Modulation Instability in Biased Photorefractive–Photovoltaic Crystals *

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We show the modulation instability of broad optical beams in biased photorefractive–photovoltaic crystals under steady-state conditions. This modulation instability growth rate depends on the external bias field, the bulk photovoltaic effect, and the ratio of the optical beam intensity to that of the dark irradiance. Under appropriate conditions, this modulation instability growth rate is the modulation instability growth rate studied previously in biased photorefractive–nonphotovoltaic crystals, and the modulation instability growth rate in open- and closed-circuit photorefractive–photovoltaic crystals can be predicted.

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Photorefractive (PR) solitons have been a subject of intense study over the past few years because of their possible applications for optical switching and routing.[1−11] So far, several generic types of steady-state PR solitons have been known, such as screening solitons,[5,6] which occur when an external bias voltage is appropriately applied to a PR-nonphotovoltaic crystal, and photovoltaic (PV) solitons,[7,8] which exist in PR materials with appreciable PV coefficients. Very recently, we have shown theoretically that the application of an external field enables steady-state solitons,[9] soliton pairs,[10] and vector solitons [11] in PR-PV crystals. In particular, solitons in biased PR-PV crystals can change into screening solitons when the bulk PV effect is neglectable, and can change into PV solitons in the absence of external field.[9] However, modulation instability (MI) is an ordinary process that is inherent to most nonlinear systems.[12] It occurs in the parameter region where solitons are observed, and it is considered as a precursor of soliton formation. In the spatial domain, MI manifests itself as the filament of a broad optical beam through the spontaneous growth of spatial-frequency sidebands.[13] The MI of quasi-plane-wave optical beams has been predicted and observed in biased PR-nonPV crystals.[13,14] In biased PR-PV crystals, we should also pay attention to MI.

In this Letter, we investigate the modulation instability of broad (quasi-plane-wave) optical beams in biased PR-PV crystals under steady-state conditions. This modulation instability growth rate depends on the external bias field, the bulk PV effect, and the ratio of the optical beam intensity to that of the dark irradiance. Moreover, our analysis indicates that the MI growth rate is the MI growth rate studied previously in biased PR-nonPV crystals when the bulk PV effect is neglectable for short circuits, and this MI growth rate predicts the MI growth rate in open- and closed-circuit photorefractive-photovoltaic crystals when the external bias field is absent.

First, let us consider an optical beam that propagates in a PR-PV crystal along the z-axis and is allowed to diffract only along the x-direction. Moreover, let us assume that the polarization of the optical beam and the external bias electric field are parallel to the c-axis. For convenience of the analysis, let the PR-PV crystal be $LiNbO_3$ with its optical c-axis oriented along the x-direction. Under these conditions, the perturbed extraordinary refractive index n_e' (along the c-axis) is given by $n_e'^2 = n_e^2 - n_e^4 r_{33} E_{sc}$,[6] where n_e is the unperturbed extraordinary index of refraction, r_{33} is the electro-optic coefficient, and $\boldsymbol{E}_{sc} = E_{sc}\boldsymbol{i}$ is the space-charge field induced in this PR-PV crystal, $\boldsymbol{i}$ is the unit vector along the x-direction. The optical field $\boldsymbol{E} = E\boldsymbol{i}$ is then expressed in terms of a slowly varying envelope U: that is, $\boldsymbol{E} = \boldsymbol{i}(2\eta_0 I_d/n_e)^{1/2} U(x,z)\exp(ikz)$, where I_d is the dark irradiance that phenomenologically accounts for the thermal generation of electrons in the conduction band, $k = k_0 n_e = (2\pi/\lambda_0)n_e$ is the wavevector, η_0 is the vacuum intrinsic impedance, and λ_0 is the free-space wavelength of the lightwave employed. In this case, we can readily obtain the following paraxial equation of diffraction:[6,10]

$$i\frac{\partial U}{\partial z} + \frac{1}{2k}\frac{\partial^2 U}{\partial x^2} - \beta E_{sc} U = 0, \tag{1}$$

where $\beta = k_0 n_e^3 r_{33}/2$. The power density has been scaled with respect to the dark irradiance I_d, i.e. $I = |U|^2 I_d$. Under strong bias conditions and for relatively broad beam configurations, the space-charge field is

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approximately given by [11]

$$E_{sc} = E_0 \frac{1}{1+|U|^2} - E_p \frac{|U|^2}{1+|U|^2}, \tag{2}$$

where $U(x,z)$ is assumed to be a bright-like beam, i.e. $|U|^2 = 0$ at $x \to \pm\infty$, and E_0 is the value of space-charge field at $x \to \pm\infty$. In Eq. (2), $E_p = \kappa\gamma N_A/e\mu$ is the PV field constant, γ is the recombination rate coefficient, μ is the electron mobility, N_A is the density of negatively charged acceptor, e is the electric charge, and κ is the PV constant. The constant field E_0 can be computed from the potential condition $V_0 = -\int_{-l/2}^{l/2} E_{sc} dx + RJS$ and is approximately given by $E_0 = -(V_0\chi + E_p\delta\chi - RJS\chi)$,[11] where $\chi = 1/\int_{-l/2}^{l/2}[1/(1+|U|^2)]dx$, $\delta = -\int_{-l/2}^{l/2}[|u|^2/(1+|U|^2)]dx$, S is the surface area of the electrodes, R is the external resistance, l is the width of the crystal between the electrodes, $\boldsymbol{J} = J\boldsymbol{i}$ is the electric current density, and V_0 is the electro-motive force of the source.

For broad planar beams, the envelope U is expected to remain relatively constant over a large range of x. Therefore, the diffraction term $(2k)^{-1}(\partial^2 U/\partial x^2)$ can be neglected in Eq. (1). By substituting $(2k)^{-1}(\partial^2 U/\partial x^2) = 0$ into Eq. (1), we find that its broad beam solution takes the form

$$U = r^{1/2} \exp\Big[-i\beta\Big(E_0 \frac{1}{1+r} - E_p \frac{r}{1+r}\Big)\Big], \tag{3}$$

where the positive quantity r represents the ratio of the optical beam intensity to that of the dark irradiance. To investigate the stability of this solution, let us perturb the stationary result of Eq. (3) in the following way:

$$U = [r^{1/2} + \sigma(x,z)] \exp\Big[-i\beta(E_0 \frac{1}{1+r} - E_p \frac{r}{1+r})\Big], \tag{4}$$

where $\sigma(x,z)$ represents the weak complex perturbation and $|\sigma(x,z)| \ll r^{1/2}$. By substituting Eq. (4) into Eq. (1) and by linearizing in σ, we obtain the evolution equation

$$i\frac{\partial\sigma}{\partial z} + \frac{1}{2k}\frac{\partial^2\sigma}{\partial x^2} + \beta(E_0+E_p)\frac{1}{(1+r)^2}(\sigma+\sigma^*) = 0. \tag{5}$$

We now assume that the spatial perturbation $\sigma(x,z)$ takes the form

$$\sigma(x,z) = a(z)\exp(ipx) + b(z)\exp(-ipx). \tag{6}$$

The above equation shows that $\sigma(x,z)$ is composed of two sideband plane waves. Moreover, p in the above equation is the spatial frequency of the sideband plane wave. Substitution Eq. (6) into Eq. (5) yields the following coupled differential equations:

$$i\frac{da}{dz} - \frac{p^2}{2k}a + \beta(E_0+E_p)\frac{r}{(1+r)^2}(a+b^*) = 0, \tag{7a}$$

$$i\frac{db}{dz} - \frac{p^2}{2k}b + \beta(E_0+E_p)\frac{r}{(1+r)^2}(b+a^*) = 0. \tag{7b}$$

Equation (7) can then be decoupled into an equivalent set of ordinary differential equations:

$$\frac{d^2a}{dz^2} = \Big[\frac{p^2}{k}\beta(E_0+E_p)\frac{r}{(1+r)^2} - \frac{p^4}{4k^2}\Big]a, \tag{8a}$$

$$\frac{d^2b}{dz^2} = \Big[\frac{p^2}{k}\beta(E_0+E_p)\frac{r}{(1+r)^2} - \frac{p^4}{4k^2}\Big]b. \tag{8b}$$

The MI gain can be obtained from Eq. (8) and is given by

$$\Gamma = \mathrm{Re}\Big\{\Big[\frac{p^2}{k}\beta(E_0+E_p)\frac{r}{(1+r)^2} - \frac{p^4}{4k^2}\Big]^{1/2}\Big\}. \tag{9}$$

The above equation clearly shows that the MI gain is possible when $(E_0+E_p) > 0$. As an example, we consider the following examples: $\lambda_0 = 0.5\,\mu$m, $T = 300\,$K, $R = 0$, and $l = 1\,$cm. The $LiNbO_3$ parameters are taken to be $n_e = 2.2$, $r_{33} = 30\times10^{-12}\,$m/V, $N_A = 10^{22}\,\mathrm{m}^{-3}$, $\varepsilon_r = 30$ and $E_p = 40\,$kV/cm. For this set of values, we find that $\beta = 2.0\times10^{-3}\mathrm{V}^{-1}$ and $k = 2.76\times10^7\mathrm{m}^{-1}$. Figure 1 depicts the MI gain Γ as a function of the dimensionless ratio p/k for $V_0 = \pm20000\,$V when $r = 1$. Obviously, the maximum MI gain as well as the corresponding spatial-frequency increase with V_0. Moreover, from Eq. (9) the MI gain is a maximum when $p/k = \pm[\beta(E_0+E_p)]^{1/2}[r^{1/2}/(1+r)]$, and is given by

$$\Gamma_{\max} = \frac{k_0 n_e^3 r_{33}(E_0+E_p)}{2}\frac{r}{(1+r)^2}. \tag{10}$$

The above equation clearly reveals that the maximum MI gain $\Gamma_{\max}$ attains a maximum when $r = 1$ and increases linearly with (E_0+E_p). On the other hand, some PV materials (for example, $BaTiO_3$) possess PV constants κ (i.e. E_p) that can change their sign under polarization rotation.[8] Thus, the MI gain Γ can be changed by rotating the polarization of the light.

It is also very interesting to discuss the properties of MI in biased PR-PV crystals. The MI studied previously in biased PR-nonPV crystals can be directly obtained from this MI growth rate under appropriate conditions. When the bulk PV effect is neglectable for short circuits ($R = 0$), i.e. $E_p = 0$ and $E_0 = E_{0S} = -V_0\chi$, Eqs. (1) and (9) yield the following results:

$$i\frac{\partial U}{\partial z} + \frac{1}{2k}\frac{\partial^2 U}{\partial x^2} - \beta E_{0S}\frac{U}{1+|U|^2} = 0, \tag{11}$$

$$\Gamma = \mathrm{Re}\Big\{\Big[\frac{p^2}{k}\beta E_{0S}\frac{r}{(1+r)^2} - \frac{p^4}{4k^2}\Big]^{1/2}\Big\}. \tag{12}$$

Equations (11) and (12) are the same dimensionless parameters and are in the similar forms to Eqs. (15) and (17) in Ref. [13], in which MI of quasi-plane-wave optical beams in biased PR-nonPV crystals is investigated for short circuits.

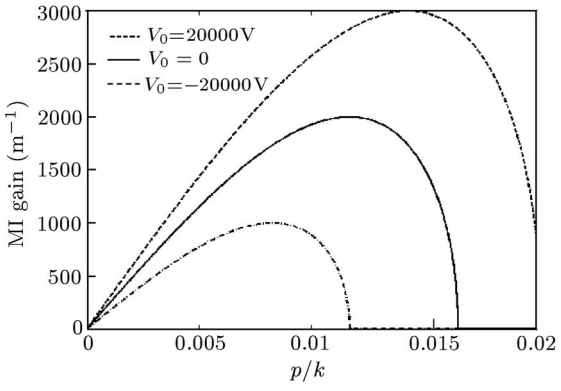

Fig. 1. MI gain as a function of p/k when $V_0 = \pm 20000$ V and $r = 1$.

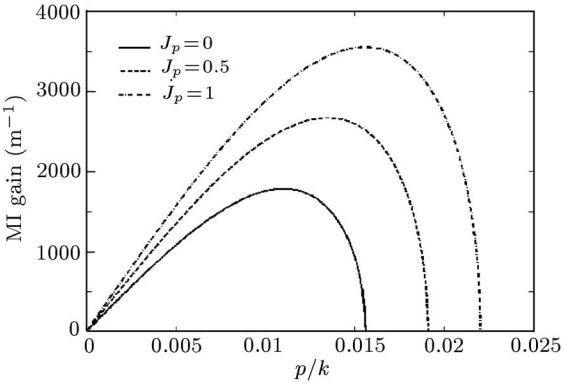

Fig. 2. MI gain as a function of p/k in the open- and closed-circuit $LiNbO_3$ crystal when $r = 1$.

The MI in open- and closed-circuit PR-PV crystals can be obtained from the MI in biased PR-PV crystals in the absence of the external field. The absence of the external field in PR-PV crystals signifies either an open circuit ($J = 0$) or closed circuit ($J \neq 0$).[9,10] Here the electric current density J is give by $J_p = -(E_p\delta\chi - RJS)/E_p$,[11] where $J_P = J/(\hat{s}I_dN_D\kappa)$, and $\hat{s}$ is the photoexcitation cross section. Hence, when the external bias field is absent, i.e. $E_0 = E_{0p} = -(E_p\delta\chi - RJS\chi)$, our physical system becomes the physical system of MI in open- and closed-circuit PR-PV crystals. In this case, Eqs. (1) and (9) yield the following results:

$$i\frac{\partial U}{\partial z} + \frac{1}{2k}\frac{\partial^2 U}{\partial x^2} - \beta E_p(J_P - 1)\frac{|U|^2}{1+|U|^2}U = 0, \tag{13}$$

$$\Gamma = \mathrm{Re}\left\{\left[\frac{p^2}{k}\beta E_p(J_P+1)\frac{r}{(1+r)^2} - \frac{p^4}{4k^2}\right]^{1/2}\right\}. \tag{14}$$

Equations (13) and (14) predict MI in open- and closed-circuit PR-PV crystals. Figure 2 depicts the MI gain given by Eq. (14) as a function of p/k in the open- and closed-circuit $LiNbO_3$ crystal when $r = 2$. From Fig. 2 it is apparent that the maximum MI gain as well as the corresponding spatial-frequency increase with J_P.

In conclusion, we have shown the MI of broad optical beams in biased PR-PV crystals under steady-state conditions. This MI growth rate depends on the external bias field, the bulk PV effect, and the ratio of the optical beam intensity to that of the dark irradiance. Obviously, this may allow one to externally control the spatial period of the spontaneously generated filaments. Our analysis indicates that this MI growth rate is the MI growth rate studied previously in biased PR-nonPV crystals when the bulk PV effect is neglectable for short circuits, and the MI growth rate in open- and closed-circuit PR-PV crystals is predicted when the external bias field is absent.

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