Density Dependence of the Beta Decay Width in Astronuclear Matter *

YUAN Wei(袁玮)[1], LIU Yu-Xin(刘玉鑫)[1,2,3,4**]

[1] *Department of Physics, Peking University, Beijing 100871*

[2] *Key Laboratory of Heavy Ion Physics of Ministry of Education, Peking University, Beijing 100871*

[3] *Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080*

[4] *Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000*

(Received 3 February 2004)

By combining the four-fermion interaction theory and the linear σ–ω model, we propose an approach to describe the density dependence of the β-decay width (the half-life time of the decay) in infinite and charge-neutral nuclear matter. We find that the half-life time of the β-decay decreases rapidly with the increase of the nuclear matter density.

PACS: 21. 65. +f, 23. 40. −s, 24. 10. Jv

Beta decay is a very important process with responsibility for the species evolution in astrophysical nuclear matter. Therefore, great efforts have been made to study the mechanism of the beta decay in astrophysical circumstance. Up to now, both experimental observation and theoretical results reveal that the beta decay width is very sensitive to temperature and density of astrophysical circumstance. Particularly, in the case of finite nucleus, once the environment temperature is high enough, there will be certain numbers of nuclei in excited states. The contribution of such excited states to beta decay will remarkably change the half-life time of the nucleus compared with the experimental observation only related to the ground state. For example, the half-life time of beta decay of ground-state ^{99}Tc is about 200 thousand years; once the environment temperature reaches 3.0×10^6 K, the half-life time of ^{99}Tc will be reduced to about 10 years.[1] In addition, the distribution of electrons outside a nucleus works on either nuclear reaction [2] or beta decay width of the nucleus considerably. The existence of electrons of course reduces the phase-space of the final states of beta decay. On the other hand, since electrons are mostly excited to continuum states due to the high temperature, the phase-space available for creating an electron in the unfilled outer atomic orbits become larger than that of neutral atoms. This means that bound-state beta decay may occur.[3−5] Furthermore, if the matter was compressed to very high density, the bound state would vanish and the bound-state decay could then not occur. If the matter is compressed even further so that atomic nuclei overlap to form superdense nuclear matter, how the β-decay width (or the half-life) depends on the density is imperative to be investigated. Therefore, in this Letter we study the density dependence of the beta decay width in astrophysical nuclear matter.

Considering a charge-neutral system made up of protons, neutrons and electrons, if we suppose that the interaction between different kinds of particles, for example between a proton and a neutron, can be fully replaced by the mean fields of mesons, the Hamiltonian of the system can be given by

$$\hat{H} = \hat{H}_p + \hat{H}_n + \hat{H}_e \,. \tag{1}$$

The statistical density matrix of the nuclear matter is

$$\hat{\rho} = \exp\Big\{-\beta\Big[\sum_B (\hat{H}_B - \mu_B \hat{N}_B) + (\hat{H}_e - \mu_e \hat{N}_e)\Big]\Big\}, \tag{2}$$

where $\hat{H}_B$ ($\hat{H}_e$) is the Hamiltonian of the nucleons (electron), $\hat{N}_B$ ($\hat{N}_e$) is the baryon (electron) number operator, μ_B (μ_e) is the chemical potential of the baryon (electron). The energy eigenstate of the nuclear matter can be written as

$$|\alpha\rangle = |\alpha_p, \alpha_n, \alpha_e\rangle \,. \tag{3}$$

The relative probability for us to find $|\alpha\rangle$ in the ensemble is

$$\langle\alpha|\hat{\rho}|\alpha\rangle = \exp\Big\{-\beta\Big[\sum_B (E_B^\alpha - \mu_B N_B^\alpha) + (E_e^\alpha - \mu_e \hat{N}_e^\alpha)\Big]\Big\}. \tag{4}$$

Since the energy eigenstates are noncoherently mixed in the ensemble, the usual method might be to deal with each state individually and then evaluate the average value of the ensemble. However, such a method seems to be too complicated to accomplish the practical calculation. Therefore we attempt to translate such a statistical ensemble to a pure quantum state.

* Supported by the National Natural Science Foundation of China under Grant Nos 10075002 and 10135030, the Major State Basic Research Development Programme of China under Grant No G2000077400, and the Foundation for University Key Teacher by the Ministry of Education of China.

** To whom correspondence should be addressed. Email: liuyx@phy.pku.edu.cn

©2004 Chinese Physical Society and IOP Publishing Ltd

We introduce a set of new basis states $\{|\alpha\rangle'\}$ that are homological to $\{|\alpha\rangle\}$, along the way of finite temperature field theory (see, for example, Ref. [6]). These new basis states look like an observer to nuclear matter, so it might be referred to as measure basis. With such a measure basis, the ensemble can be expressed by

$$|T,\rho_n,\rho_p\rangle = \sum_\alpha \exp\Big\{-\frac{\beta}{2}\Big[\sum_B (E_B^\alpha - \mu_B N_B^\alpha) + (E_e^\alpha - \mu_e N_e^\alpha)\Big]\Big\}|\alpha\rangle' \otimes |\alpha\rangle\,. \tag{5}$$

Considering the case of pure mean-field approximation, we can approximately express the Hamiltonian in a diagonal form

$$\hat{H}_B = \sum_k E_B(k) a_B^\dagger(k) a_B(k)\,. \tag{6}$$

An arbitrary energy eigenstate of the nuclear matter reads as

$$\begin{aligned}|\alpha\rangle &= |\alpha_p,\alpha_n,\alpha_e\rangle \\ &= |k_1^p\cdots k_{N_p}^p; k_1^n\cdots k_{N_n}^n; k_1^e\cdots k_{N_e}^e\rangle\,.\end{aligned} \tag{7}$$

To maintain the charge neutrality, we should have $N_p = N_e$. The relative probability of the state $|\alpha\rangle$ in the ensemble is then

$$\langle\alpha|\hat{\rho}|\alpha\rangle = \exp\Big\{-\beta\sum_P\Big[\sum_{\{\alpha_P\}}(E_P^{\alpha_P}-\mu_P)\Big]\Big\}, \tag{8}$$

where P refers to the particles proton, neutron and electron. Meanwhile, the pure state of the ensemble is

$$|T,\rho_n,\rho_p\rangle = \sum_\alpha \exp\Big\{-\frac{\beta}{2}\cdot\sum_P\Big[\sum_{\{\alpha_P\}}(E_P^{\alpha_P}-\mu_P)\Big]\Big\}|\alpha\rangle'\otimes|\alpha\rangle\,. \tag{9}$$

Making use of the fermion distribution function

$$n_{P,\alpha_P}(k) = \frac{1}{\exp[\beta(E_P^{\alpha_P}(k)-\mu_P)]+1}\,, \tag{10}$$

we can obtain

$$|T,\rho_n,\rho_p\rangle = \prod_{k,P}\big[\sqrt{1-n_P(k)}\otimes|k,P,0\rangle' + \sqrt{n_P(k)}\otimes|k,P,1\rangle' a_P^\dagger(k)\big]|0\rangle\,, \tag{11}$$

where $|k,P,0\rangle'$ and $|k,P,1\rangle'$ come from the decomposition of the measure basis, and the index 0 or 1 represents that the single particle state $|k,P\rangle$ is unoccupied or occupied, respectively. Equation (11) shows clearly that the particles distribute in each single-particle state independently and noncoherently according to the fermion distribution. In addition, the right-hand side of Eq. (11) has been normalized, and the arbitrary phase factor of each term is unimportant due to the decomposed measure basis.

If we neglect the weak interaction for the time being and consider the charge-neutral postulation, the independent statistical parameters are $T, \rho_n,\ \rho_p = \rho_e$ and the electromagnetic interaction can be neglected. In the calculation of nuclear matter structure, we are usually interested in stationary states. Stationarity implies that all the time derivatives as well as the spatial components of the current densities vanish. The latter also leads to the vanishing of the spatial components of the fields themselves, since their source terms are just these densities. The fields left are thus σ, ω_0. In the simplest model, namely the linear σ–ω model, the Lagrangian for the stationary system of nucleon N can be given by

$$L_{RMF} = \bar{\psi}_N(i\gamma_\mu\partial^\mu - M_N)\psi_N - \frac{1}{2}m_\sigma^{*\,2}\sigma^2 + \frac{1}{2}m_\omega^{*\,2}\omega_0^2 - g_\sigma\bar{\psi}_N\sigma\psi_N - g_\omega\bar{\psi}_N\gamma_0\omega^0\psi_N\,, \tag{12}$$

where m_σ^* and m_ω^* are the Debye screening masses of σ-meson and ω-meson, respectively; g_σ and g_ω are the coupling constants between the meson and nucleon. Taking the Lagrangian equation we can easily obtain

$$\sigma = -\frac{g_\sigma}{m_\sigma^{*\,2}}(\rho_p^s + \rho_n^s)\,, \tag{13}$$

$$\omega_0 = \frac{g_\omega}{m_\omega^{*\,2}}(\rho_p + \rho_n)\,, \tag{14}$$

where the densities are defined by

$$\rho_N^s = \langle T,\rho_p,\rho_n|\bar{\psi}_N\psi_N|T,\rho_p,\rho_n\rangle\,, \tag{15}$$

$$\rho_N = \langle T,\rho_p,\rho_n|\psi_N^\dagger\psi_N|T,\rho_p,\rho_n\rangle\,, \tag{16}$$

where $N = p,n$ represents proton and neutron, respectively.

The equation of motion of the nucleon can be given as

$$(i\gamma_\mu\partial^\mu - M_N^*)\psi_N - g_\omega\omega_0\gamma_0\psi_N = 0\,, \tag{17}$$

where

$$M_N^* = M_N + g_\sigma\sigma\,. \tag{18}$$

Defining an effective chemical potential

$$\nu_N = \mu_N - g_\omega\omega_0\,, \tag{19}$$

and taking Eqs. (17) and (18) into account, we have the distribution function

$$n_N = \frac{1}{\exp[(E_N^*(\boldsymbol{p})-\nu_N)/T]+1}\,, \tag{20}$$

$$n_e = \frac{1}{\exp[(E_e(\boldsymbol{p})-\mu_e)/T]+1}\,. \tag{21}$$

Meanwhile, the Debye screening masses m_ω^* and m_σ^* can be fixed as [7–9]

$$m_\omega^* = \sqrt{m_\omega^2 + \Sigma_\omega}\,, \text{ with } \Sigma_\omega = g_\omega^2\Big(\frac{\rho_p^s}{2M_p^*} + \frac{\rho_n^s}{2M_n^*}\Big)\,, \tag{22a}$$

$$m_\sigma^* = \sqrt{m_\sigma^2 + \Sigma_\sigma}\,, \text{ with } \Sigma_\sigma = g_\sigma^2\Big(\frac{\rho_p^s}{M_p^*} + \frac{\rho_n^s}{M_n^*}\Big)\,. \tag{22b}$$

Furthermore, the densities in Eqs. (15) and (16) can be evaluated with Eq. (11). In the case of pure mean-field approximation, we have

$$\psi_N = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}}\sqrt{\frac{M_N^*}{E^*(k)}}\sum_{r=1,2}[C_r(\boldsymbol{k})U_r(\boldsymbol{k})e^{ikx} + D_r^\dagger(\boldsymbol{k})V_r(\boldsymbol{k})e^{-ikx}]\,, \tag{23}$$

$$\bar\psi_N = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}}\sqrt{\frac{M_N^*}{E^*(k)}}\sum_{r=1,2}[C_r^\dagger(\boldsymbol{k})\bar U_r(\boldsymbol{k})e^{-ikx} + D_r(\boldsymbol{k})\bar V_r(\boldsymbol{k})e^{ikx}]\,, \tag{24}$$

where $r = 1, 2$ refers to spin up and spin down, respectively. Meanwhile the densities are given by

$$\rho_N^s = \langle T, \rho_p, \rho_n|\bar\psi_N\psi_N|T, \rho_p, \rho_n\rangle = 2\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{M_N^*}{E^*(\boldsymbol{k})}n_N(\boldsymbol{k})\,, \tag{25}$$

$$\rho_N = \langle T, \rho_p, \rho_n|\psi_N^\dagger\psi_N|T, \rho_p, \rho_n\rangle = 2\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}n_N(\boldsymbol{k})\,. \tag{26}$$

Up to now, Eqs. (13), (14), (18)–(22), (25), and (26) constitute a self-consistent scheme corresponding to the pure mean-field case, which can be used for the numerical calculation.

Employing the four-fermion interaction to describe the weak interaction, we have the Lagrangian

$$L_{\text{weak}} = \frac{G_f}{\sqrt{2}}\bar\psi_p\gamma_\mu(1+\gamma_5)\psi_n\bar\psi_e\gamma_\mu(1+\gamma_5)\psi_{\nu_e} + \text{H.c.} \tag{27}$$

In vacuum, i.e. in the case that the background of nucleus vanishes, the beta decay width can easily be evaluated with the usual quantum field theory method, and it gives

$$\Gamma(k_n) = \frac{G_f^2}{8\pi^5 E_n^*}\int d^3k_p d^3k_e d^3k_{\nu_e}\frac{1}{E_p^*E_e^*E_{\nu_e}^*}\delta^{(4)} \cdot (k_n - k_p - k_e - k_{\nu_e})(k_n\cdot k_{\nu_e})(k_p\cdot k_e). \tag{28}$$

In nuclear matter, the initial state of decay is $|i\rangle = |T, \rho_n, \rho_p\rangle$ and the renormalized final state is $|f\rangle = N^{-1/2}C_p^\dagger(\boldsymbol{k}_p, \sigma_p)C_e^\dagger(\boldsymbol{k}_e, \sigma_e)D_{\nu_e}^\dagger(\boldsymbol{k}_{\nu_e}, \sigma_{\nu_e})C_n(\boldsymbol{k}_n, \sigma_n)|i\rangle$. With Eqs. (11)–(14), (18)–(22), (25), and (26), the β decay amplitude in the case of the pure mean-field approximation can be obtained to be

$$\langle f|\hat S^{(1)}|i\rangle = (-i)\frac{G_f}{\sqrt{2}}\sqrt{\frac{M_n^*}{E_n^*(\boldsymbol{k}_n)}}\sqrt{\frac{M_p^*}{E_p^*(\boldsymbol{k}_p)}} \cdot\sqrt{\frac{M_e}{E_e(\boldsymbol{k}_e)}}\frac{(2\pi)^4}{V^2}\delta^{(4)}(k_n - k_p - k_e - k_{\nu_e}) \cdot N^{1/2}(\boldsymbol{k}_n, \boldsymbol{k}_p, \boldsymbol{k}_e)[\bar U_p\gamma_\mu(1+\gamma_5)U_n] \cdot[\bar U_e\gamma_\mu(1+\gamma_5)U_{\nu_e}]\,. \tag{29}$$

and

$$\Gamma(k_n) = \frac{G_f^2}{8\pi^5 E_n^*}\int d^3k_p d^3k_e d^3k_{\upsilon_e}\frac{N(\boldsymbol{k}_n, \boldsymbol{k}_p, \boldsymbol{k}_e)}{E_p^*E_e^*E_{\nu_e}^*} \cdot\delta^{(4)}(k_n - k_p - k_e - k_{\nu_e})(k_n\cdot k_{\nu_e})(k_p\cdot k_e)\,, \tag{30}$$

where the normalization factor N is

$$N = n_n(\boldsymbol{k}_n)[1 - n_p(\boldsymbol{k}_p)][1 - n_e(\boldsymbol{k}_e)]\,. \tag{31}$$

With the conventional parameters $M_n = 939.565\,\text{MeV}$, $M_p = 938.272\,\text{MeV}$, $m_\sigma = 522\,\text{MeV}$, $m_\omega = 782\,\text{MeV}$, $g_\sigma = 7.62$ and $g_\omega = 8.03$ and $G_f = 1.16639\times10^{-11}\,\text{MeV}^{-2}$, we accomplish the calculation at a fixed temperature $T = 10\,\text{MeV}$, and various total nuclear matter density. The obtained results are listed in Table 1.

From this table one can recognize easily that the half-life time of the β-decay in astrophysical nuclear matter is shorter than that in finite nucleus. Meanwhile, with the increase of the nuclear matter density, the half-life time becomes even shorter.

With the formalism of four-fermion interaction, we

Table 1. Calculated results of the density ($\rho = \rho_n + \rho_p$ in units of the normal density ρ_0) dependence of effective masses of the nucleons and mesons and the half-life time (in minutes) of the beta decay in hot stellar matter.

ρ/ρ_0	ρ_p/ρ	ν_p	ν_n	μ_e	M_n^*	M_p^*	m_σ^*	m_ω^*	$\tau_{1/2}$
1.2	1.7%	835.49	750.44	84.67	763.92	762.63	626.76	852.32	3820
3.6	3.2%	795.00	639.67	155.33	626.54	625.24	777.55	996.09	1427
6.0	4.5%	816.72	609.92	207.51	577.25	575.96	901.51	1119.54	943
8.4	5.4%	849.48	602.49	247.23	552.54	551.24	1004.92	1225.35	722
10.8	6.0%	882.74	603.12	278.76	537.72	536.43	1094.25	1317.68	596
13.2	6.5%	914.81	607.95	306.36	527.47	526.14	1163.92	1390.54	449

have investigated the half-life time of β-decay in dense nuclear matter in the linear σ–ω model of the relativistic mean field approximation. The numerical result in the case of neglecting the pairing effect of nucleons has been obtained. The obtained result shows obviously that the half-life time of the β-decay in dense astronuclear matter is shorter than that in finite nuclei and it decreases with the increase of nuclear matter density.

To understand the reason for such a changing characteristic of the half-life time, we recall the evaluating formalism. Even though Eqs. (20), (21), (30), and (31) show that the renormalization factor N may reduce the decay width, the corrected mass of nucleon in nuclear matter enhances the width drastically. As a result, the half-life time decreases with the increase of the nuclear matter density due to the reduction of the nucleon mass. Furthermore, since the renormalization factor N and the corrected mass of nucleon depend also on the pairing correlation between nucleons, the pairing correlation may affect the β-decay width in astronuclear matter. Investigation of the pairing correlation effect on the β-decay width is in progress.

References

[1] Takahashi K, Mathews G J and Bloom S D 1986 *Phys. Rev.* C **33** 296
[2] Liolios T E 2001 *Phys. Rev.* C **63** 045801
[3] Jung M et al 1992 *Phys. Rev. Lett.* **69** 2164
[4] Bosch F et al 1996 *Phys. Rev. Lett.* **77** 5190
[5] Takahashi K, Boyd R N, Mathews G J and Yokoi K 1987 *Phys. Rev.* C **36** 1522
[6] Su R K 2004 *Statistical Physics* 2nd edn (Beijing: Higher Education Press)
[7] Sun B X, Lü X F, Shen P N and Zhao E G 2003 *Mod. Phys. Lett.* A **18** 1485
[8] Sun B X, Lü X F, Shen P N and Zhao E G 2003 *Int. J. Mod. Phys.* E **12** 543
[9] Sun B X, Lü X F, Li L, Ning P Z, Shen P N and Zhao E G 2002 Sum rules on quantum hadrodynamics at finite temperature and density *Preprint* arXiv:nucl-th/0209041