Testing the Dissipative Type of a Qubit Interacting with Environment *

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(Received 27 February 2003)

We propose a method to test the correctness of the coupling model of a qubit interacting with environment and to determine the type of dissipation. The environment is modelled by a bath of oscillators with infinite degrees of freedom and the qubit-bath coupling is chosen to be a general dissipation-decoherence form. The proposed method can be realized in current experiments.

PACS: 03. 67. Lx, 03. 65. Bz, 89. 70. +c

It is well known that quantum computers can solve some problems more rapidly than classical ones. Some problems, such as factoring and quantum simulation, can apparently be solved exponentially faster on a quantum computer than on a conventional digital computer.[1−3] Other problems, such as database search,[4−6] can be solved polynomially faster on a quantum computer. One of the main reasons to make quantum computers more powerful than classical counterparts is that quantum computation or quantum algorithm can exploit quantum coherence to perform parallel calculations, which cannot be utilized in classical computation.[7−11] However, quantum systems are very frail and the interaction of a quantum memory with its environment destroys the quantum coherence of the stored information, a process called the *decoherence*.[12,13] Decoherence may be induced by the pure dephasing[14−16] as well as amplitude damping,[17−19] and sometimes the latter is the main source of decoherence. In order to preserve coherence in quantum computation, one can encode the quantum information into a *safe corner* of a larger memory space, called the *decoherence-free subspace*, or DFS.[20−23] DFSs have been shown to require an asymptotically small overhead for large systems[23] and to support universal fault-tolerant quantum logic.[24,25] These properties suggest that DFSs will be intrinsic to future quantum computing architectures. Logic gates on DFS-encoded qubits have been proposed in the context of cavity electrodynamics,[26] solid state quantum computation[25] and ion-trapped quantum computation.[27] Also, recent experiments have demonstrated the immunity of DFSs of two photons and two atoms to collective dephasing.[28,29]

In general, DFS builds upon the concrete form of a quantum memory interacting with its environment. The different interaction form induces different DFS. If we know more about this interaction, simpler codes can be found. Therefore, experimentally testing and determining the coupling form of a quantum memory with its environment is very important. In Ref. [20], Duan and Guo have constructed a DFS by pairing each qubit of a register with an ancilla and assuming that the two qubits in each pair are set to be close so that they interact with the same modes of the environment. However, their scheme works well only in the case that the coupling detail between the paired qubit and environment is known, because the encoding and decoding depend on the type of dissipation. A problem naturally arises: is it practical for Duan and Guo coupling model? If it is right, how can we know the type of dissipation experimentally? These are the motivation.

According to the model in Ref. [20], let us consider a qubit which can be described by Pauli spin operator σ_α ($\alpha = x, y, z$), interacting with a bath of oscillators with infinite degrees of freedom. The whole Hamiltonian describing dissipation of the qubit, including the phase damping and the amplitude damping, has the following form ($\hbar = 1$):

$$H = \frac{\omega_0 \sigma_z}{2} + \sum_k \omega_k b_k^+ b_k + \sum_k (\alpha\sigma_x + \beta\sigma_y + \gamma\sigma_z) g_k (b_k + b_k^+), \quad (1)$$

where ω_0 and ω_k are the frequencies of the qubit and the bath mode k respectively, and $b_k^+(b_k)$ is the bosonic creation (annihilation) operator for the kth bath mode. The efficiencies α, β, and γ are real and their ratio $\alpha{:}\beta{:}\gamma$ is determined by the type of dissipation. For example, if $\alpha = \beta = 0$, it describes the phase damping; and if $\gamma = 0$, it is the amplitude damping.

* Supported by the National Natural Science Foundation of China under Grant Nos 19734006 and 10075018, the EYTF of the Ministry of Education of China, Chinese Academy of Sciences, STF of Hunan Province, Scientific Research Fund of Education Bureau of Hunan Province, and the National Natural Science Foundation of China via Institute of Theoretical Physics, Chinese Academy of Sciences.

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We can restrict these efficiencies satisfying

$$\alpha^2+\beta^2+\gamma^2=1 \tag{2}$$

and let g_k characterize the coupling strength.

Our aim is to test the rationality of the coupling Hamiltonian (1) and then determine the type of dissipation, i.e., the ratio $\alpha{:}\beta{:}\gamma$ experimentally. To reach this, let us firstly solve the corresponding quantum motional equation. Introducing the above Hamiltonian into interaction picture, one has

$$H_I=\sum_k \hat{O}g_k(b_k^+ \mathrm{e}^{\mathrm{i}\omega_k t}+b_k\mathrm{e}^{-\mathrm{i}\omega_k t}) \tag{3}$$

with the qubit operator $\hat{O}$ defined as

$$\hat{O}=(\alpha-\mathrm{i}\beta)\mathrm{e}^{\mathrm{i}\omega_0 t}\sigma_+ +(\alpha+\mathrm{i}\beta)\mathrm{e}^{-\mathrm{i}\omega_0 t}\sigma_- +\gamma\sigma_z. \tag{4}$$

In the common Pauli representation, $\hat{O}$ is a 2×2 matrix satisfying $\mathrm{Tr}\hat{O}=0$, which has eigenvalues: $\sqrt{\alpha^2+\beta^2+\gamma^2}=\pm 1$. We can use unitary transformation

$$U=\begin{bmatrix} a\mathrm{e}^{\mathrm{i}\omega_0 t/2} & b\mathrm{e}^{-\mathrm{i}(\varphi-\omega_0 t/2)} \\ b\mathrm{e}^{\mathrm{i}(\varphi-\omega_0 t/2)} & -a\mathrm{e}^{-\mathrm{i}\omega_0 t/2}\end{bmatrix} \tag{5}$$

with $a=(\gamma+1)/\sqrt{(\gamma+1)^2+\alpha^2+\beta^2}$, $b=\sqrt{\alpha^2+\beta^2}/\sqrt{(\gamma+1)^2+\alpha^2+\beta^2}$, and $\tan\varphi=\beta/\alpha$ to diagonalize $\hat{O}$ as $U^+\hat{O}U=\sigma_z$. Then the effective interaction Hamiltonian (3) becomes

$$\widetilde{H}_I=\sum_k \sigma_z g_k(b_k\mathrm{e}^{\mathrm{i}\omega_k t}+b_k\mathrm{e}^{-\mathrm{i}\omega_k t}), \tag{6}$$

and the corresponding propagator can be evaluated as

$$\widetilde{U}=\exp\Big\{\sigma_z\sum_k[b_k^+\xi_k(t)-b_k\xi_k^*(t)]\Big\}, \tag{7}$$

where $\xi_k(t)=\frac{g_k}{\omega_k}(1-\mathrm{e}^{\mathrm{i}\omega_k t})$ is a complex function.

Now we assume that, in the interaction picture, the initial state of the system plus its environment is

$$\rho_{SB}(0)=\rho_S(0)\otimes\rho_B(0), \tag{8}$$

where $\rho_S(0)$ is the initial state of the qubit, and $\rho_B(0)=\prod_k(1-\mathrm{e}^{\beta\omega_k})\mathrm{e}^{-\beta\omega_k\hat{n}_k}$ with $\beta=1/(k_BT)$ and k_B the Boltzmann constant is the initial state of bath. The evolution of the above initial state is governed by the unitary transformations (5) and (7) which can be written as

$$\rho_{SB}(t)=U\widetilde{U}U^+\rho_{SB}(0)U\widetilde{U}^+U^+. \tag{9}$$

Tracing over the bath degrees of freedom, then the reduced density-matrix elements of the qubit are

$$\begin{aligned}\rho_{00}(t)=&\rho_{00}(0)-2a^2b^2(1-\mathrm{e}^{-\Gamma(t)})[\rho_{00}(0)-\rho_{11}(0)]\\&-2ab(a^2-b^2)(1-\mathrm{e}^{-\Gamma(t)})\\&\cdot\mathrm{Re}[\rho_{10}(0)\mathrm{e}^{\mathrm{i}(\varphi-\omega_0 t)}],\end{aligned} \tag{10}$$

$$\begin{aligned}\rho_{11}(t)=&\rho_{11}(0)+2a^2b^2(1-\mathrm{e}^{-\Gamma(t)})[\rho_{00}(0)-\rho_{11}(0)]\\&+2ab(a^2-b^2)(1-\mathrm{e}^{-\Gamma(t)})\\&\cdot\mathrm{Re}[\rho_{10}(0)\mathrm{e}^{\mathrm{i}(\varphi-\omega_0 t)}],\end{aligned} \tag{11}$$

$$\begin{aligned}\rho_{01}(t)=&ab(a^2-b^2)(1-\mathrm{e}^{-\Gamma(t)})[\rho_{00}(0)-\rho_{11}(0)]\\&\cdot\mathrm{e}^{\mathrm{i}(\varphi-\omega_0 t)}+\rho_{01}(0)[(a^4+b^4)\\&\cdot\mathrm{e}^{-\Gamma(t)}+2a^2b^2]+2\rho_{10}(0)a^2b^2(1-\mathrm{e}^{-\Gamma(t)})\\&\cdot\mathrm{e}^{\mathrm{i}2(\varphi-\omega_0 t)},\end{aligned} \tag{12}$$

where $\Gamma(t)=\sum_k[2|\xi_k(t)|^2\coth(\frac{\beta\omega_k}{2})]$. It is clear that $\rho_{00}(t)+\rho_{11}(t)=1$.

The solutions of Eqs. (10)–(12) imply some profound physical meanings. (i) For pure phase damping $\alpha=\beta=0$ which means $a=1$ and $b=0$, then $\rho_{00}(t)=\rho_{00}(0)$, $\rho_{11}(t)=\rho_{11}(0)$, $\rho_{01}(t)=\rho_{01}(0)\mathrm{e}^{-\Gamma(t)}$, the clear decoherence occurs in the non-diagonal matrix elements. (ii) For amplitude damping $\gamma=0$ which means $a=b=1/\sqrt{2}$, then

$$\rho_{00}(t)=\{1+[\rho_{00}(0)-\rho_{11}(0)]\mathrm{e}^{-\Gamma(t)}\}/2, \tag{13}$$

$$\rho_{11}(t)=\{1-[\rho_{00}(0)-\rho_{11}(0)]\mathrm{e}^{-\Gamma(t)}\}/2, \tag{14}$$

$$\begin{aligned}\rho_{01}(t)=&[(1+\mathrm{e}^{-\Gamma(t)})\rho_{01}(0)+(1-\mathrm{e}^{-\Gamma(t)})\rho_{10}(0)\\&\cdot\mathrm{e}^{\mathrm{i}2(\varphi-\omega_0 t)}]/2,\end{aligned} \tag{15}$$

which indicates that amplitude damping not only causes energy dissipation, but also decoherence [$\rho_{01}(t)$ decays as time]. (iii) There is a concept needed to be clarified: in the existence of dissipation, the coherence of a system does not always reduce. For example, for the qubit initial state of $\rho_{00}(0)=1$, $\rho_{11}(0)=\rho_{01}(0)=0$, Eq. (12) gives $\rho_{01}(t)=ab(a^2-b^2)(1-\mathrm{e}^{-\Gamma(t)})\mathrm{e}^{\mathrm{i}(\varphi-\omega_0 t)}$. If $a\neq b$, it means a gain of qubit coherence. However, the gained coherence belongs to 'bad' information or noise coming from environment which must be eliminated by some plans.

Now we can describe how to test the correctness of the coupling model (1) and determine the type of dissipation. According to Eq. (10) or (11), owing to the density-matrix elements $\rho_{00}(t)$ and $\rho_{11}(t)$ can be measured experimentally, if we prepare a number of initial states of the qubit, and for each of the initial states, let the qubit interact with environment for a fixed time and followed by a measurement of the end state, we can then obtain an enough number of equations for the variables α, β and γ. From these equations, we can then test the correctness of coupling model (1) and determine the ratio $\alpha{:}\beta{:}\gamma$. For example, if we prepare the initial states of the qubit as

$$\rho_s(0)=A_n|0\rangle+B_n|1\rangle \tag{16}$$

with $n=1,2,\cdots N$ and A_n, B_n being real, we can then obtain the following N equations with respect to

variables α, β and γ,

$$\rho_{00}(T) = A_n^2 - 2a^2b^2[1 - \mathrm{e}^{-\Gamma(T)}](A_n^2 - B_n^2) + 2ab(b^2 - a^2)[1 - \mathrm{e}^{-\Gamma(T)}]A_nB_n\cos\varphi, \tag{17}$$

where T is the time for the qubit exposed in the environment between each preparation and measurement. Because of the limitation (2), there are only two independent variables in α, β and γ. Therefore, the set of Eq. (17) contain only three independent variables: two of them come from α, β, γ, the other is the dimensionless time $\Gamma(T)$. In principle, we can determine α, β, γ by combining relation (2) and any three equations of Eq. (17) after eliminating $\Gamma(T)$. If the coupling model (1) is correct, then the values of α, β, γ determined by different equations are consistent. We can then determine the ratio α:β:γ, i.e., the type of dissipation. If the values of α, β, and γ determined by different equations are inconsistent, then we can say that the coupling model (1) is wrong. Of course, because of the experimental errors, we can only expect the consistent of the results within a certain area in practice.

It is worthwhile to point out that the proposed method here may be realized in current experiments. For example, in the trapped ion system, we can stably trap one ion for a long time,[30] the fidelity of initial quantum state-preparation can be at least greater than 0.97[31] and the read-out of quantum states has efficiency close to one.[32,33] These provide favourable conditions for the test of our method. With the improvement of experimental precision, our method may provide a judgement for testing the above coupling model.

In conclusion, We have proposed a method to test the correctness of the coupling model of a qubit interacting with environment and to determine the type of dissipation. The test relies on the initial-state preparation and end state measurements which completely locate in current experimental technologies.

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