Stochastic Multiresonance for Periodically Modulated Noise in a Single-Mode Laser *

WANG Jun(王俊)[1,3], CAO Li(曹力)[1,2,3], WU Da-Jin(吴大进)[2,3]

[1]*State Key Laboratory of Laser Technology, Huazhong University of Science and Technology, Wuhan 430074*
[2]*CCAST (World Laboratory), PO Box 8730, Beijing 100080*
[3]*Department of Physics, Huazhong University of Science and Technology, Wuhan 430074*

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Multiplicatively introducing the noise and the signal into the gain-noise model of a single-mode laser system, we detect the two forms of stochastic resonance (the single-peak form stochastic resonance and stochastic multiresonance) in the curve of the dependence of signal-to-noise ratio upon the noise correlation time. Moreover, when the correlation coefficient and the intensities of both noises are changed, the two forms of stochastic resonance alternate.

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Since Benzi *et al.*[1] originally discovered the phenomenon of stochastic resonance (SR) in 1981, it has attracted wide investigations.[2−6] In recent years, there have been considerable developments in the process of exploring the essence of SR. Compared with the traditional concept, the new progresses in theories and experiments lie in the following three aspects. Firstly, the restriction from the three necessary factors (nonlinearity system, noise, and periodic signal),[5,6] which were thought of as three essential ingredients for the onset of SR, was broken through because SR was observed without external signal or in linear systems.[7−9] Secondly, the traditional SR was confined to the single-peak form, i.e., there was only one resonance peak in SR.[5,6] The multi-peak form SR, namely, stochastic multiresonance, was detected firstly by Vilar *et al.*[10] in 1997. Now it becomes a focus in the investigations of SR.[11] Finally, in the traditional works, the dependence of signal-to-noise ratio (SNR) on noise intensity was generally used to quantify the characteristic quantity of SR.[2,6,10,12] In 1998, Barzykin *et al.*[14] proposed that, in the broad sense, SR can also be characterized by the dependence of SNR to noise correlation time. Especially in the first aspect, SR occurring in the linear systems was studied, in detail, in theory and experiment. In 1996, Berdichevsky *et al.*[13] studied a linear system driven by multiplicative noise, and found that when the multiplicative noise is dichotomous, SR can occur in the linear system. Then they further considered a linear system driven by correlated multiplicative and additive noise.[15] Later, we found that SR can occur in a linear system only driven by additive noise, considering the cross-correlation between pump noise and quantum noise and using the linear approximation method.[16]

In all the above-mentioned works, the noise and the signal were introduced additively in stochastic systems. However, the noise and the signal were introduced multiplicatively by Dykman *et al.*[17] when they studied SR in an asymmetric bistable potential, i.e., the noise was modulated by the signal. Moreover, in the practical physical systems, periodically modulated noise is very common, for examples, at the output of amplifier in optics or radio astronomy, the amplification factor varies periodically with time.[17] Therefore, it is very important to investigate the modulated noise for the practical application. Based on our previous works,[16] in this Letter, we introduce the noise and the signal multiplicatively into a single-mode laser system driven by correlated pump noise and quantum noise, and firstly find stochastic multiresonance in a laser system. By studying a gain-noise model of the single-mode laser driven by coloured correlated pump noise and quantum noise, each of which is coloured, we calculate the laser intensity correlation function, power spectrum and SNR, and detect that, not only the single-peak form SR, but also stochastic multiresonance appear in the dependence of SNR upon noise correlation time. As a result of adopting the linear approximation method, the analytic results can available without the restriction of the adiabatic approximation method.[2,6,11,12]

The equation of intensity for the gain-noise model of a single-mode laser with input signal is given by[16]

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2KI + \frac{2\Gamma}{1+\beta I} I + D + \frac{2I}{1+\beta I} A\cos(\Omega t)\xi(t) + 2\sqrt{I}\eta(t), \quad (1)$$

where the pump noise $\xi(t)$ and the quantum noise $\eta(t)$ are correlated in the following form

$$\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0,$$

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$$\langle \xi(t)\xi(t')\rangle = \frac{Q}{2\tau}\exp\Big(-\frac{|t-t'|}{\tau}\Big),$$
$$\langle \eta(t)\eta(t')\rangle = \frac{D}{2\tau}\exp\Big(-\frac{|t-t'|}{\tau}\Big),$$
$$\langle \xi(t)\eta(t')\rangle = \langle \eta(t)\xi(t')\rangle = \frac{\lambda\sqrt{DQ}}{2\tau}\exp\Big(-\frac{|t-t'|}{\tau}\Big), \quad (-1\le\lambda\le 1). \tag{2}$$

In Eqs. (1) and (2), I is the laser intensity; K represents the loss coefficient; $\beta = \tilde{A}/\Gamma$, $\tilde{A}$ and Γ represent the self-saturation and gain coefficients; Q and D are the intensities of the multiplicative and the additive noise respectively; λ is the noise correlation coefficient; A is the amplitude of a periodic signal, Ω is the frequency; and it is assumed that the self-correlation and cross-correlation time of the pump noise and quantum noise are the same with the value of τ.

Let

$$I = I_0 + \varepsilon(t), \tag{3}$$

where $I_0 = (\Gamma - K)/\beta K$ is the deterministic steady-state intensity, and $\varepsilon(t)$ is the perturbational term. We linearize Eq. (1) around the deterministic steady-state intensity I_0, thus the linear equation of the laser intensity is found to be

$$\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t} = -\gamma\varepsilon(t) + D + \frac{2I_0}{1+\beta I_0}A\cos(\Omega t)\xi(t) + 2\sqrt{I_0}\eta(t), \tag{4}$$

where $\gamma = 2K(\Gamma - K)/\Gamma$. By virtue of Eq. (4), we have

$$\langle\varepsilon(t)\rangle = \frac{D}{\gamma}(1 - \mathrm{e}^{-\gamma t}). \tag{5}$$

According to the intensity correlation function defined by

$$B(t) = \lim_{t'\to\infty}\overline{\langle I(t'+t)I(t')\rangle}, \tag{6}$$

where

$$\overline{\langle I(t'+t)I(t')\rangle} = \frac{\Omega}{2\pi}\int_{t'}^{t'+2\pi/\Omega}\langle I(t'+t)I(t')\rangle\mathrm{d}t'.$$

Integrating Eq. (4) and combining with Eqs. (3) and (6), we can have the analytical solution of $B(t)$ (only consider the case of $\gamma \neq \tau^{-1}$). Finally, in order to obtain the expression of SNR, we have to work out the output power spectra of the signal and the noise firstly. Thus, we translate the intensity correlation function $B(t)$ into the power spectrum $S(\omega)$ by Fourier transform

$$S(\omega) = S_1(\omega) + S_2(\omega), \tag{7}$$

where $S_1(\omega)$ and $S_2(\omega)$ are the output power spectra of the signal and the noise, respectively. The SNR is defined as the ratio of the output power of the signal P_s and the broadband noise output at $\omega = \Omega$. We have

$$SNR = \frac{P_s}{S_2(\omega=\Omega)}, \tag{8}$$

where

$$P_s = \int_0^\infty S_1(\omega)\mathrm{d}\omega = \frac{\pi\Lambda^2 Q(\gamma^2+\Omega^2-1/\tau^2)}{4\tau\kappa_+\kappa_-},$$

$$S_2(\omega=\Omega) = \frac{2\gamma}{\kappa}\Bigg\{\frac{Q\Lambda^2}{2\tau}\Bigg[\frac{\gamma}{4\kappa}\Big(\frac{\gamma-1/\tau}{\kappa_-} - \frac{\gamma+1/\tau}{\kappa_+}\Big) + \frac{\Omega^2}{4\kappa}\Big(\frac{1}{\kappa_-} - \frac{1}{\kappa_+}\Big) - \frac{\gamma-1/\tau}{4\gamma\kappa_-} - \frac{2DI_0}{\gamma(\tau^2\gamma^2-1)}\Bigg] + \frac{\Lambda\lambda\sqrt{DQI_0}}{\tau}\Bigg[\frac{2\gamma}{4\gamma^2+\Omega^2}\Big(\frac{\gamma-1/\tau}{\kappa_-} - \frac{\gamma+1/\tau}{\kappa_+} + \frac{2}{\tau\gamma^2-1/\tau}\Big) + \frac{\Omega^2}{4\gamma^2+\Omega^2}\Big(\frac{1}{\kappa_+} - \frac{1}{\kappa_-}\Big)\Bigg]\Bigg\} + \frac{2DI_0}{(1+\tau^2\Omega^2)(\gamma^2-1/\tau^2)}$$

with

$$\kappa = \gamma^2+\Omega^2, \quad \Lambda = 2AI_0/(1+\beta I_0),$$
$$\kappa_+ = (\gamma+1/\tau)^2+\Omega^2, \quad \kappa_- = (\gamma-1/\tau)^2+\Omega^2.$$

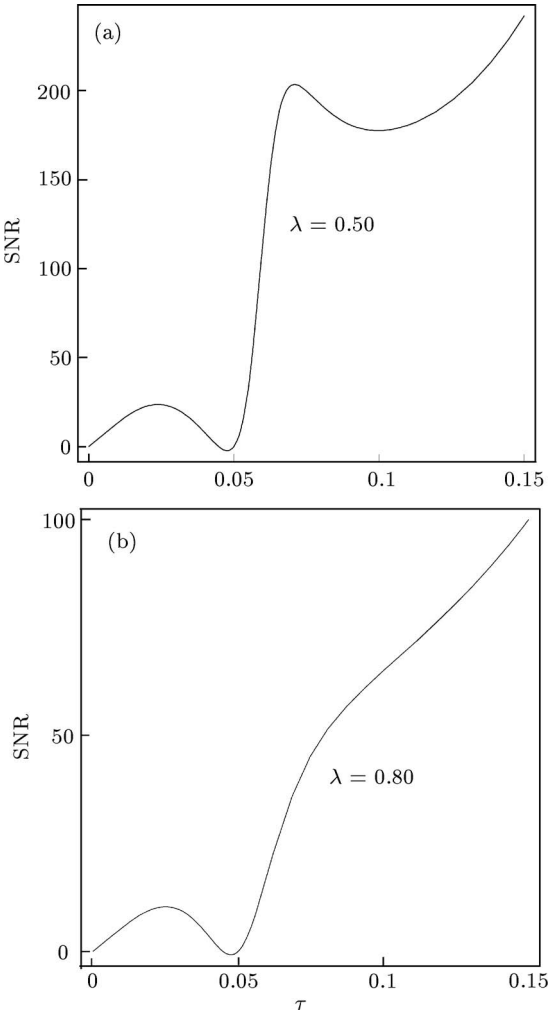


Fig. 1. Signal-to-noise ratio (SNR) versus the noise correlation time τ for (a) $\lambda = 0.5$ and (b) $\lambda = 0.8$ with $Q = 0.8$, $D = 0.1$, $I_0 = 1.0$, $\beta = 1.0$, $\gamma = 20$, $A = 1.0$, and $\Omega = 10$.

According to the expression of SNR of Eq. (8), the curves of SNR as a function of the noise correlation time τ are plotted in Figs. 1–3. From them, we can see the single-peak and the multi-peak SRs. Moreover, if the correlation coefficient between noises and the intensities of both noises is changed, the transition between the single-peak form and the multi-peak form SR can take place, i.e., we can determine the form of SR by controlling the parameters of the noises.

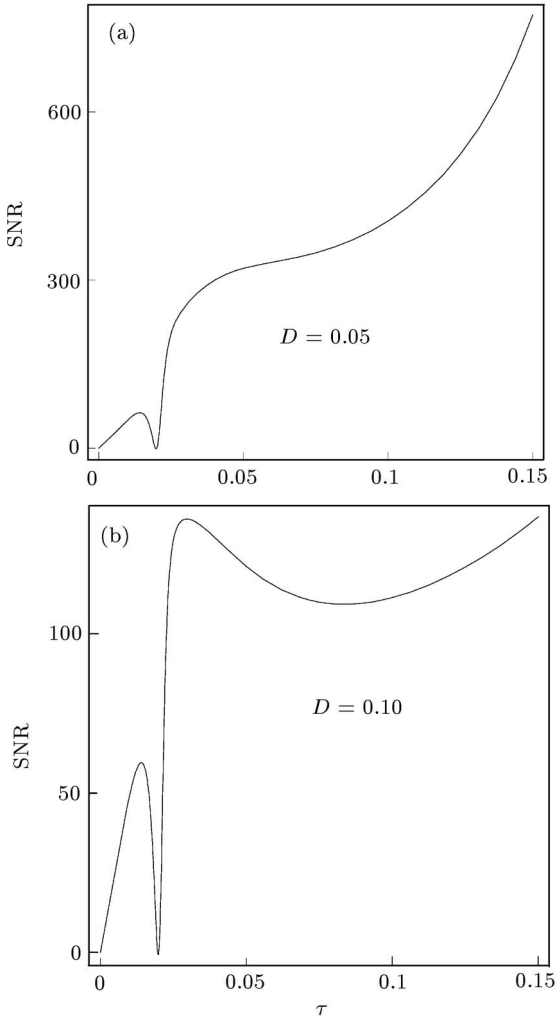


Fig. 2. SNR versus the noise correlation time τ for (a) $D = 0.05$ and (b) $D = 0.10$ with $Q = 1.0$, $\lambda = 0.5$, $I_0 = 1.0$, $\beta = 1.0$, $\gamma = 50$, $A = 1.0$, and $\Omega = 10$.

For the effect on the form of SR by the correlation coefficient λ, we can choose λ as the parameter. Figure 1 depicts the curve of the dependence of SNR upon the noise correlation time τ. When $0.45 < \lambda \leq 0.60$, there are two peaks in the curve (i.e. stochastic multiresonance), and the right peak is higher than the left one, as shown in Fig. 1(a). When $0.60 < \lambda \leq 1.00$, the curve shown in Fig. 1(b) is changed into the single-peak form SR.

For the effect on the form of SR by the intensity of quantum noise D, we can choose D as the parameter. Figure 2 depicts the curve of the dependence of SNR upon the noise correlation time τ. When $0.04 < D \leq 0.06$, the curve exhibits the single-peak form SR, as shown in Fig. 2a. When $D > 0.06$, as shown in Fig. 2(b), there are two maxima and two minima in the curve, i.e., the curve exhibits the stochastic multiresonance.

For the effect on the form of SR by the intensity of pump noise Q, we can choose Q as the parameter. Figure 3 depicts the curve of the dependence of SNR upon the noise correlation time τ. When $0.04 < Q \leq 1.06$, there are two maxima and two minima in the curve, that is, the curve exhibits the stochastic multiresonance, as shown in Fig. 3(a). When $Q > 1.06$, the curve exhibits the single-peak form SR, as shown in Fig. 3(b).

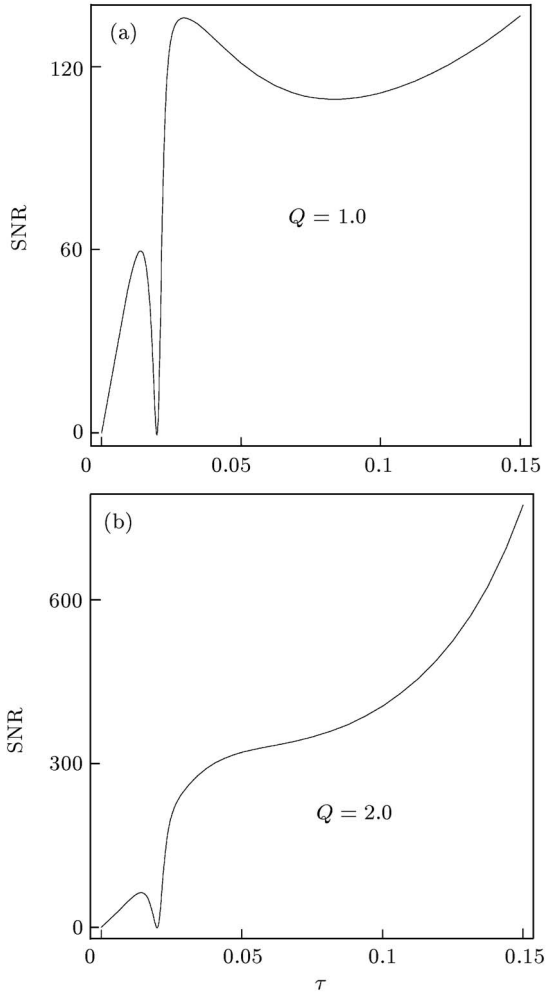


Fig. 3. SNR versus the noise correlation time τ for (a) $Q = 1.0$ and (b) $Q = 2.0$ with $D = 0.1$, $\lambda = 0.5$, $I_0 = 1.0$, $\beta = 1.0$, $\gamma = 50$, $A = 1.0$, and $\Omega = 10$.

It is emphasized that, the conclusions drawn by the linear approximation method is correct and reliable, when the values of parameters are within the valid range for this approximation method. The valid

range is determined by the condition of the normalized variance of the laser intensity $B(0) \ll 1$.[18] Here $B(0)$ is smaller than 10^{-2}, hence the conclusions we obtained are reliable.

Next, we draw a comparison between our previous works[16] and this works. In the former, the noise and the signal were introduced additionally into the single-mode laser system, while the noise and the signal were introduced multiplicatively in the latter. Hence, they represent different physical process. Furthermore, there is only the single-peak form SR in the former, while both the single-peak form and stochastic multiresonance appear in the latter. In the former, SR occurs in the curve of dependence of SNR upon the intensities of pump noise and quantum noise, and dose not in the curve of the dependence of SNR upon the noise correlation time. On the contrary, SR in the latter appears in the curve of the dependence of SNR upon the noise correlation time, and dose not in the curve of dependence of SNR upon the intensities of pump noise and quantum noise. SR in the former occurs in the case of $\lambda < 0$, while SR occurs in the case of $\lambda > 0$ in the latter.

In summary, due to the noise and the signal being multiplicatively introduced into the single-mode laser system, we firstly detect the two forms of SR in the single-mode laser. It is believed that the results not only supply new phenomena to the investigations of SR, but also propose a new method to optimize the design of laser amplifiers.

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