on $\partial M/\partial t$ and $\partial Q/\partial t$; (iii) Vaidya radiation is not a quantum process or effect; (iv) both $\partial M/\partial t$ and $\partial Q/\partial t$ are independent variables; (v) there is no direct relation between Vaidya radiation and temperature T.

(6) To approach the extreme RN black hole we must add electric charge to the black hole very slowly in order to keep the state of the black hole from deviating far from the equilibrium state. Near the extreme state we must add charge to the RN black hole infinitely slowly, so that the extreme state cannot reach a finite time interval, and the absolute zero Hawking temperature can never reach a finite time interval.

(7) The reason why we use the approximation Eq. (5) is that the eigenvalue of $r^{-2}{\partial_\theta}^2 + r^{-2}\sin^{-2}\theta{\partial_\varphi}^2$ is $l(l+1)/r^2 \approx l^2/r_{\rm o}^2$. The eigenvalue of $\partial_y^2 + \partial_z^2$ is $k_y^2 + k_z^2 = k^2$.

Many authors use the approximation $l(l+1) \approx l^2$ in their calculations, so our approximation is the same as those authors.

The article entitled Total Quantum Statistical Entropy of Fermi Field in Kerr–Newman Black Hole Spacetime is submitted elsewhere.

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