Anomalous Behaviour of the Spectra in Young's Double-Slit Interference Experiments *

PU Ji-Xiong(蒲继雄)**, CAI Chao(蔡超)

Department of Electronic Science and Technology, Huaqiao University, Quanzhou 362011

(Received 9 January 2004)

It is shown that an anomaly of the spectrum takes place in the interference field when Young's double-slit interference experiment is illuminated by spatially coherent polychromatic light. Specifically, drastic spectral changes (i.e. the spectral shifts exhibit a rapid transition) occur in the vicinity of the dark fringes. The potential applications of this spectral anomaly are also considered.

PACS: 42. 25. Hz, 42. 25. Kb, 42. 25. Fx

In recent years, the points in an optical field at which the intensity has zero value have attracted great interest, because the phase at the points becomes undetermined and the structure of the field in the neighbourhood of such points has a rather complex structure: for example, exhibiting dislocations and optical vortices.[1−3] Studies of the phenomena associated with phase singularities have gradually developed into a new branch of physical optics, referred to as singular optics.[3] Most of the publications concerned with singular optics have dealt with monochromatic waves. Recently Wolf and his co-workers showed that drastic spectral changes take place in the vicinity of intensity zeros in the focal region of the polychromatic scalar field,[4,5] and experimental studies of the singular behaviour near the singular points were also carried out.[6] These spectral anomalies also occur in the vicinity of the dark rings of a Fraunhofer diffraction pattern.[7] It seems that a close link exists between the spectral anomalies near the singular points and the spectral switches.[8−10] The spectral anomalies related to the spectral switches have been investigated in Refs. [11,12].

Young's interference experiment is one of the most fundamental experiments in all physics, and is widely applied to physical optics and quantum optics. In the past decade, great care has been taken to examine spectral properties in Young's interference experiment in which the incident light is of polychromatic partially coherent light.[13−15] It is well known that for illumination by partially coherent light, the points at which the field amplitude has zero value vanish. This means that the singular points disappear. In this case, some interesting phenomena of the spectra may be erased. In this Letter we demonstrate that when Young's double-slit interference experiment is illuminated by spatial completely coherent polychromatic light, the anomalous behaviour of the spectra occur in the interference field. Specifically, spectral anomalies also take place in the critical directions, associated with the dark fringes for a fixed frequency.

Let us first consider that a monochromatic wave of frequency ω, with spectral density $S^{(0)}(\omega)$, is incident upon double slits, as shown in Fig. 1. The separation of the two slits is equal to $2a$, and the widths of both the slits are equal and are very small compared to the separation, so that the amplitude transmission function of the plane in which the two slits are located is

$$E(x, z = 0, \omega) = \sqrt{S^{(0)}(\omega)}[\delta(x - a) + \delta(x + a)], \quad (1)$$

where δ is a delta function. The transverse coordinates at the $z = 0$ plane are denoted as x and y. The distance of the observation point from one slit is R_1 or R_2.

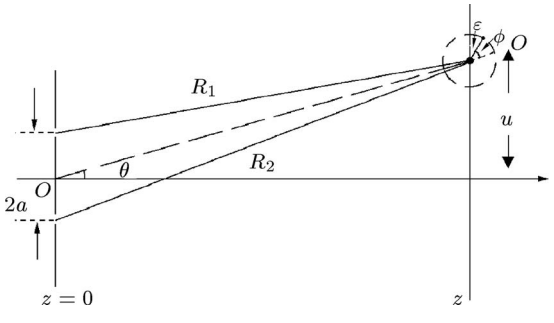


Fig. 1. A schematic for Young's two-slit interference experiment.

It is assumed that the observation plane is located at the far field, denoted as the z plane. The field at a point Q beyond the screen is given by

$$E(u, z, \omega) = \Big(-\frac{ik}{2\pi z}\Big)^{1/2} \exp(ikz) \int E(x, z = 0, \omega) \cdot \exp\Big(-i\frac{ku}{z}x\Big)\, dx, \quad (2)$$

* Supported by the Fujian Natural Science Foundation of China.

** Email: jixiong@hqu.edu.cn

©2004 Chinese Physical Society and IOP Publishing Ltd

where $k = \omega/c$ with c being the speed of light in vacuum. The transverse coordinates at the z plane are u and v.

Substituting Eq. (1) into Eq. (2) and after direct integral calculation, we obtain the field in the observation screen, i.e.

$$E(\theta, z, \omega) = 2\Big(-\frac{ikS^{(0)}(\omega)}{2\pi z}\Big)^{1/2} \exp(ikz)\cos(k\theta a). \tag{3}$$

In Eq. (3), we have made the approximation $\theta \approx u/z$, which is the observation angle, viewed from the central point O of the two slits.

The spectral intensity in the interference field is

$$S(\theta, \omega) = |E(\theta, z, \omega)|^2 = \frac{\omega S^{(0)}(\omega)}{\pi c z}[1 + \cos(2\omega\theta a/c)]. \tag{4}$$

It is supposed that the spectrum of the incident field on Young's double-slit experiment consists of a single line of Gaussian profiles, centred at frequency ω_0 and with the rms width Γ, i.e.

$$S^{(0)}(\omega) = S_0 \exp\{-(\omega - \omega_0)^2/2\Gamma^2\}, \tag{5}$$

where S_0 is a constant.

It is readily found from Eq. (4) that the direction θ at which the zero intensity occurs at the central frequency ω_0 is given by

$$\theta_n = (2n-1)\theta_1, \quad n = 0, \pm 1, \pm 2, \pm 3 \cdots, \tag{6}$$

where

$$\theta_1 = \frac{\pi c}{2\omega_0 a} \tag{7}$$

is the direction angle of the first-order dark fringe for the frequency ω_0, which is called the first-order critical direction. The dark fringes corresponding to the direction angle $\theta_n = (2n-1)\theta_1$ are just the positions at which the phase is singular for the frequency ω_0, and the angle θ_n is referred to as the nth-order critical direction.

The incident spectrum is a single Gaussian line (see Eq. (5)) with $\omega_0 = 10^{-15}\,\mathrm{s}^{-1}$ and $\Gamma = 0.01\omega_0$, which is plotted in Figs. 2 and 3. The spectra in the interference field in the first critical direction and the other two directions (very close to the first critical one on the left and right) are displayed. We find that in the first critical direction, the spectrum is split into two lines, whereas in the immediate neighbourhood of the critical direction, it is either redshifted or blueshifted with respect to the spectrum of the incident light. This spectral anomaly is the consequence of the interference of different spectral components of the light.

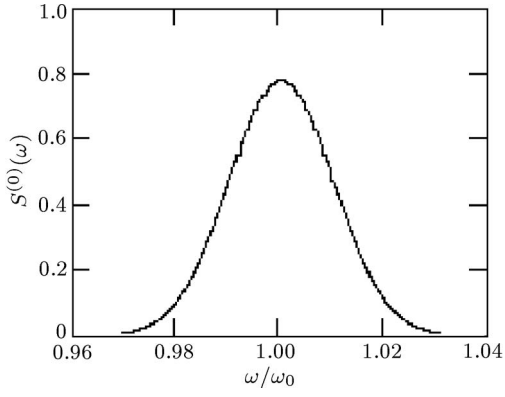


Fig. 2. Gaussian spectrum of the incident wave with central frequency $\omega_0 = 10^{15}\,\mathrm{s}^{-1}$. The rms width of the spectral line was taken to be $\Gamma = 0.01\omega_0$.

Let us also examine the behaviour of the spectrum on a small circular loop of the radius $\varepsilon \ll z$, centred in the first-order critical direction. In Fig. 4, the spectrum is plotted for different values of polar angle ϕ as is indicated in Fig. 1. We can see that when $\phi = 0$, the spectrum is split into two lines, one with redshift and the other with blueshift. The magnitudes of the maximum of the line shifted towards the blue are slightly greater. When $\phi = \pi/8$ and $\phi = \pi/2$, the spectrum is blueshifted with respect to the spectrum of the incident light, whereas when $\phi = \pi$, the spectrum is just the same as that in the case of $\phi = 0$. When $\phi = 3\pi/2$ and $\phi = 15\pi/8$, the spectrum is redshifted with respect to the spectrum of the incident light. From the spectral changes indicated above, we find that the spectral line towards the blue is dominated. Such behaviour is different from the spectrum of the focused wave on a circle centred at a singular point [3] or the critical diffraction angle in the Fraunhofer diffraction.[7]

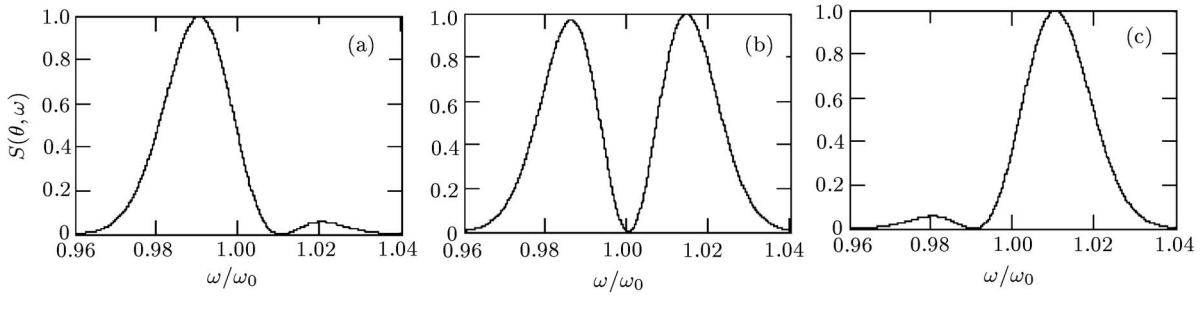


Fig. 3. Spectra of the interference field in the far zone in the vicinity of the first critical direction θ_1: (a) $\theta = 0.99\theta_1$, (b) $\theta = 1.0\theta_1$, (c) $\theta = 1.01\theta_1$. The spectrum of the incident light is a single Gaussian line, with $\omega_0 = 10^{15}\,\mathrm{s}^{-1}$ and $\Gamma = 0.01\omega_0$.

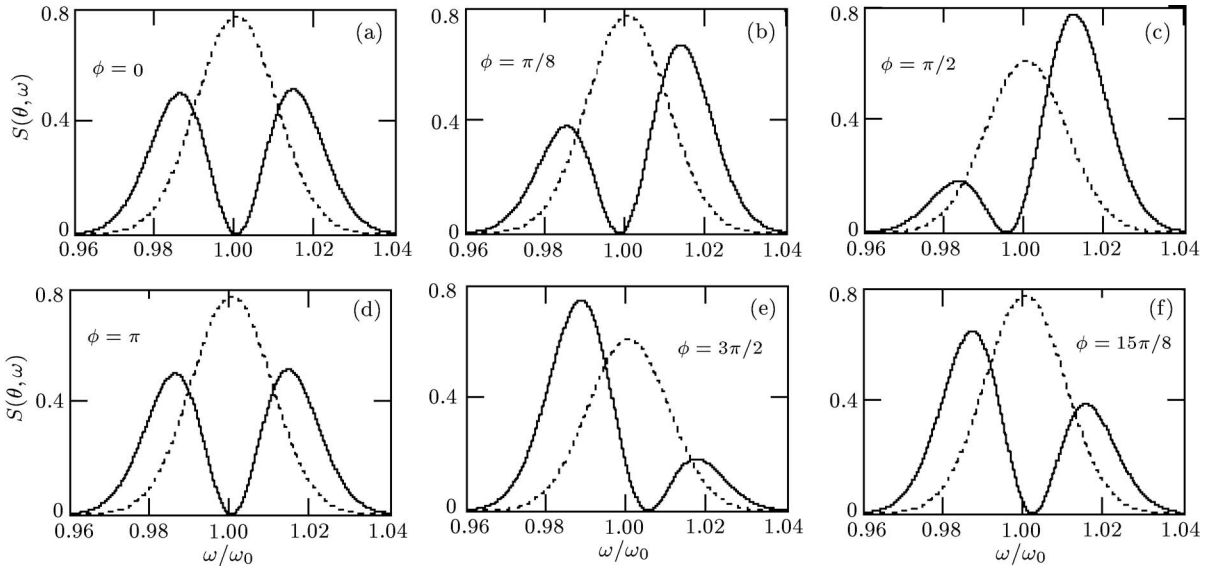


Fig. 4. Spectral changes in the neighbourhood of the zero point Q of the first-order dark fringe of the interference pattern at frequency ω_0. The spectrum is displayed for different values of the angle ϕ, denoted in Fig. 1. The numerical parameters are chosen so that $\varepsilon/z\theta_1 = 0.005$. The dotted curve represents the spectrum of the incident light, which is a single Gaussian line, with $\omega_0 = 10^{15}\,\mathrm{s}^{-1}$ and $\Gamma = 0.01\omega_0$.

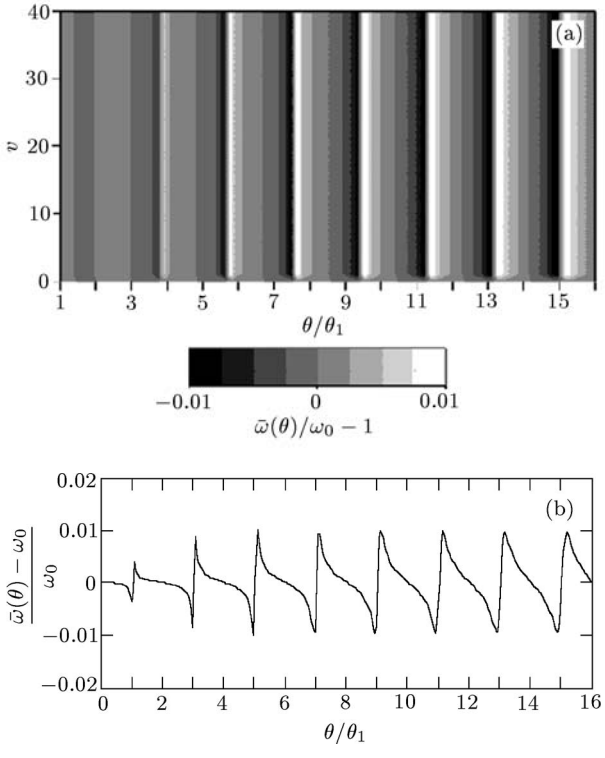


Fig. 5. (a) The contour plot of the mean frequency $\bar{\omega}(\theta)$ of the spectrum in the far zone of the interference field as a function of the direction angle θ and v with v being one of the transverse coordinates in the z plane. (b) The change of value $\bar{\omega}(\theta)/\omega_0 - 1$ as a function of the direction angle θ, corresponding to Fig. 5(a). Other numerical parameters are the same as in Fig. 3.

The spectral anomaly can be characterized by the mean frequency for different observation angles, which is defined as

$$\bar{\omega}(\theta) = \frac{\int \omega' S(\theta, z, \omega') d\omega'}{\int S(\theta, z, \omega') d\omega'}. \tag{8}$$

The mean frequency is plotted as a function of the direction angle θ in Fig. 5 for $\omega_0 = 10^{-15}\,\mathrm{s}^{-1}$ and $\Gamma = 0.01\omega_0$. The contour in Fig. 5(a) is more black and more white as the spectrum is more redshifted or blueshifted, respectively. The change of $\bar{\omega}(\theta)/\omega_0 - 1$ with respect to the direction angle θ is displayed in Fig. 5(b), corresponding to Fig. 5(a). It is seen that although there are some changes of the spectrum at any direction angle, the most drastic spectral changes occur at the critical angles given by Eq. (6). These drastic changes arise from the intensity zeros in the interference fringes for the central frequency ω_0.

In conclusion, we have shown that when a spatial completely coherent polychromatic light is incident upon Young's double slits, spectral anomalies occur in the critical directions, at which the dark fringes for the centred frequency ω_0 take place, of the far-zone of the interference field: i.e., at these critical directions, a single Gaussian spectrum is split into two lines, and on one side of the vicinity of these directions, the spectrum is redshifted, and on the other side of the vicinity the spectrum is blueshifted. Such a spectral anomaly may have applications in free-space communications, in which the blueshift could be associated with logic gate 1, and the redshift could be associated with 0. As is well known, the interference phenomena in Young's double-slit experiment will keep the self-similarity in the far-zone of the interference field. It is this self-similarity of the far-zone of the interference field that can make it possible to transmit this infor-

mation over quite long distances. Further investigation on the spectral anomalies and their applications in the free-space communications will be carried out.

References

[1] Nye J F and Berry M V 1974 *Proc. R. Soc. Lond.* A **336** 165

[2] Kessler D A and Freund I 2003 *Opt. Lett.* **28** 111

[3] Soskin M S and Vasnetsov M V 2001 *Progress in Optics* ed Wolf E (Amsterdam: Elsevier) **42** 219

[4] Gbur G, Visser T D and Wolf E 2002 *Phys. Rev. Lett.* **88** 013901

[5] Gbur G, Visser T D and Wolf E 2002 *J. Opt. Soc. Am.* A **19** 1694

[6] Popescu G and Dogariu A 2002 *Phys. Rev. Lett.* **88** 183902

[7] Ponomarenko S A and Wolf E 2002 *Opt. Lett.* **27** 1211

[8] Pu J X, Zhang H H and Nemoto S 1999 *Opt. Commun.* **162** 57

[9] Pu J X and Nemoto S 2000 *IEEE J. Quantum Electron.* **36** 1407

[10] Kandpal H C and Vaishya A S 2002 *IEEE J. Quantum Electron.* **38** 336

[11] Foley J T and Wolf E 2002 *J. Opt. Soc. Am.* A **19** 2510

[12] Pu J X, Nemoto S and Lu B D 2003 *J. Opt. Soc. Am.* A **20** 1933

[13] James D F V and Wolf E 1991 *Phys. Lett.* A **157** 6

[14] Kandpal H C, Vaishya J S, Chander M, Saxena K and Joshi K C 1992 *Phys. Lett.* A **167** 120

[15] Pan L Z and Lu B D 2001 *IEEE J. Quantum Electron.* **37** 1377