Wave Dispersion and Attenuation in Partially Saturated Sandstones *

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We investigate the wave dispersion and attenuation in partially water-saturated sandstones based on the improved Biot/squirt (BISQ) model in which the saturation is introduced. Numerical experiments indicate that the phase velocity of the fast P-wave decreases as the saturation increases in the low-frequency range (10^2–10^4 Hz), and reaches the minimum at the full-saturation state. The behaviour of the phase velocity varying with the saturation in the high-frequency range (10^4–10^6 Hz), however, is opposite to that in the low-frequency range. The peak value of P-wave attenuation increases with increasing saturation, and is the maximum at the fully saturated state. Numerical models and experiments show that the improved BISQ model is better than the traditional Gassmann-Biot model.

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Recently the research on wave propagation in partially saturated porous media has become a hot focus, which is one of the keys in finding the residual oil.[1] Although the Biot theory has been developed and widely applied, it is well known that the Biot theory, based on the Biot-flow mechanism of pore fluids, cannot adequately explain the large velocity dispersion and the strong attenuation in many rocks.[2,3] Many researchers have shown that the local squirt-flow of pore fluids is responsible for observed large velocity dispersion and strong attenuation.[3,4] The squirt-flow mechanism has been traditionally investigated by examining fluid flow in an individual crack.[5] Therefore, the application of the squirt theory is strongly limited. In 1993, Dvorkin and Nur offered an isotropic model for the Biot/squirt (BISQ) flow in rocks by which both the Biot-flow mechanism and the squirt-flow mechanism can be simultaneously treated.[3] This method combines the macroscal parameters with the microscal fluid properties, and adequately explains the higher dispersion and strong attenuation in many rocks. In recent years, the BISQ theory has been widely developed and many results have been obtained.[6–9]

However, in the above-mentioned theories, the traditional Biot theory and the BISQ theory are only suitable for investigating wave propagation in a full-saturated porous medium. Thus, wave propagation in partially saturated rocks is one of main problems needed to be solved for finding residual oil. As an important parameter reflecting characteristics of the reservoirs, the saturation has special meaning during the process of monitoring exploitation. In this Letter, we concentrate on the relationship between the parameters of petrophysical property and the velocity dispersion and attenuation of different waves by using the improved BISQ model in which the saturation is introduced, and the wave propagation in partially saturated porous media.

Following the three-dimensional BISQ model in the isotropic porous medium and using the Fourier transform, we can derive the following formulae of the phase velocities and inverse attenuation quality factors of S-wave, fast P-wave and slow P-wave,[9]

$$V_S = \sqrt{\mu}/\mathrm{Re}(\sqrt{\rho_x}), \tag{1}$$

$$Q_S^{-1} = 2\mathrm{Im}\sqrt{\rho_x}/\mathrm{Re}(\sqrt{\rho_x}), \tag{2}$$

$$V_{P^{+,-}} = \sqrt{2}/\mathrm{Re}(\sqrt{b \pm c}), \tag{3}$$

$$Q_{P^{+,-}}^{-1} = 2\mathrm{Im}(\sqrt{b \pm c})/\mathrm{Re}(\sqrt{b \pm c}), \tag{4}$$

where

$$\rho_x = (1-\phi)\rho_s + (1-\theta)\phi\rho_f,$$

$$b = \frac{\rho_x}{\lambda+2\mu} + \frac{\rho_f(\alpha-\phi\theta)^2}{\phi\theta(\lambda+2\mu)} + \frac{\rho_f}{FS_q\theta},$$

$$c^2 = b^2 - \frac{4\rho_f\rho_x}{FS_q\theta(\lambda+2\mu)},$$

$$\theta = \Big(\frac{\rho_a/\rho_f+\phi}{\phi} + i\frac{\eta\phi}{\omega\rho_f k}\Big)^{-1},$$

$$F = \Big(\frac{1}{K_f} + \frac{\alpha-\phi}{\phi K_s}\Big)^{-1},$$

$$S_q = 1 - \frac{2J_1(\lambda_q R)}{\lambda_q R J_0(\lambda_q R)}, \quad \lambda_q^2 = \frac{\rho_f\omega^2}{F\theta},$$

$$\alpha = \frac{2(1+\nu)}{3(1-2\nu)} \cdot \frac{\mu}{(1/K + 1/K_s)} = 1 - \frac{K}{K_s}.$$

The equivalent medium theory[10] is an effective method to describe the partially saturated porous medium. The theory gives some equivalent coefficients

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to describe the porous properties by converting the partially saturated porous medium into a fully saturated solid by using the wave scattering theory. For the gas-water saturated system of inhomogeneous saturation, the effective bulk modulus (the Voigt average fluid bulk modulus) K_f^* and equivalent density ρ_f^* are defined as[11,12]

$$K_f^* = S_w K_w + (1 - S_w) K_g, \tag{5}$$

$$\rho_f^* = S_w \rho_w + (1 - S_w) \rho_g, \tag{6}$$

where K, ρ, and S are the bulk modulus, the density and the saturation. The subscripts w and g denote *water* and *gas* of the pore fluids. For a oil-water saturated system, we only need to replace the bulk modulus of water by that of oil.

Considering the gas-water saturated system and substituting Eqs. (5) and (6) into Eqs. (1)–(4), we can introduce the saturation into these equations of the phase velocity and attenuation. In this way, we have offered an improved BISQ model that can be used to investigate the velocity dispersion and attenuation of wave propagation in partially saturated porous medium.

In order to investigate the effects of the saturation on wave propagation in partially saturated rocks, we predict the phase velocity and attenuation in isotropic porous media by using the improved BISQ model.

We choose the parameters of our model to be the same as those presented in Ref. [3]. In other words, the porosity of the rock is $\phi = 0.15$. The bulk modulus of the skeleton in drained conditions is $K = 16\,\mathrm{GPa}$, and its Poisson ratio is $\nu = 0.15$. The density and the bulk modulus of the solid phase are $\rho_s = 2650\,\mathrm{kg/m^3}$ and $K_s = 38\,\mathrm{GPa}$. The density of the pore fluid is $\rho_f = 1000\,\mathrm{kg/m^3}$, and its viscosity η is 0.01 ps. The additional coupling density is $\rho_a = 420\,\mathrm{kg/m^3}$. The characteristic squirt-flow length is chosen by $R = 1\,\mathrm{mm}$. The permeability of the skeleton is $k = 1.25\,\mathrm{mD}$. Figure 1 shows the effects of saturation and frequency on the phase velocity and attenuation in partially saturated porous medium.

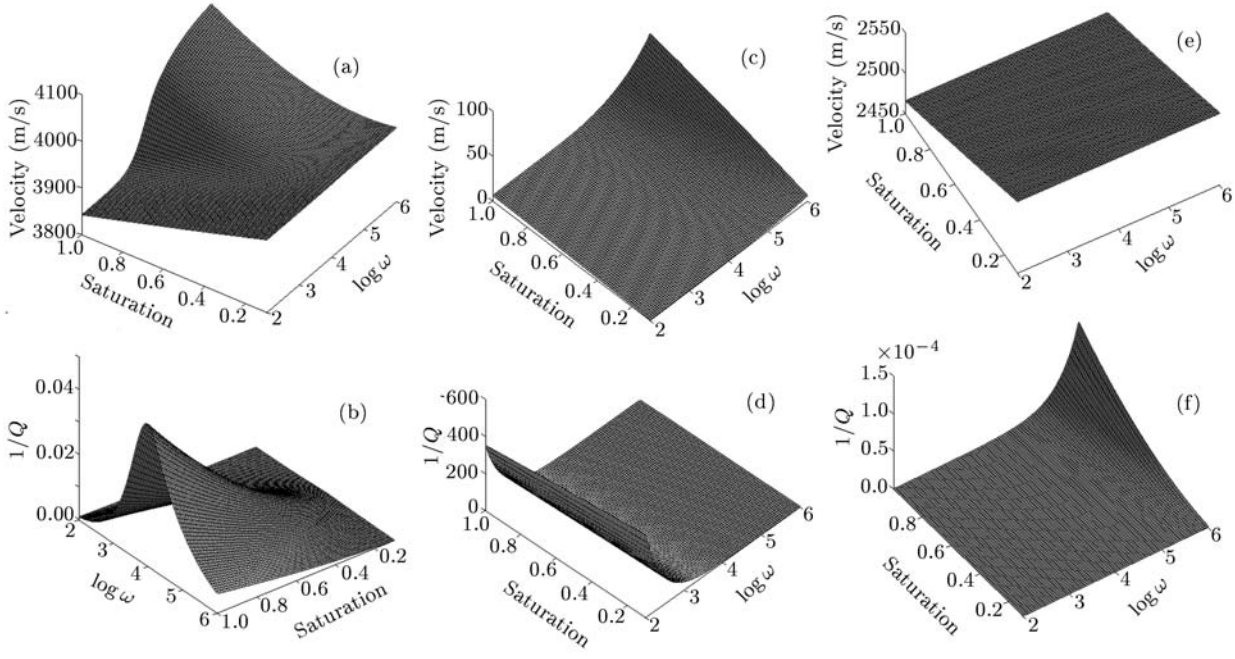

Fig. 1. Effects of saturation and frequency on the velocity dispersion and attenuation with (a) phase velocity and (b) attenuation of the fast P-wave, (c) phase velocity and (d) attenuation of the slow P-wave, and (e) phase velocity and (f) attenuation of the S-wave.

Figure 1(a) shows that the phase velocities of the fast P-wave (V_{P^+}) are significantly different in different frequency ranges. The value of V_{P^+} decreases with increasing saturation in the low-frequency range. The behaviour of its phase velocity varying with the saturation in the high-frequency range, however, is just opposite to that in the low-frequency range, and V_{P^+} reaches the maximum at the full-saturated state. For the case of the high saturation ($S > 70\%$), the phase velocity of the fast P-wave speedily increases around the frequency of $10^4\,\mathrm{Hz}$, but the frequency has little effect on V_{P^+} in the low-saturated ($S < 30\%$) sandstone. The behaviour of attenuation for the fast P-wave ($Q_{P^+}^{-1}$) agrees with the experimental results given by Murphy *et al.*[13] Figure 1(b) clearly shows that $Q_{P^+}^{-1}$ increases with increasing saturation and reaches the maximum at the full-saturated state and frequency about $10^4\,\mathrm{Hz}$. The physical significance of the result is that the motion of the fluids relative to the solids becomes more rapid because of the abrupt

change of the phase velocity of the fast P-wave around this frequency, and further enhances the fluid/solid interaction resulting in large losses of energy of the two-phase system. In Figs. 1(c) and 1(d), we observe that the slow-P velocity increases as the saturation increases, and its attenuation is larger than that of the fast P-wave and S-wave especially in the frequency range of the seismic exploration (around 10^2 Hz). This is the reason why it is difficult to observe the slow P-wave in practical seismic records. Figure 1(e) displays that the surface of velocity for S-wave (V_S) is smooth. The value of V_S decreases as the saturation increases, and it is independent of the frequency. The attenuation of S-wave (Q_S^{-1}) is about 1/100 to 1/1000 of that of the fast P-wave, and Q_S^{-1} is less than 10^{-5} in most of the frequency range [Fig. 1(f)].

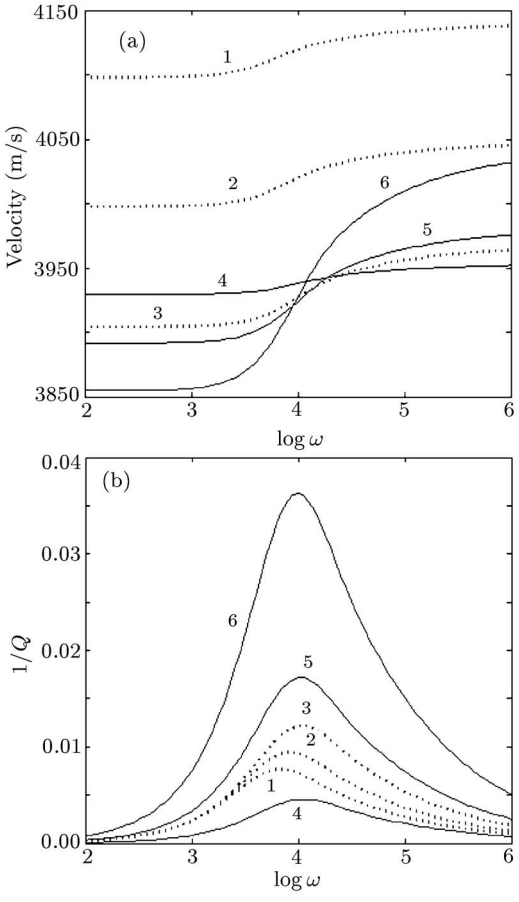

Fig. 2. Effects of frequency on (a) phase velocity and (b) attenuation (Q^{-1}) of the fast P-wave. The dashed lines 1, 2, and 3 denote the results for different porosity $\phi_1 = 25\%$, $\phi_2 = 20\%$, and $\phi_3 = 15\%$ at the same saturation $S = 0.5$. The solid lines 4, 5, and 6 denote the results for different saturation $S_4 = 0.3$, $S_5 = 0.6$, and $S_6 = 0.9$ at the same porosity $\phi = 15\%$.

Next, we take the fast P-wave as example to investigate the effects of saturation and porosity on velocity dispersion and attenuation at the changing frequency by our model, which has the same parameters as those mentioned above except for saturation and porosity. In Fig. 2(a), the velocity (dashed lines) increases as porosity increases at a given saturation ($S = 0.5$), but for different porosities the trends of velocity versus frequency keep accordance with each other. At the same porosity ($\phi = 15\%$, solid lines), the velocities, however, differ greatly in the high- and low-frequency ranges at a high saturation. During the process of petroleum exploitation, the result may be directly applied in approximately estimating the saturation of the reservoir by combining seismic data at the seismic frequency (e.g. around 10^2 Hz) with those of sonic logging (around 10^4 Hz). Figure 2(b) shows that for a constant saturation ($S = 0.5$, dashed lines), the peak of the attenuation slightly shifts towards lower frequencies with increasing porosity, and its value decreases as porosity increases. The physical significance is that the squirting motion of the pore fluid becomes easier to relax at lower frequencies for higher porosity. For the case of porosity equal to 15% (solid lines), the value of the attenuation peak increases with increasing saturation. This indicates that the fast P-wave has the property of strong attenuation in a saturated porous medium with no or little gas.

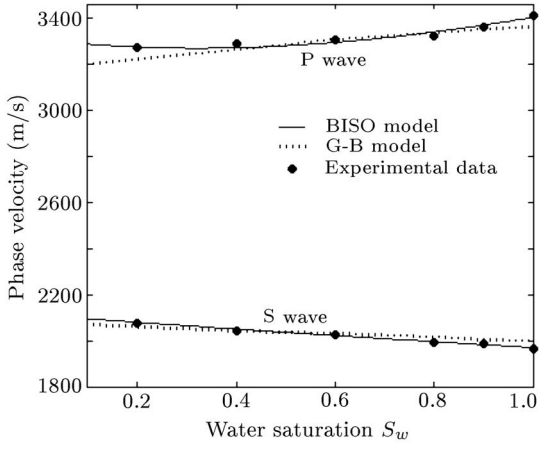

Fig. 3. Comparison of the theoretical predicted results and the experimental results (phase velocities).

To test the improved BISQ model, we simulate the experimental results of King *et al.*[12] by using our model. Therefore, the improved-BISQ parameters, chosen to be the same as those used in Ref. [12], are: $K_s = 40$ GPa, $\rho_s = 2550\,\mathrm{kg/m^3}$, $K_f = 2.25$ GPa, $\rho_f = 1000\,\mathrm{kg/m^3}$, $k = 2$ mD, $R = 15$ mm, and $\omega = 600$ KHz. We select the additional coupling density $\rho_a = 420\,\mathrm{kg/m^3}$.[6] Using the genetic algorithm, we obtain the other parameters by matching the experimental data.[12] Those parameters are: $\phi = 27.4\%$, $\nu = 0.156$, and $K = 9.23$ GPa. The model is a gas-liquid saturated system. At low-saturation ($S < 10\%$) state, the capillary pressure and adsorption mechanism dominate the phase velocity.[14] Therefore, we consider that the saturation changes from 10% to

100%.

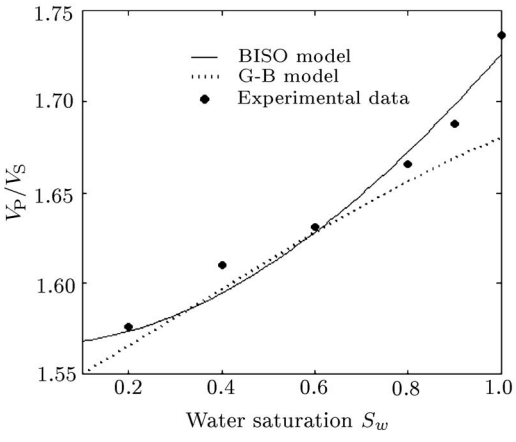

Fig. 4. Ratio of P- and S-wave versus saturation.

The curves of the phase velocity and the ratios of P-wave and S-wave are given in Figs. 3 and 4 where the solid, dashed and dotted lines denote the improved BISQ-predicted results, the results by the traditional Gassmann-Biot (G-B) model, and the experimental data,[12] respectively. In Fig. 3, the BISQ-predicted results for the S-wave are evidently concordant with the experimental data, and our predictions of the P-wave velocity are better than the results of the G-B model especially in the case of low saturation ($S < 40\%$) and high saturation ($S > 90\%$). It is clearly that our predicted results of V_P/V_S are more consistent with the trend of the experimental data than those of the traditional G-B model especially in the high-saturation ($S > 60\%$) case (Fig. 4).

In conclusions, based on the idea of the equivalent medium, an improved BISQ model including the saturation has been proposed, and the wave propagation in partially water-saturated porous medium has been simulated by the model. The numerical simulation results show that the phase velocity of the fast P-wave decreases with increasing saturation in the frequency range from 10^2 to about 10^4 Hz, but its velocity increases with increasing saturation and reaches the maximum at the full-saturated state in the frequency range from about 10^4 to 10^6 Hz. The frequency, however, has little effect on the phase velocity of the fast P-wave in the low water-saturated ($S < 30\%$) sandstone. There is a transition zone of the fast P-wave attenuation from low to high frequencies in the high water-saturated ($S > 70\%$) case. The value of attenuation for the fast P-wave increases with increasing saturation and reaches the maximum at the full-saturated state and frequency about 10^4 Hz. The phase velocity of the slow P-wave increases with increasing saturation and frequency. The improved BISQ model may be directly applied to the oil exploitation such as dynamic reservoir monitoring.

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