Magnetic Extraction of Energy from Accretion Disc Around a Rotating Black Hole *

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An analytical expression for the jet power extracted from the disc (JPEFD) is derived based on an equivalent circuit in black hole magnetosphere with a mapping relation between the radial coordinate of the disc and that of unknown astrophysical load. It turns out that this JPEFD is comparable with two other JPEFDs derived in the Poynting flux and hydrodynamic regimes, respectively. In addition, the relative importance of this JPEFD relative to the Blandford–Znajek power is discussed. It is shown that the BZ power is generally dominated by the JPEFD except in some extreme cases. Furthermore, we show that the JPEFD derived in our model can be well fitted with the jet power of 3C 273.

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It is well known that jets exist in many astronomical cases, such as active galactic nuclei, quasars, and young stellar objects. The association of jets with magnetized accretion discs or magnetized central objects (black holes or stars) is strongly supported by the recent observations of Hubble Space Telescope (HST), Chandra, and VLBI.[1,2] Different theoretical models have been proposed for the acceleration and collimation of jets, which can be divided into two main regimes, the Poynting flux regime and the hydromagnetic regime.[3−6] Both regimes are related to a poloidal magnetic field threading the disc, from which energy and angular momentum are extracted. In the Poynting flux regime, energy is extracted in Poynting flux (i.e. purely electromagnetic energy), but in the form of magnetically driven material winds in the latter regime.

The jet power extracted from the disc (JPEFD) has been discussed either in the hydrodynamic regime or in the Poynting flux regime. Recently, Cao discussed JPEFD in the hydrodynamic regime (henceforth $P_{\rm MHD}$), and found that $P_{\rm MHD}$ is mainly determined by the strength and the configuration of the poloidal magnetic field.[7] Lee discussed the JPEFD in the Poynting flux regime (henceforth $P_{\rm EM}^{\rm I}$), and found that the Poynting flux caused by a rotating magnetic field with Keplerian angular velocity can balance the energy and angular momentum conservation of a stationary accretion flow.[8] In this Letter, we derive the JPEFD (henceforth $P_{\rm EM}^{\rm II}$) by using an equivalent circuit in black hole (BH) magnetosphere, and compare $P_{\rm EM}^{\rm II}$ with $P_{\rm MHD}$ and $P_{\rm EM}^{\rm I}$, respectively. The geometric units $G = c = 1$ are used.

In order to derive the expression of $P_{\rm EM}^{\rm II}$, we make the following assumptions:

(1) The disc is thin, Keplerian, stable and perfectly conducting, which lies in the equatorial plane of the BH with the inner boundary being at the marginally stable orbit with the radius $r_{\rm in} = r_{\rm ms}$.[9]

(2) The configuration of the poloidal magnetic field of the BH-disc system is shown in Fig. 1. The poloidal magnetic field lines thread both the BH horizon and the surrounding disc, connecting the disc with the remote astrophysical load. The shape of each field line threading the disc at radius r has an analogy with that in the asymptotic jet model.[10] It is assumed that the asymptotic radius of each field line r' is smaller than the radius R_L of the light cylinder. The asymptotic radius r' is related to the disc radius r by

$$r' = \lambda r. \tag{1}$$

(3) The astrophysical load is axisymmetric, being located even in a plane $\boldsymbol{P}$ with some height above the disc. The surface resistivity σ_L of the load obeys the relation $\sigma_L = \alpha_Z \sigma_H = 4\pi\alpha_Z$, where $\sigma_H = 4\pi = 377\,\Omega$ is the surface resistivity of the BH horizon, and $\alpha_Z = 1$ is taken in the calculation.

(4) The poloidal magnetic field on the disc varies as a power law as given in Refs. [11–13],

$$B_D^p \propto \xi^{-n}, \tag{2}$$

where $\xi \equiv r/r_{\rm ms}$ is the radial parameter of the disc defined in terms of the radius of inner edge of the disc. The field threading the BH is just a continuation of the field threading the disc, and the following relation is satisfied,[7]

$$B_H^p = \varsigma B_D^p(r_{\rm ms}), \tag{3}$$

where B_H^p and $B_D^p(r_{\rm ms})$ are the magnetic fields at the horizon and the inner edge of the disc, respectively. Some authors[14,15] argued that the magnetic field threading the BH should not be significantly stronger than the field threading the inner disc. Here we assume $\varsigma = 3$ in the calculations.

Based on the work of MacDonald *et al.*,[16] we propose an equivalent circuit to calculate the disc power, as shown in Fig. 2. The segments PS and QR represent two adjacent magnetic surfaces consisting of the field lines connecting the disc and the load, and

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the segments PQ and RS represent the disc surface and the load sandwiched by the two surfaces, respectively. The quantities ΔZ_A and $\Delta\varepsilon_D$ (the subscript 'i' is omitted) are the resistance of the load and emf due to the rotation of the disc, respectively. The disc load is neglected in the calculation due to its perfect conductivity.

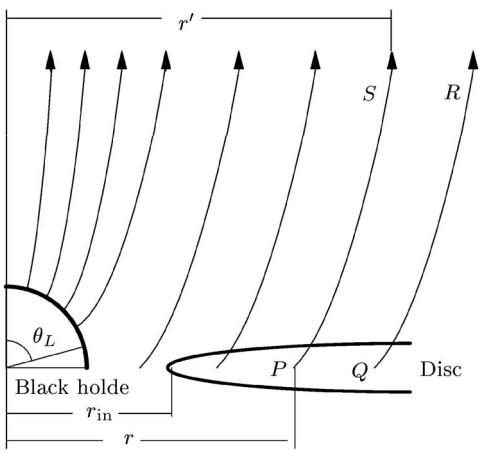


Fig. 1. Configuration of poloidal magnetic field threading the BH and its surrounding disc.

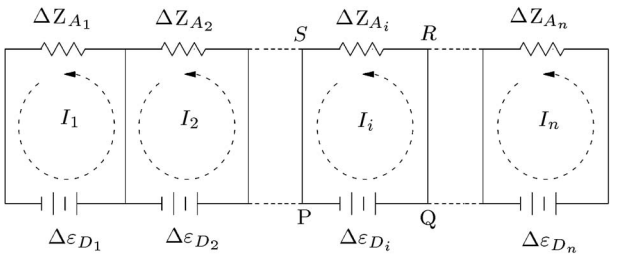


Fig. 2. Equivalent circuit for magnetic extraction of energy from accretion disc around a rotating BH.

Based on assumptions (1) and (2), the load resistance ΔZ_A between the two adjacent magnetic surfaces can be written as

$$\Delta Z_A = \sigma_L \frac{dr'}{2\pi r'} = 2\frac{dr}{r}. \tag{4}$$

The following equations are used in deriving the disc power, which are almost the same as given in deriving the Blandford–Znajek (BZ) power and MC power in our previous study:[12,13]

$$\Delta P^{\rm II}_{\rm EM} = (I^p)^2 \Delta Z_A, \quad I^p = \Delta\varepsilon_D/\Delta Z_A,$$
$$\Delta\varepsilon_D = (\Delta\Psi_D/2\pi)\Omega_D, \tag{5}$$

where I^p is the current in each loop, and $\Delta\Psi_D$ is the magnetic flux between the two adjacent magnetic surfaces,

$$\Delta\Psi_D = B^p_D 2\pi(\varpi\rho/\sqrt{\Delta})_{\theta=\pi/2}dr, \tag{6}$$

The quantity Ω_D is the angular velocity of the disc at the place where the magnetic flux penetrates, and it reads

$$\Omega_D = \frac{1}{M(\xi^{3/2}\chi^3_{\rm ms} + a_*)}. \tag{7}$$

The concerned Kerr metric coefficients are expressed by[17]

$$\varpi = (\Sigma/\rho)\sin\theta, \quad \Sigma^2 \equiv (r^2+a^2)^2 - a^2\Delta\sin^2\theta,$$
$$\rho^2 \equiv r^2 + a^2\cos^2\theta, \quad \Delta \equiv r^2 + a^2 - 2Mr. \tag{8}$$

Combining Eqs. (2) and (3), we have the poloidal magnetic field on the disc expressed by

$$B^p_D = \varsigma^{-1} B^p_H \xi^{-n}. \tag{9}$$

Incorporating Eqs. (5)–(9), we have

$$\Delta P^{\rm II}_{\rm EM}/P_0 = f(a_*, \xi, n)d\xi, \tag{10}$$

where the function $f(a_*, \xi, n)$ is expressed by

$$f(a_*,\xi,n) = \left[\chi^8_{\rm ms}(1+\xi^{-2}\chi^{-4}_{\rm ms}a^2_* + 2a^2_*\xi^{-3}\chi^{-6}_{\rm ms})\xi^{-2n+3}\right]$$
$$\cdot\left[2\alpha_Z\varsigma^2(1+\xi^{-2}\chi^{-4}_{\rm ms}a^2_* - 2\xi^{-1}\chi^{-2}_{\rm ms})(\xi^{3/2}\chi^3_{\rm ms}+a_*)^2\right]^{-1}. \tag{11}$$

The quantity P_0 is defined as

$$P_0 = B^2_H M^2 \approx B^2_4 (M/M_\odot)^2 \times 6.59\times 10^{28}\,{\rm erg\cdot s^{-1}}, \tag{12}$$

where B_4 and $M/M_\odot$ are the strength of B^p_H and the BH mass in units of 10^4 Gauss and one solar mass, respectively. Integrating Eq. (10) over the radial parameter ξ, we have the expression for the disc power,

$$P^{\rm II}_{\rm EM}(a_*,\xi,n)/P_0 = \int_1^\xi f(a_*,\xi',n)d\xi', \tag{13}$$

Cao[7] derived the JPEFD in the hydromagnetic regime as follows:

$$L_d = \frac{1}{2}\int (B^p_D)^2 [r\Omega_D(r)]^\alpha f(\alpha,\gamma_j) r dr, \tag{14}$$

$$f(\alpha,\gamma_j) = (\gamma_j - 1)\gamma_j^\alpha(\gamma_j^2-1)^{-(\alpha+1)/2}, \tag{15}$$

where the function $f(\alpha,\gamma_j)$ depends on the self-similar index $\alpha > 1$ and the Lorentz factor of the jet γ_j. Since both $r\Omega_D(r)$ and $f(\alpha,\gamma_j)$ are always less than unity, the upper limit to the disc power L_d can be written as

$$L_d < P_{\rm MHD} = \frac{1}{2}\int (B^p_D)^2 r^2 \Omega_D(r) dr. \tag{16}$$

The expression of the poloidal magnetic field is expressed as follows:[8]

$$B^p_D(r) \propto (r/M)^{-3/4}(M/M_\odot)^{-1/2}A^{-1}BE^{1/2}, \tag{17}$$

where A, B, and E are the general relativistic correction factors given in Ref. [9]. Combining Eq. (16) with Eq. (17), we obtain

$$P_{\rm MHD}(a_*,\xi)/P_0 = \frac{1}{2\varsigma^2}\int_1^\xi \frac{\chi^3_{\rm ms}\xi'^{-1}}{K^2(1+\xi'^{-3/2}\chi^{-3}_{\rm ms}a_*)}d\xi', \tag{18}$$

where $K = (A^{-1}BE^{1/2})_{r_{\rm in}}/(A^{-1}BE^{1/2})_r$.

Lee[8] derived the JPEFD $P^{\rm I}_{\rm EM}$ in the Poynting flux regime as follows:

$$P^{\rm I}_{\rm EM} = 4\{[-u_0(r_{\rm out})] - [-u_0(r_{\rm in})]\}[B^p_D(r_{\rm in})r_{\rm in}]^2. \tag{19}$$

Setting the inner and outer edge of the disc at $r_{\rm in} = r_{\rm ms}$ and $r_{\rm out} = \xi r_{\rm ms}$, we have

$$P^{\rm I}_{\rm EM}(a_*, \xi)/P_0 = 4\varsigma^{-2}\chi^4_{\rm ms}(E^\dagger - E^\dagger_{\rm ms}), \qquad (20)$$

where $E^\dagger = -u_0(\xi r_{\rm ms})$ and $E^\dagger_{\rm ms} = -u_0(r_{\rm ms})$ are the specific energies of the disc matter corresponding to r and $r_{\rm ms}$, respectively.[9]

We compare our disc power $P^{\rm II}_{\rm EM}$ with $P_{\rm MHD}$ and $P^{\rm I}_{\rm EM}$. To facilitate the comparison between $P^{\rm II}_{\rm EM}$ and $P_{\rm MHD}$, we take $n = 3/4$ in Eq. (13), which is the same as that given in Eq. (17). By using Eqs. (13), (18), and (20), we have the curves of the three JPEFDs varying with the radial parameter ξ for the given values of a_* as shown in Fig. 3.

In order to compare the above JPEFDs more clearly, we define $\eta_1 \equiv P^{\rm II}_{\rm EM}/P_{\rm MHD}$ and $\eta_2 \equiv P^{\rm II}_{\rm EM}/P^{I}_{\rm EM}$, and have the contours of η_1 and η_2 with different values in $\xi - a_*$ parameter spaces as shown in Figs. 4(a) and 4(b), respectively.

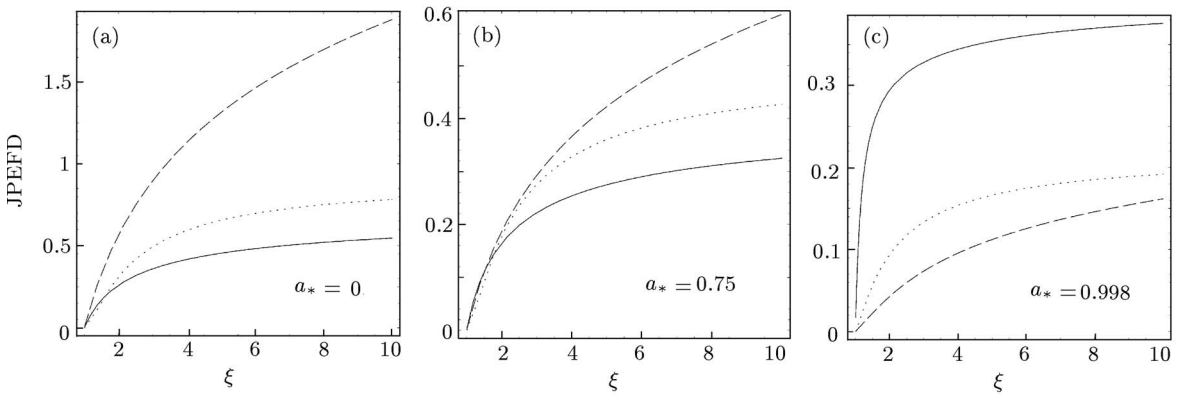


Fig. 3. Characteristics of $P^{\rm II}_{\rm EM}$ (solid), $P_{\rm MHD}$ (dashed) and $P^{\rm I}_{\rm EM}$ (dotted) versus ξ for $1 < \xi < 10$ with different values of a_*.

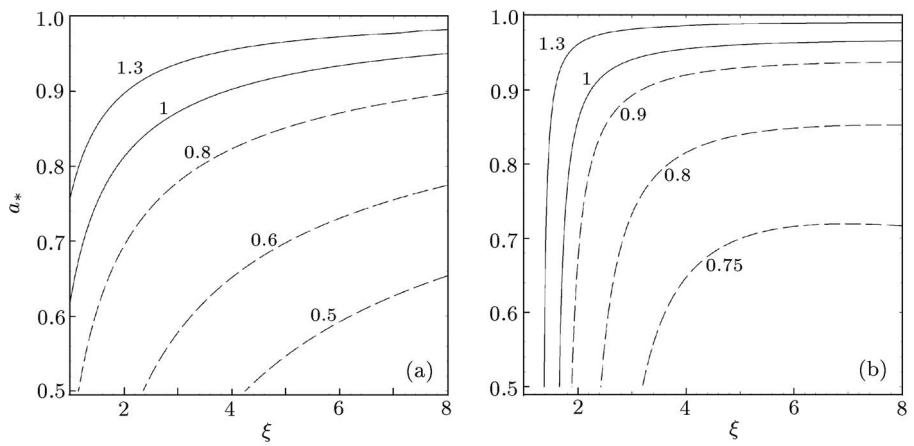


Fig. 4. Contours of the ratios of the JPEFDs in $a_* - \xi$ parameter space, (a) $\eta_1 \equiv P^{\rm II}_{\rm EM}/P_{\rm MHD}$, (b) $\eta_2 \equiv P^{\rm II}_{\rm EM}/P^{\rm I}_{\rm EM}$.

Inspecting Figs. 3 and 4, we have the following results:

(1) If the BH spin is not extremely high, the three JPEFDs satisfy the relation $P^{\rm II}_{\rm EM} < P^{\rm I}_{\rm EM} < P_{\rm MHD}$. However, this order reverses to $P_{\rm MHD} < P^{\rm I}_{\rm EM} < P^{\rm II}_{\rm EM}$, when the BH spin approaches unity,

(2) The jet power $P^{\rm II}_{\rm EM}$ dominates over $P^{\rm I}_{\rm EM}$ and $P_{\rm MHD}$ in two cases: (i) the region is very close to the inner edge of the disc; (ii) the BH spin is very high.

Recently some authors have argued that the BZ power was overestimated, and it is generally dominated by the electromagnetic power output of the inner region of the disc, provided that the poloidal magnetic field threading the BH does not differ significantly in strength from that threading the disc.[14,15] Here we discuss the importance of the JPEFD relative to the BZ power. The BZ power has also been derived based on an equivalent circuit in BH magnetosphere in our previous study:[12,13]

$$\frac{P_{BZ}}{P_0} = 2a_*^2 \int_0^{\theta_L} \frac{k(1-k)\sin^3\theta d\theta}{2-(1-q)\sin^2\theta}, \qquad (21)$$

where $q \equiv \sqrt{1-a_*^2}$ is a function of the BH spin, and $k \equiv \Omega_F/\Omega_H$ is the ratio of the angular velocity of the magnetic field lines to that of the horizon. The angular coordinate θ varies from 0 to θ_L on the BH horizon as shown in Fig. 1. In our calculation, $k = 0.5$ and $\theta_L = 0.45\pi$ are assumed in Eq. (21). In order to

compare the JPEFD with the BZ power clearly, we define the ratio of the JPEFD to the BZ power as $\eta = P_{\rm EM}^{\rm II}/P_{BZ}$. For the given values of power-law index n, the contours of η are shown in $\xi - a_*$ parameter space in Fig. 5.

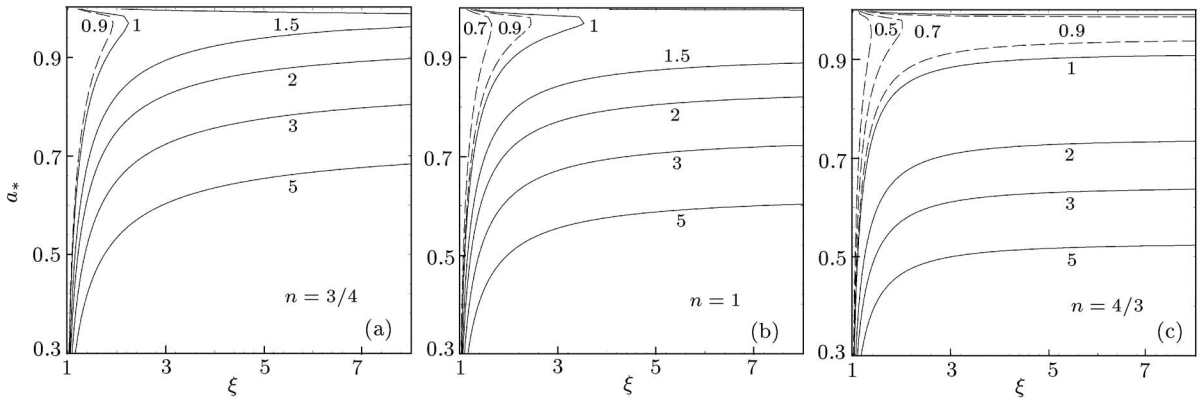


Fig. 5. Contours of constant values of η in $\xi - a_*$ parameter space for (a) $n = 3/4$, (b) $n = 1$, and (c) $n = 4/3$.

Table 1. Values of $P_{\rm EM}^{\rm II}$ according to the 3C 273 jet power for different values of the BH spin a_*.

a_*	0	0.3	0.5	0.75	0.998
B_H (G)	2.945×10^4	3.015×10^4	3.157×10^4	3.546×10^4	5.540×10^4
$P_{\rm EM}^{\rm II}$ (erg·s^{-1})	7.820×10^{46}	6.966×10^{46}	6.669×10^{46}	6.724×10^{46}	1.900×10^{47}

We can find from Fig. 5 that $P_{\rm EM}^{\rm II}$ will generally dominate over the BZ power, except that the BH spin a_* is extremely high. This result is consistent with those given in Refs. [14,15].

In our model, different magnetic flux surfaces anchor at different foot point radii r on the disc, which correspond to different light cylinder radii R_L. It is reasonable that the asymptotic radius of the magnetic flux surface is not greater than its light cylinder radius, i.e. $r' \leq R_L$. The light cylinder radius satisfies the condition

$$R_L \Omega_D = 1. \tag{22}$$

Incorporating Eqs. (7) and (22), we obtain the location of the light cylinder radius of magnetic flux surface,

$$R_L = M[(\sqrt{\xi}\chi_{\rm ms})^3 + a_*]. \tag{23}$$

Here 3C 273 is the brightest quasar on the celestial sphere. We intend to fit the jet power of 3C 273 by using $P_{\rm EM}^{\rm II}$ expressed by Eq. (13). The magnetic field B_H depends on the accretion rate $\dot{M}_D$ by the following relation, which is derived by considering the balance between the magnetic pressure on the horizon and the ram pressure of the innermost part of an accretion flow:[18]

$$(B_H^p)^2/(8\pi) = P_{ram} \sim \rho c^2 \sim \dot{M}_D/(4\pi r_H^2). \tag{24}$$

From Eq. (24), the strength of magnetic field on BH horizon is given by

$$B_H = \sqrt{2\dot{M}_D/(M^2(1+q)^2)}. \tag{25}$$

By using Eqs. (13) and (25), we have the values of the jet power of 3C 273 as listed in Table 1, where $\xi = 10$ is the upper limit of integration in Eq. (13). In the calculation, the BH mass and accretion rate are taken as $5 \times 10^8 M_\odot$ and $\dot{M} = 5 M_\odot$/yr, respectively, as given in Ref. [19]. It turns out that the values of $P_{\rm EM}^{\rm II}$ in Table 1 are approximately consistent with the jet power of the 3C 273.[20]

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