Influence of Traxiality on the Signature Inversion in Odd–Odd Nuclei *

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The nature of signature inversion in the $\pi g_{9/2}\nu h_{11/2}$ bands of odd–odd $^{98,102}Rh$ nuclei is studied. Calculations are performed by using a triaxial rotor plus two-quasiparticle model and are compared with the experimentally observed signature inversions. The calculations reproduce well the observations and suggest that, in these bands, the signature inversion can be interpreted mainly as a competition between the Coriolis and the proton–neutron residual interactions in low K space. The triaxiality applied in the Hamiltonian enlarges the amplitudes of high spin signature zigzags at small triaxial deformation and might push the signature inversion point to higher spin at large triaxial deformation.

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The $\pi g_{9/2}\nu h_{11/2}$ bands and the three-quasiparticle bands containing this configuration in the γ-soft $A \approx 100$ nuclei have been previously studied extensively in connection with the concept of configuration dependent triaxiality.[1] It has been pointed out by several authors that these bands have much smaller signature splitting than the one-quasiparticle $\pi g_{9/2}$ bands. The measured differences between the signature splittings were attributed to different γ triaxiality in the different configurations. However, the relative position and the crossing of the two signature branches of the bands have not been discussed in these works.

There is plenty of physics in the nuclei of $A \approx 100$.[2] Signature inversion in this region has been reported earlier only in the case of the odd–odd ^{98}Rh nucleus.[3] Our recent studies on the $^{100-103}$Rh isotopes[4–6] and a close inspection of the known $\pi g_{9/2}\nu h_{11/2}$ bands of the Rh ($Z = 45$) and Ag ($Z = 47$) isotopes revealed that the signature splitting effects, earlier considered as quenchings of signature splitting, are not only quenchings but signature inversions. Indeed, the energetically favoured signature at low spins in these $\pi g_{9/2}\nu h_{11/2}$ bands is the $\alpha = 1$ branch (odd spins), instead of the expected $\alpha = 0$ branch (even spins). The systematic occurrence of signature inversion in this mass region is discussed in Refs. [5,7] together with attempts to understand its behaviour qualitatively.

Among many attempts (e.g. Refs. [8–11]) for interpreting the mechanism of signature inversion in odd–odd nuclei, a model using an axially symmetric rotor plus two quasi-particle model has already been successfully applied for the description of signature inversions in the $A \approx 80$ and $A \approx 160$ mass regions.[12,13] According to this model the signature inversion is caused by the competition between the Coriolis and the proton–neutron residual interactions in low K space (a similar signature inversion mechanism for ^{160}Tm and ^{162}Tm can be seen in Ref. [14], although the theoretical method is different from that in Ref. [12]). This model has also been successfully applied to the $\pi g_{9/2}\nu h_{11/2}$ bands in the odd–odd ^{98}Rh and ^{102}Rh nuclei.[15]

The main aim of the present paper is twofold. One is to examine if inclusion of a triaxial core in the model Hamiltonian enables us to give a better description of the signature inversion in the above-mentioned two odd–odd Rh nuclei. The other is to examine the role of the triaxiality in this model on the pattern of the signature inversion. In order to see the influence of the triaxiality on the signature inversion easily, in this Letter we perform the calculations of two isotopes ^{98}Rh and ^{102}Rh which can also be treated by the axial symmetry Hamiltonian.

The present triaxial rotor plus two quasi-particles model is a generalized version of the axial rotor plus two quasi-particles model described in Refs. [12,13,15]. The γ convention is the same as that in Ref. [16].

The model Hamiltonian H for an odd–odd nucleus reads

$$H = H_R + H_{sp} + H_{sn}, \tag{1}$$

where H_R is the Hamiltonian of the rotor, H_{sp} and H_{sn} are the Hamiltonians of the quasi-proton and quasi-neutron located in two single j orbits outside the rotor, and

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$$H_R = \sum_{k=1}^{3} \frac{1}{2g_k}(I_k - j_{nk} - j_{pk})^2. \quad (2)$$

This can be rewritten as

$$\begin{aligned} H_R = & \Big(\frac{1}{4g_1} + \frac{1}{4g_2}\Big)[(I^2 - I_3^2) + (j_n^2 - j_{n3}^2) \\ & + (j_p^2 - j_{p3}^2)] + \Big(\frac{1}{4g_1} + \frac{1}{4g_2}\Big) \\ & \cdot C_2[(j_{n-}j_{p+} + j_{n+}j_{p-})] - \Big(\frac{1}{4g_1} + \frac{1}{4g_2}\Big) \\ & \cdot [C_n(I_+ j_{n-} + I_- j_{n+}) + C_p(I_+ j_{p-} + I_- j_{p+})] \\ & - \frac{1}{2}\Big(\frac{1}{4g_2} - \frac{1}{4g_1}\Big)[(I_+^2 + I_-^2) + (j_{n+}^2 + j_{n-}^2) \\ & + (j_{p+}^2 + j_{p-}^2)] - \Big(\frac{1}{4g_2} - \frac{1}{4g_1}\Big) \\ & \cdot [C_4(j_{n+}j_{p+} + j_{n-}j_{p-})] \\ & + \Big(\frac{1}{4g_2} - \frac{1}{4g_1}\Big)C_5[(I_+ j_{n+} + I_- j_{n-}) + (I_+ j_{p+} \\ & + I_- j_{p-})] + \frac{1}{2g_3}(I_3^2 + j_{n3}^2 + j_{p3}^2) \\ & + \frac{1}{g_3}[-(I_3 j_{n3} + I_3 j_{p3}) + j_{n3}j_{p3}], \end{aligned} \quad (3)$$

where g_k $(k = 1, 2, 3)$ is the moment of inertia with irrotational flow form about the kth axis:

$$g_k = \frac{4}{3} J \sin^2\Big(\gamma - \frac{2\pi}{3}k\Big), \quad (4)$$

where J is a parameter which turns g_k back to the axial symmetry case when $\gamma = 0$. If $g_1 = g_2$ and all terms including g_3 are skipped, the Hamiltonian in Eq. (3) becomes that of an axial rotor as in our previous work.[15] Note the existence of C_2, C_n, C_p and C_4, C_5 terms added in Eq. (3), where $(C_2 - 1)$ is called the residual n–p interaction strength,[13,15] and C_n and C_p are the Coriolis attenuation factors for valence neutron and proton, respectively. The two factors C_n and C_p are called C_3 when they are set equal to that in the case of ^{102}Rh discussed in the following. $C_4 = C_5 = 1.0$ are added *only* for the results analysis; they are the coefficients for turning on or switching off the corresponding off-diagonal matrix elements having phase factor $(-1)^{I-j_p-j_n}$ (see discussion below).

The sum of the two single particle Hamiltonians is

$$H_{sp} + H_{sn} = \sum_{\nu_p} e_{\nu_p}\alpha_{\nu_p}^+\alpha_{\nu_p} + \sum_{\nu_n} e_{\nu_n}\alpha_{\nu_n}^+\alpha_{\nu_n} \quad (5)$$

with

$$e_{\nu_i} = \sqrt{(\varepsilon_{\nu_i} - \lambda_i)^2 + \Delta_i^2} \quad (i = p, n) \quad (6)$$

being the energy of a single quasi-particle, and λ_i and Δ_i the parameters of Fermi energy and energy-gap, respectively; ε_i is the single particle eigen-energy in a triaxial deformed field produced by the core[16]

$$V_{s_i}(\beta, \gamma) = k_0\beta r^2\Big[\cos\gamma Y_{20} + \sqrt{\frac{1}{2}}\sin\gamma(Y_{22} + Y_{2-2})\Big], \quad (i = p, n) \quad (7)$$

where γ is the angle of the triaxial deformation, the meaning of other symbols being the same as in the axial case.[12] The eigenstates corresponding to ε_i in the basis of spherical j shell, the bases of the model space for diagonalizing the Hamiltonian (1) and the eigenvalue equation of H are all the same as in the axial Hamiltonian case.[12]

Table 1. Values of parameters for yrast band calculations in the two odd–odd ^{98,102}Rh nuclei.

Nucleus	$g(\kappa^{-1})$	$\lambda_n(\kappa)$	C_2	C_p	C_n	$\gamma(^\circ)$	κ (keV)
^{98}Rh	20.00	−0.91	1.15	1.0	0.80	1 2	1800
^{102}Rh	36.00	−0.88	1.1	0.68	0.68	10	1770

For the calculations of the odd–odd ^{98,102}Rh nuclei, we used the following values: $j_p = g_{9/2}$, $j_n = h_{11/2}$ and $\Omega_p = -1/2, 3/2, -5/2, \cdots - 9/2$, $\Omega_n = 1/2, -3/2, 5/2, \cdots - 11/2$ because of time reversal symmetry. We have chosen $\Delta_p = \Delta_n = 0.46\kappa$, $\lambda_p = -0.45\kappa$, where κ is the energy unit[10] and $K = -5, \cdots + 5$. Values of the other parameters for the two isotopes are listed in Table 1.

All the parameters are those used for the axial Hamiltonian case[15] except for the energy unit κ because the addition of a triaxial deformation to the core of the nucleus affects the parameter κ, which is proportional to the axially symmetric deformation β.

In order to justify the parameters used here, let us consider the nucleus ^{102}Rh as an example. Its calculated energy spectrum from spin $I = 10 = j_n + j_p$ to $I = 26$ is compared with the experimental band as shown in Fig. 1.

Except for an energy starting point which has been deduced from the results of the calculation, it appears that the theoretical energy spectrum vs spin I agrees qualitatively with the experimental data.

As seen in Figs. 2(a) and 2(b), the calculations with a triaxial Hamiltonian reproduce quite well the experimental amplitudes and phases of the staggering at low-, medium- and high-spin ranges in ^{98}Rh and ^{102}Rh. Figures 2(c) and 2(d), identical to Fig. 1 of Ref. [15], are presented to compare the results of calculations made using theories with triaxial and axial symmetric Hamiltonians. Comparing Figs. 2(a) and 2(b) with Figs. 2(c) and 2(d) respectively, it is obvious that the introduction of triaxial deformation improves the agreement between experiments and theory for ^{98}Rh and ^{102}Rh, especially in the higher spin range where the staggering is larger.

To interpret the mechanism of the signature inversion, we notice that only matrix elements with phase coefficient $(-1)^{I-j_p-j_n}$ can contribute to the signature zigzag behaviour. Therefore we will examine the contributions of the different terms which possess this phase factor.

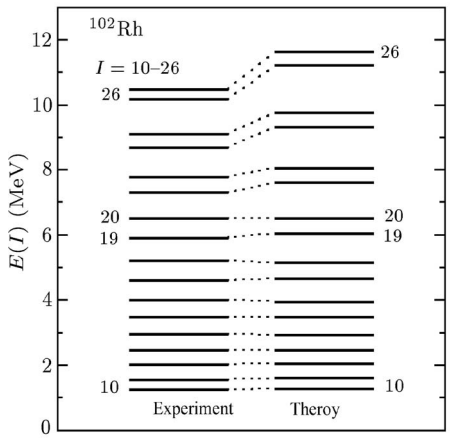


Fig. 1. Comparison of the results of calculations using a Hamiltonian with triaxial deformation (right column) for the $\pi g_{9/2}\nu h_{11/2}$ band in the odd–odd ^{102}Rh nucleus with experimental data (left column) from Ref. [6] The comparison is made for spin values I extending from $I = 10 = j_n + j_p$ to $I = 26$.

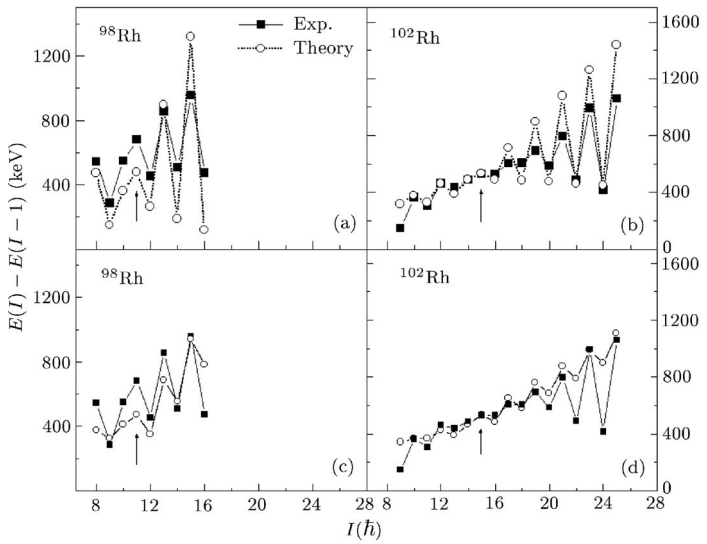


Fig. 2. Comparison of the calculation results using a Hamiltonian with triaxial deformation ((a)and (b)) and a Hamiltonian with axial deformation ((c)and (d)), respectively, (dashed lines) for $\pi g_{9/2}\nu h_{11/2}$ bands in the two odd–odd nuclei ^{98}Rh and ^{102}Rh with experimental data (solid lines) from Refs. [3,6]. The vertical line in each box indicates the location of the calculated inversion spin. The subfigures (c) and (d) are similar to Fig. 1 of Ref. [15].

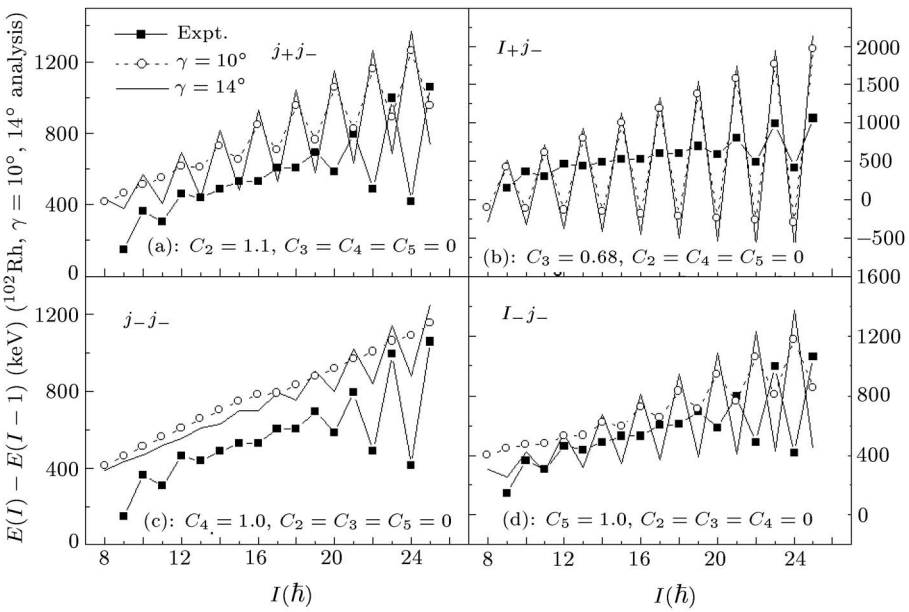


Fig. 3. Comparison of experimental data for $\pi g_{9/2}\nu h_{11/2}$ bands in ^{102}Rh from Ref. [6] with calculations using a triaxial deformation Hamiltonian with $\gamma = 10°$ and $\gamma = 14°$ and varying C_n terms in Eq. (3).

Let us consider the nucleus ^{102}Rh as an example and analyse the results in Fig. 3. We first switch off all the matrix elements possessing phase factor $(-1)^{I-j_p-j_n}$, and then reintroduce them, one by one. For example, in Fig. 3(a), only matrix elements which we called the n–p interaction terms, $j_{n-}j_{p+}$ and $j_{n+}j_{p-}$ with the simple form j_+j_-, are used in the calculation and all other terms of matrix elements having the same phase factor are switched off by setting $C_3 = C_4 = C_5 = 0$ (with $C_p = C_n = C_3$). The calculation results shown in Fig. 3(a) suggest the following rules: (i) The n–p interaction term makes the signatures always inverted because they are out of phase with the experimental zigzags in the high spin region where the signature is known to have normal oscillations. (ii) The amplitudes of the inverted signature splitting caused by n–p interaction are proportional to spin I, i.e. the larger the spin, the greater the

amplitudes of the abnormal signature splitting. (iii) The amplitudes of the inverted signature splitting induced by n–p interaction with $\gamma = 14^\circ$ are larger than that with $\gamma = 10^\circ$. This means that the amplitudes of abnormal signature oscillation are also proportional to the triaxial deformation angle γ: i.e., the larger the γ deformation, the greater the amplitudes of the abnormal signature splitting.

The above-mentioned rules (ii) and (iii) also hold for the cases shown in Figs. 3(b)–3(d) but the result given in rule (i), i.e. the normality or abnormality of the phase of the staggering, are not the same for the different subfigures. The Coriolis terms, i.e. I_+j_- terms in Fig. 3(b), and the $j_{n-}j_{p-}$, $j_{n+}j_{p+}$ terms, i.e. j_-j_- coming only from triaxial deformation in Fig. 3(c), make the signature vibration normal. The other set of terms having the phase factor and coming only from triaxiality, i.e. I_-j_- terms including I_-j_{n-}, I_+j_{n+} and I_-j_{p-}, I_+j_{p+} in Fig. 3(d), induces abnormal signature oscillation such as the n–p interaction term in Fig 3(a).

The comparison of the amplitudes of the oscillations presented in the four subfigures (note the scales of the vertical Y axis) illustrates that on the one hand, both the I_+j_- and j_-j_- sets of terms in Figs. 3(b) and 3(c), respectively, make the signature vibration normal but the influence of the Coriolis terms in Fig. 3(b) is much larger than that of the j_-j_- terms in Fig. 3(c). On the other hand, the sets of the terms which make the signature oscillation abnormal, i.e. n–p interaction in Fig. 3(a) and I_-j_- terms in Fig. 3(d), produce effects with similar and medium strength amplitudes.

In the axial Hamiltonian case where there are no j_-j_- and I_-j_- terms, the possible mechanism of signature inversion comes from the competition between the n–p interaction and the Coriolis force in low K space as previously discussed.[13,15] The magnitude of the abnormal staggering due to the n–p interaction is very small in the axial case, $\gamma = 0^\circ$, and basically does not increase with increasing spin I, hence it can push the signature inversion point in a limited magnitude of spin. If the inversion point is in the position of very high spin, the axial Hamiltonian calculation would have difficulty in reproducing the experimental data.

When the core of the nucleus becomes triaxially deformed, two more sets of terms are involved in the competition, one for normal signature vibration, i.e. j_-j_- in Fig. 3(c), and one for abnormal signature oscillation, i.e. I_-j_- in Fig. 3(d). As the effect of the (I_-j_-) term is much stronger than that of the (j_-j_-) term, the signature inversion point will be pushed to higher spin. Therefore, the bigger the triaxial deformation is, the higher the position of the signature inversion point is, because of the rule (iii) discussed above. This indicates that the calculations performed for very high spin of inversion point might take a large γ triaxiality parameter to reproduce the experiment.

The values of $\gamma = 12^\circ$ for ^{98}Rh and $\gamma = 10^\circ$ for ^{102}Rh are small as their inversion points are at lower spins (at about $I = 11$ and 15 respectively). In order to reproduce the larger experimental staggering, we have chosen a larger γ value for ^{98}Rh than for ^{102}Rh. Though the smaller moment of inertia in ^{98}Rh itself causes a larger staggering as discussed in Refs. [7,15], we obtained a better agreement with the experiment using a larger γ value due to the effect of rule (iii). The main improvement brought by a triaxial Hamiltonian [Figs. 2(a) and 2(b)] relative to an axial Hamiltonian [Figs. 2(c) and 2(d)] concerns the amplitude of the staggering as quoted in the rules (ii) and (iii).

It should be noticed that in Fig. 3, the main phase competition happens between the n–p interaction [Fig. 3(a)] and the Coriolis force [Fig. 3(b)] especially with small γ triaxial deformation, as the amplitudes decrease with decreasing γ much faster in the triaxial I_-j_- case [Fig. 3(d)] than that in the n–p interaction case [Fig. 3(a)]. Figures 3(c) and 3(d) comprise all the sets of terms with phase factor $(-1)^{I-j_n-j_p}$ accompanying only with triaxiality. From the difference of the amplitudes in Figs. 3(c) and 3(d) we can see that the triaxial deformation can mainly help the abnormal signature vibration to push the signature inversion point to higher spin in the competition between normal and abnormal zigzags. Therefore, the triaxial deformation could play an important role in the situation where the signature inversion point is at high spin.

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