Energy Density in Quark–Gluon Plasma *

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We study the energy density in quark–gluon plasma. At the very high temperature, the quark matter is a hot and dense matter in the colour deconfinement condition, and quarks can coalescent diquarks. Energy density of this system is worked out and compared with the energy density in the other two kinds of situations. Possible energy density is about $\epsilon_0 \approx 2.4\,\mathrm{GeV/fm^3}$ *according to our estimation for quark matter including diquarks.*

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Based on the theory of standard model, the structures of the vacua of the strong and electroweak interactions were investigated. To study the vacuum structure, one has to excite the vacuum under the condition of high-energy density and high temperature. It was suggested that in a volume of nucleus size, the hadronic matter phase transits into the phase of quark–gluon plasma under the condition of high-energy density and high temperature.

The quark–gluon plasma phase cannot be observed directly. The directly observable objects are leptons and photons emitted by quark–gluon plasma and hadrons emitted by the hadronization of the quark–gluon plasma phase, which occurs as the phase transition from quark–gluon plasma phase to hadronic matter phase. Based on the theoretical analysis, several kinds of experimental phenomena have been proposed as the possible signals of formation of quark–gluon plasma.[1,2]

One significant difference between gluon plasma phase and hadronic matter phase is their local energy densities.[3] The energy density in quark–gluon plasma for the temperature above the critical temperature T_c should be larger than $2\,\mathrm{GeV/fm^3}$, which is much larger than the energy density of the ordinary hadronic matter phase. Theoretical estimation suggested that,[4,6] for example, if $T_c = 200\,\mathrm{MeV}$, then $\varepsilon_{HM} = 0.613\,\mathrm{GeV/fm^3}$ and $\varepsilon_{QGP} = 3.253\,\mathrm{GeV/fm^3}$, the ratio of ε_{QGP} to ε_{HM} is about 5.31. If one obtains an estimation of $\varepsilon > 3\,\mathrm{GeV/fm^3}$ from the experimental data of a certain heavy-ion collision, the quark–gluon plasma phase probably appears in this collision.[12] This kind of large energy density will manifest itself as large transverse energy density of rapidity $\mathrm{d}E_T/\mathrm{d}\eta$ and large multiplicity density of rapidity $\mathrm{d}n/\mathrm{d}\eta$ in the relativistic heavy-ion collision.[7,8]

In the high-energy heavy ion research, it was assumed that the quark matter formed in the high-energy heavy-ion collisions is similar to a quasi-stationary quark–gluon plasma,[11,13] consisting of massless gluons and quarks and antiquarks with current mass. We now consider a quark–gluon system including diquark in a volume V with the temperature T, and compare the result with those in the systems without diquark and with hadrons only.

For the normal quark matter consisting of massless gluons and quarks, the u- and d-quark masses are only several MeV, the s-quark mass is about 75 to 155 MeV.[10] In the very high temperature system, we can treat them as massless gluons and quarks (which only has a very little difference with the case considering mass). The number of quarks

$$\mathrm{d}N_q = \frac{g_q V 4\pi p^2 \mathrm{d}p}{(2\pi)^3}\left\{\frac{1}{1+\mathrm{e}^{(p-\mu_q)/T}}\right\},$$

where g_q is the number of degrees of freedom of the quark. When the net number of baryons is zero, the number densities for quarks and antiquarks are the same,

$$n_q(\mu) = \frac{g_q}{(2\pi)^3}\int_0^\infty 4\pi p^2 \mathrm{d}p \frac{1}{1+\mathrm{e}^{(p-\mu_q)/T}},$$
$$n_{\bar{q}}(\mu) = \frac{g_q}{(2\pi)^3}\int_0^\infty 4\pi p^2 \mathrm{d}p \frac{1}{1+\mathrm{e}^{(p+\mu_q)/T}},$$

and $\mu_q = 0$. In a quark matter with volume V and temperature T, the energy of massless quarks is

$$E_q = \frac{g_q V}{2\pi^2}\int_0^\infty \frac{p^3 \mathrm{d}p}{1+\mathrm{e}^{p/t}}.$$

Thus the system energy come from quarks is

$$E_q = \frac{7}{8} g_q \frac{\pi^2}{90} V T^4.$$

Similarly, the energy from gluons is

$$E_g = g_g V \frac{\pi^2}{30} T^4.$$

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For the normal quark matter, the energy density is

$$\varepsilon = g_{\rm all}\frac{\pi^2}{30}T^4,$$

where $g_{\rm all} = g_g + \frac{7}{8}(g_q + g_{\bar{q}})$ is the total number of degrees of freedom of the quark, antiquark and gluon.

In principle, the production and the emission of particles can occur throughout the evolution of the dense matter fireball. However, it has generally been assumed that the bulk of particles is produced at the final moment of the evolution when the temperature of the dense matter sinks below the deconfinement condition.[5] At the beginning, the quark–gluon plasma consists of gluons and quarks and antiquarks. Different mechanisms of flavour pair production have been explored. It is generally accepted that in the deconfined region, diquarks are primarily produced in gluon fusion reactions, and baryons in the hadronic gas come from the fusion reaction between quarks and diquark. It is obvious that there exist diquarks in the quark matter near by the hadronization. When there are diquarks in the quark–gluon plasma, like kion or pion consisting of two quarks, the diquark $|q_iq_j\rangle$ mass $m_{d_k} \neq m_{q_i} + m_{q_j}$. We let diquark mass to be

$$m_{d_k} = m_D + m_{q_i} + q_j,$$

where m_D is a parameter, $m_{d_k} > T$, so

$$\mathrm{d}N_{d_i} = \frac{g_{d_i}V4\pi p^2\mathrm{d}p}{(2\pi)^3}\Big\{\frac{1}{\mathrm{e}^{(E_i-\mu_{d_i})/T}-1}\Big\},$$

where $E_i = \sqrt{m_i^2+p^2}$ and g_{d_i} is the number of degrees of freedom of the diquark $|qq\rangle$. Then

$$n_{d_i}(\mu) = \frac{g_{d_i}}{(2\pi)^3}\int_0^\infty 4\pi p^2\mathrm{d}p\frac{1}{\mathrm{e}^{(E_i-\mu_{d_i})/T}-1},$$

and the system energy from quarks is

$$\varepsilon_{d_i} = \sum_i \frac{g_{d_i}}{2\pi^2}\int_0^\infty \frac{E_ip^2\mathrm{d}p}{\mathrm{e}^{(E_i-\mu_{d_i})/T}-1},$$

where the sum runs over all possible diquarks, because not only diquarks (antidiquarks) but also gluons and quarks (antiquarks) contribute to the energy density of the quark matter.

The diquarks have nine basis states of the form of $|q_iq_j\rangle$, where $1 \le i,\ j \le 3$. We first classify the two-quark states $|q_iq_j\rangle$ according to the irreducible representation. As can easily be seen, we only need the symmetric and antisymmetric combinations

$$(|q_iq_j\rangle + |q_jq_i\rangle)$$

and

$$\varepsilon_{lij}|q_iq_j\rangle \quad (l = 1,2,3).$$

Six symmetric states form a 6-representation, and three antisymmetric states form a 3*-representation. Thus we have the reduction

$$3 \times 3 = (3^* + 6).$$

All the established baryons and mesons should be $SU(3)$ colour singlet, but in the deconfined hot and dense quark matter, diquarks are not necessary to be colour singlet. Since each quark has a spin of 1/2 and the quark is fermions, the state function of any diquark must be symmetric under interchange of two quarks. Therefore, we can write out the number of degrees of freedom for both diquark and antidiquark.

At the very high temperature associated with massive particles, the energy density of hadron gas can similarly be estimated. For a given particle species i, if we have $T \ll m_i$, we cannot neglect the mass. In this case, we have

$$\varepsilon = \frac{1}{2\pi^2}\Big(\sum_B g_B\int_0^\infty \frac{E_ip^2\mathrm{d}p}{\mathrm{e}^{(E_i-\mu)/T}-1} + \sum_F g_F\int_0^\infty \frac{E_ip^2\mathrm{d}p}{\mathrm{e}^{(E_i-\mu)/T}+1}\Big),$$

where $E_i = \sqrt{m_i^2+p^2}$ is the energy of the ith particle, $g_{B(F)}$ is the number of degrees of freedom of each boson (fermion) and the sum runs over all boson and fermion states we choose. When $\mu = 0$, we have

$$\varepsilon = \frac{1}{2\pi^2}\Big(\sum_B g_B\int_0^\infty \frac{\sqrt{x^2+a^2}x^2\mathrm{d}x}{\mathrm{e}^{\sqrt{x^2+a^2}}-1} + \sum_F g_F\int_0^\infty \frac{\sqrt{x^2+a^2}x^2\mathrm{d}x}{\mathrm{e}^{\sqrt{x^2+a^2}}+1}\Big)T^4,$$

where $a = m_i/T$.

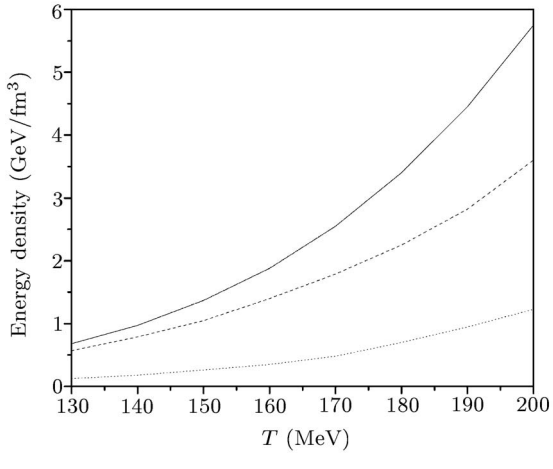


Fig. 1. Energy density at the given temperature. The solid line is the energy density of the quark–gluon plasma including diquarks, the dashed line is the energy density of the normal quark–gluon plasma, the dotted line is the energy density of the hadron gas.

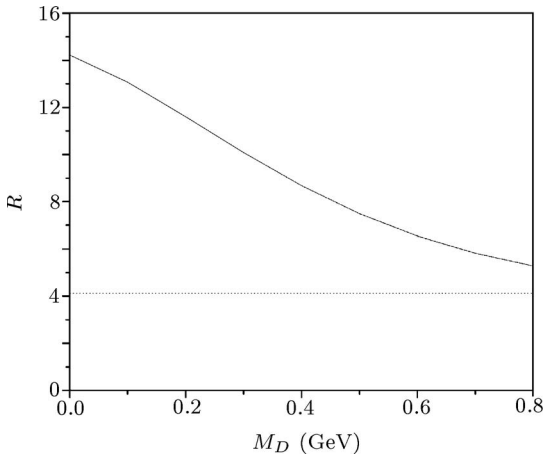


Fig. 2. Expansion times of the system at different mass parameter m_D for the quark matter including diquarks transiting into the hadronic matter, where $\mu_N = 40\,\mathrm{MeV}$. The dotted line refers to $R = 4.11$, which is the ratio of the normal quark matter to the hadronic matter.

According to the above discussion, we can compute the energy densities of there kinds of matter states at any given temperature. The result is plotted in Fig. 1.

According to our estimation,[9,12] the critical temperature T_c of phase transition between the hadronic matter phase and the quark–gluon plasma phase is about $T_c = 166.01\,\mathrm{MeV}$ when baryon chemical potential $\mu_N = 0$. At this temperature, the energy density for the hadronic matter phase is about $\varepsilon = 0.368\,\mathrm{GeV/fm^3}$, and for the normal quark–gluon plasma is $\varepsilon = 1.513\,\mathrm{GeV/fm^3}$. We define R to be the ratio of ε_{QGP} to ε_{HM}, then $R = 4.123$. The critical temperature T_c for the phase transition is 165.81 MeV when baryon chemical potential $\mu_N = 0.040\,\mathrm{GeV}$. At this temperature, the energy density for hadronic matter phase is about $\varepsilon = 0.367\,\mathrm{GeV/fm^3}$, and for the normal quark–gluon plasma is $\varepsilon = 1.510\,\mathrm{GeV/fm^3}$, and $R = 4.116$.

From Table 1 we can see that the energy density of the quark–gluon plasma including diquark is about $\varepsilon = 2.40\,\mathrm{GeV/fm^3}$ when $M_D = 0.600\,\mathrm{GeV}$, then $R = 6.54$, which means that the system will expand to about 6.5 times in volume when the phase of quark–gluon plasma transits into the hadronic matter phase.

Table 1. Energy density of the quark–gluon plasma including diquarks at different m_D when $\mu_N = 0$ or $0.040\,\mathrm{GeV}$, R is the ratio of ε_{QGP} to ε_{HM}.

μ_N	0		40 MeV	
m_D (GeV)	ε (GeV/fm^3)	R	ε (GeV/fm^3)	R
0.00	5.19	14.12	5.22	14.22
0.10	4.78	13.01	4.80	13.07
0.20	4.25	11.56	4.26	11.61
0.30	3.69	10.05	3.70	10.08
0.40	3.18	8.66	3.19	8.68
0.50	2.74	7.48	2.85	8.49
0.60	2.40	6.54	2.40	6.54
0.70	2.14	5.81	2.13	5.81
0.80	1.94	5.29	1.94	5.29

Anyway, for the reason of colour deconfinement in the quark–gluon plasma, diquarks can be included in the theory. For this kind of quark–gluon plasma, the thermodynamic quantities can be computed. Here we only show the energy density. It is also easy to calculate the production ratio of baryons and their antiparticles, which can be compared with the experimental data to verify whether there are diquarks in the very high temperature quark matter.

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